Contents

List of Figures

List of Tables

Table 1 contains an overview of all symbols used in this lab report.

Symbol	Meaning
d	Thickness of a crystal
D	Dielectric displacement
E	Electric field
H	Strength of magnetic field
I	Current
J	Intensity
k	Wave number
l	Length of one ADP crystal
ℓ	Rod length
L	Coil length
n	Refractive index
r	Distance, section 2.1
r_{41}	Electro-optical coefficient
U	Voltage
V	Verdet constant
α	Angle of rotation, Faraday effect
$\aleph, \beth, \gimel, \daleth, \eth, \digamma$	Fit parameters
$\Delta\phi$	Phase shift
β, γ	Angles measured
ϵ	Dielectric constant
ε	Angle of the half-shade polarimeter
κ	Damping factor
λ	Wave length
ϕ, φ, θ	Angle, fig. 1
s_x	Uncertainty of x

Table 1: Symbols used in this lab report.

1 Introduction

1.1 Task

The experiment is divided into two seperate tasks.

1. The first task is to analyse the Faraday effect. To do this both the Verdet constant of a heavy flint rod and the characteristic angle of a semi-shade polarimeter are to be determined.

2. In the second task the Pockels effect is to be considered. For this the electro-optical coefficient of the pockels cell is determined using the half-wave voltage which is measured using two different methods:

- By applying a sawtooth voltage.
- By applying a DC voltage modulated with a sine wave.

1.2 General theory

The theory explained in this report is based on the Staatsexamen [1] and the instructions [2].

1.2.1 Polarization of light

Generally, light is an electromagnetic wave consisting of an electric and a magnetic wave component where the magnetic field is perpendicular to the electric field. Polarization describes the oscillation state of light. Light is said to be complete polarized if there is a fixed phase relation. As the electric field is always perpendicular to the propagation direction of the wave, only two components of the field amplitude are to be considered. Therefore, three different polarizations can be distinguished: If both field components have the same phase the electromagnetic wave is said to be linearly polarized. If the phase shift amounts to $\frac{\pi}{2}$ and both components have the same amplitude the light is circular polarized. With elliptical polarization the phase and amplitude can be arbitrary, but there has to be a fixed phase relation.

For this experiment linearly polarized light is needed. In order to achieve this, polarization filters are used which only allow the field components parallel to their optical axis to pass through. Naturally, this process decreases the intensity of the light. If linearly polarized light passes through a polarization filter, where the angle between filter and field is θ, the intensity afterwards can be described with Malus law:

$$J = J_0 \cdot \cos^2(\theta). \tag{1}$$

1.2.2 Birefringence

Birefringence is an effect which occurs in optically anisotropic materials if the velocity of propagation of incident light depends both on polarization and direction of propagation. The light gets split up in two rays which are called ordinary and extraordinary. The ordinary ray behaves according to snellius' law of refraction, so the ray won't refracted at straight incidence. The extraordinary ray on the other hand is refracted by straight light incidence. As a result the two rays are exposed to deviating refractive indices and therefore the rays have a different velocity of propagation. As a consequence, the polarization of the light changes after passing through the material.

The dependence of the refractive indices and the velocity of propagation can be described by an index ellipsoid introduced by Fresnel:

$$\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1\,, \tag{2}$$

where x_1, $_2$ and x_3 denote the axes of a three dimensional right-angled coordinate system and n_1, n_2 and n_3 are the refractive indices belonging to the these three axes.

2 Faraday Effect

2.1 Physical Basis

The Faraday effect is a magnetooptic effect named after its discoverer Michael Faraday describing the rotation of the polarizing angle of an electric wave propagating through an transparent isotropic medium in the presence of a strong magnetic field. Consider a linearly polarized plane wave entering a transparent isotropic medium. Thanks to the linear polarization one can decompose the electric wave into two circularly polarizied plane waves with different directions of rotation. Because of the strong magnetic field circular an effect called *circular birefringence* occurs: Both waves interact with the electrons of the material and create a circular motion, effectively creating a magnetic dipole. Because both waves rotate in opposing directions, the direction of the magnetic field created by the induced dipole is along the propagation direction for one wave (and thus amplifying the external field) and opposed to the propagation direction for the other wave (decreasing the external field). As the external field is strengthened for one wave and weakened for the other, both waves propagate with different velocities through the material. If both waves emerge from the material, both waves can be combined again to form a linear wave. Since the propagation speed of both waves was different, a phase shift occurs at the end of the material causing a linearly polarized wave with the same amplitude but different polarization direction as the initial wave, effectively rotating the initial wave. Naturally, the angle of rotation α is directly proportional to the length ℓ of the material as well as the strength H of the magnetic field:

$$\alpha = V \cdot H \cdot \ell \tag{3}$$

The constant of proportionality V is called *Verdet constant*. Because the magnetic field isn't usually constant throughout the material one can divide the material into infinitesimal slices $\mathrm{d}z$. Each slice contributes an infinitesimal angle

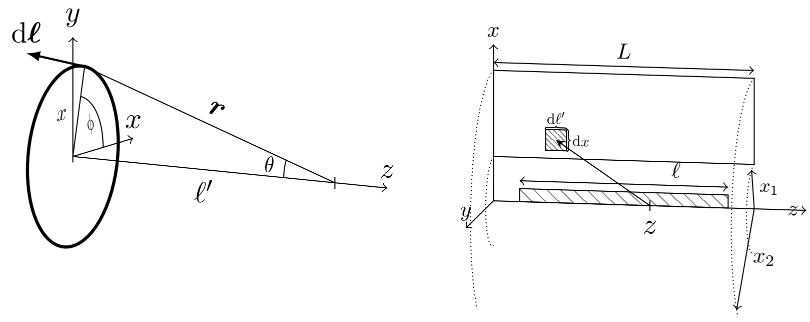
$$\mathrm{d}\alpha = V \cdot H(z)\,\mathrm{d}z \tag{4}$$

to the total angle of rotation α which in turn can be obtained by integrating eq. (4) over the total length ℓ of the material.

In the following section we would like to calculate the magnetic field of a coil using the law of Biot and Savart. Consider an infinitesimal piece of wire $\mathrm{d}\boldsymbol{\ell}$ carrying a current I. The magnetic field $\mathrm{d}H$ created by the wire observed at a distance $\boldsymbol{r}$ relative to the wire equals

$$\mathrm{d}\boldsymbol{H} = \frac{1}{4\pi}\frac{I}{r^3}\,\mathrm{d}\boldsymbol{\ell} \times \boldsymbol{r}. \tag{5}$$

This can be used to calculate the strength of the magnetic field H at axis of symmetry of a magnetic loop with radius x (cf. fig. 1(a)): Symmetry dictates that if the observer



(a) (b)

Figure 1: (a) Geometry of the loop. (b) Geometry of the coil with cross section.

is at a distance ℓ' to the center of the loop along the axis, the x- and y-components of the magnetic field must cancel. The remaining component H_z can be calculated using

$$\begin{aligned} \mathrm{d}H_z &= |\,\mathrm{d}\boldsymbol{H}\,| \sin\theta \\ &= \frac{1}{4\pi}\frac{I}{r^2}\sin\varphi\sin\theta\cdot\mathrm{d}\ell \\ &= \frac{1}{4\pi}\frac{I}{r^2}\sin\theta\cdot x\,\mathrm{d}\phi\,, \end{aligned} \tag{6}$$

where we changed the integration variable to an infinitesimal angle $\mathrm{d}\phi$ and used $\sin\varphi = 1$, where φ is the angle between $\boldsymbol{r}$ and $\mathrm{d}\boldsymbol{\ell}$. Integration and the use of $\sin\theta = x/r$ yields the magnetic field of a magnetic loop observed at a distance ℓ' from the center:

$$H(\ell') = \frac{I}{2}\frac{x^2}{r} = \frac{I}{2}\frac{x^2}{\sqrt{x^2+\ell'^2}}. \tag{7}$$

Now one is able to calculate the field of a coil by approximating the geometry as a continuous distribution of current loops each carrying a current $\mathrm{d}I$. The basic geometry is outlined in fig. 1(b). Let the inner radius of the coil be x_1, the outer radius x_2, the number of windings N and the total length of the coil L. The current $\mathrm{d}I$ flowing through one loop is the total current I multiplied by the fraction of area the loop occupies:

$$\mathrm{d}I = I\cdot N\cdot\frac{\mathrm{d}x}{x_2-x_1}\cdot\frac{\mathrm{d}\ell'}{L} \tag{8}$$

The infinitesimal magnetic field generated by each loop can now be calculated using eq. (7):

$$\mathrm{d}H = \frac{1}{2}\frac{x^2}{\sqrt{x^2+\ell'^2}}\,\mathrm{d}I = \frac{NI}{2(x_2-x_1)L}\frac{x^2}{\sqrt{x^2+\ell'^2}}\,\mathrm{d}x\,\mathrm{d}\ell' \tag{9}$$

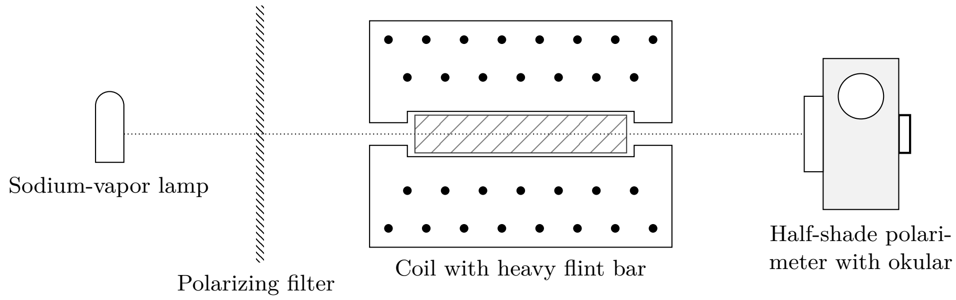


Figure 2: Schematic setup of the Faraday effect experiment. The cooling mechanism of the coil as well as the wiring is not shown.

The magnetic field of the coil can now be obtained by integrating over the area of the cross section of the coil which can be done by elementary means:

$$\begin{aligned} H(z) &= \frac{NI}{2(x_2 - x_1)L} \int_{-z}^{L-z} \int_{x_1}^{x_2} \frac{x^2}{\sqrt{x^2 + \ell'^2}} \, \mathrm{d}x \, \mathrm{d}\ell' \\ &= \frac{NI}{2(x_2 - x_1)L} \left[(L - z) \log \left(\frac{x_2 + \sqrt{(L-z)^2 + x_2^2}}{x_1 + \sqrt{(L-z)^2 + x_1^2}} \right) + z \log \left(\frac{x_2 + \sqrt{z^2 + x_2^2}}{x_1 + \sqrt{z^2 + x_1^2}} \right) \right] \end{aligned} \tag{10}$$

Now, one can calculate the total angle of rotation α: Using eq. (4), one obtains

$$\alpha = V \cdot \int_{\frac{L-\ell}{2}}^{\frac{L+\ell}{2}} H(z) \, \mathrm{d}z \, . \tag{11}$$

where ℓ is the total length of the material within the coil.

2.2 Setup and Procedure

The experiment setup of the Faraday effect and its execution is are explained in the following sections.

2.2.1 Setup

The basic structure of the Faraday effect experiment is sketched in fig. 2. The light of the sodium-vapor lamp (shown leftmost in the figure) hits a polarizing filter creating linearly polarized light. The beam travels through a heavy flint rod which is located inside a coil and is interspersed by its magnetic field. The field strength can be controlled by adjusting the current flowing through the coil, which can be set from $-5\,\mathrm{A}$ to $5\,\mathrm{A}$. An external cooling mechanism (not shown in the figure) ensures a constant coil temperature throughout the experiment. After passing through the rod the light hits an analyzer inside an half-shade polarimeter where its polarizing angle α can be measured.

The half-shade polarimeter uses a nicol as an auxiliary prism which is rotated at an angle ε relative to the polarizer. As the nicol rotates the polarizing angle of a part of the incoming beam, different regions (belonging to different parts of the beam) can be distinguished. Since the human eye is better able to compare intensities relatively to one another than to judge the intensity absolutely, the analyzer angle α is measured at which both areas have the same brightness. These angles correspond to the intersections of the curves in fig. 3.

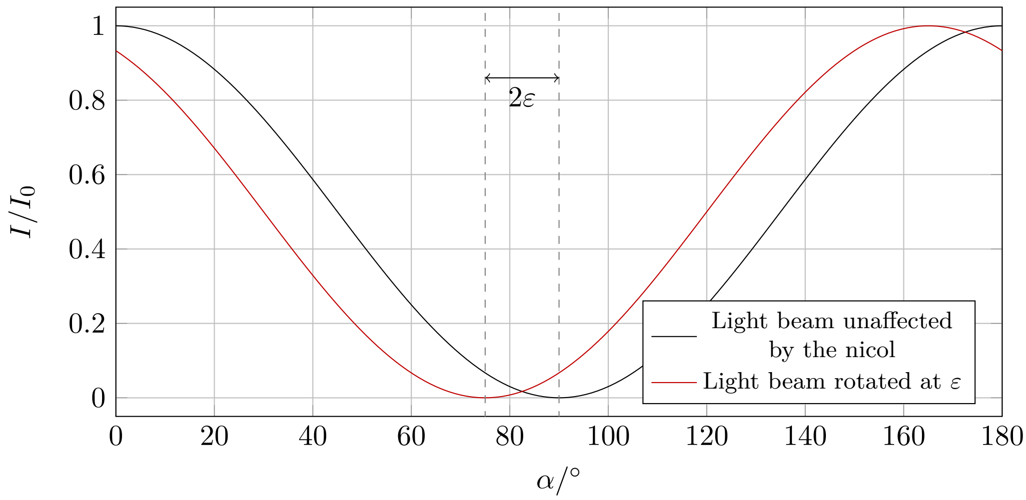


Figure 3: Intensity curves of both beam parts at the analyzer with respect to the analyzer angle α according to Malus' Law.

2.2.2 Procedure

Before starting the actual experiment, both the coil cooling and the sodium-vapor lamp need to be switched on for a few minutes. After that, the first measurements can be taken: Without supplying current to the coil (so that there's no magnetic field present) one measures the angle at the analyzer at the half-shade polarimeter at which both regions appear to be equally illuminated. As described above, two positions of equal illumination exist (cf. intersections in fig. 3); since the human eye can distinguish darker regions better, the measurements of this position are to be preferred. Now the current is increased in equal steps up to 5 A and the measurement of the analyzer angle is carried out just like described above. Both the coil current I and the analyzer angle α is to be noted. After reaching 5 A, the current is switched off and the polarity of the coil voltage is reversed. Now one can take analyzer angle meausurements for negative currents down to -5 A.

After all measurements are taken, the angle shift of ε is to be determined. By adjusting the analyzer to the point where each of the regions appears to be completely dark (corresponding to the minima in fig. 3), one can infer 2ε from the minimum angles. During further analysis, the angle at which the inner region appears completely dark will be called β, the angle at which the outer region appears dark will be called γ.

As the analyzer setting allowed for a bit of leeway (especially in the brighter region of equal illumination), we decided to measure α in a slightly different way: Instead of measuring one angle where both regions are equally illuminated, we determined the two angles at which the regions where just barely distinguishable from eachother. By taking the mean of these two angles one should not only arrive at a good estimate for α but also obtain a rough guess for the statistical error by taking the derivation of the mean and one of the bounds (cf. section 2.3.2).

For the sake of convenience we took both darkness angle measurements every time right after measuring the angles of equal illumination.

2.3 Analysis

2.3.1 Determination of the Field Integral

In order to determine the Verdet constant V from measurements of current I and polarization angle α, one has to compute the integral in eq. (11). As the H-field depends on the current I linearly (cf. eq. (10)) and the coil geometry is given, one can can evaluate the integral divided by the current numerically. In our calculations we refer to the coil geometry given in the manual [2, p. 6]:

$$\begin{aligned}
\text{Rod length:} \quad \ell &= 150\,\text{mm} \\
\text{Coil length:} \quad L &= 175\,\text{mm} \\
\text{Inner radius:} \quad x_1 &= 10\,\text{mm} \\
\text{Outer radius:} \quad x_2 &= 75\,\text{mm} \\
\text{Windings:} \quad N &= 3600
\end{aligned}$$

Using the method `quad` from the Python module `scipy.integrate`, the integral yields

$$\int\limits_{\frac{L-\ell}{2}}^{\frac{L+\ell}{2}} \frac{H(z)}{I}\,\mathrm{d}z = 2554.85, \tag{12}$$

where H is the magnetic field as of eq. (10). The error reported by `quad` reads 1.41×10^{-10} and will be neglected in the further analysis. Substituting the integral in eq. (11), one arrives at

$$\alpha = V \cdot 2554.85 \cdot I. \tag{13}$$

In addition to the calculation above, the manual of the experiment suggests comparing the rather tedious calculation with the approximation of the magnetic field using

$$H_{\text{approx}} = \frac{N \cdot I}{L}. \tag{14}$$

Using the given geometry, the magnetic field modulo current reads

$$\frac{H_{\text{approx}}}{I} = \frac{N}{L} \approx 20.571\,\frac{1}{\text{m}}. \tag{15}$$

In comparison, according to eq. (10), the magnetic field in the mid of the coil amounts to

$$H\left(\tfrac{L}{2}\right) \approx 18.350\,\frac{1}{\text{m}}. \tag{16}$$

However, the above approximation becomes more inexact when considering other positions within the coil: If one were to integrate over the whole rod length using H_{approx}, the integral becomes

$$\int_{\frac{L-\ell}{2}}^{\frac{L+\ell}{2}} \frac{H_{\text{approx}}}{I}\,\mathrm{d}z = \ell \cdot \frac{N}{L} \approx 3085.71, \tag{17}$$

which differs by about 20.8 % from the result in eq. (13), leading us to use the latter value instead.

2.3.2 Determination of the Verdet Constant Using the Intensity Minima

In this section we attempt to determine the Verdet constant using the measurements of the angles $\alpha_{\text{dunkel}}^{\text{min}}, \alpha_{\text{dunkel}}^{\text{max}}$ of the dark setting of equal illumination. By defining the mean of these bounding angles one arrives at a good estimate for the “true” angle of equal illumination:

$$\alpha_{\text{dunkel}} := \frac{\alpha_{\text{dunkel}}^{\text{max}} + \alpha_{\text{dunkel}}^{\text{min}}}{2}. \tag{18}$$

Furthermore, we estimate the error on the mean as the derivation of α_{dunkel} from one of its bounds:

$$s_{\alpha_{\text{dunkel}}} = \alpha_{\text{dunkel}} - \alpha_{\text{dunkel}}^{\text{min}} = \alpha_{\text{dunkel}}^{\text{max}} - \alpha_{\text{dunkel}}. \tag{19}$$

The calculated angles of equal illumination are listed alongside their respective measurements in table 6.

Now one can plot the values for the data pairs $(I, \alpha_{\text{dunkel}})$ (cf. fig. 4) and carry out a linear regression of the form

$$\alpha_{\text{dunkel}} = \aleph \cdot I + \beth, \tag{20}$$

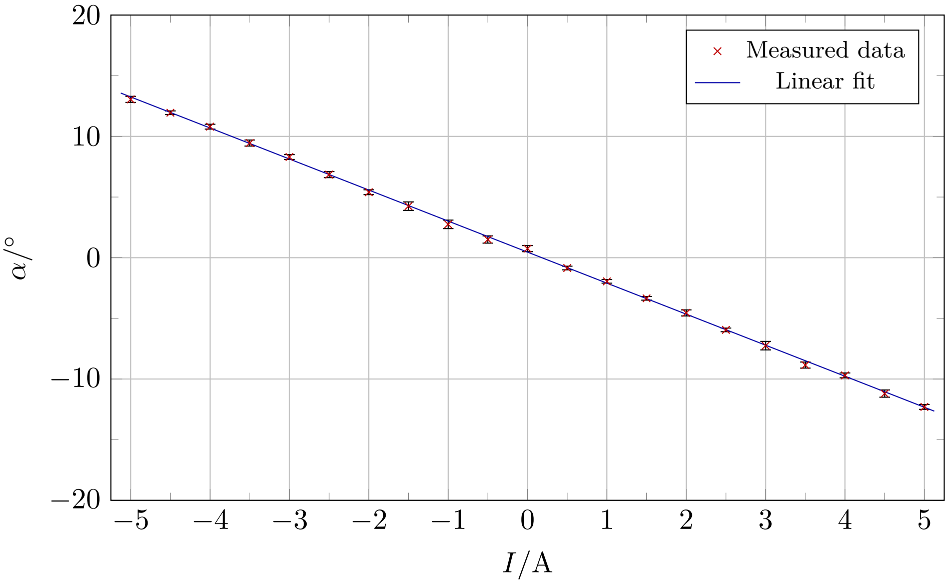


Figure 4: Plot of the polarizing angle, with respect of the current using the darker regions.

where $\aleph$ and $\beth$ are parameters of the regression. Using the method `curve_fit` from the Python module `scipy.optimize`, one arrives at

$$\begin{aligned} \aleph &= (-2.5580 \pm 0.0010)\,\frac{°}{\mathrm{A}}, \\ \beth &= (0.46 \pm 0.03)°. \end{aligned} \tag{21}$$

Because the angle α_{dunkel} is just the angle of polarization of the electromagnetic wave shifted by an offset, the Verdet constant V can be calculated by comparing eqs. (13) and (20) and using eq. (21):

$$\begin{aligned} V &= -\frac{\aleph}{2554.85} \\ &= (1.001 \pm 0.004) \times 10^{-3}\,\frac{°}{\mathrm{A}} \\ &= (4.7805 \pm 0.0190) \times 10^{-2}\,\frac{'}{\mathrm{Oe\,cm}} \end{aligned} \tag{22}$$

As the given literary value is positive, the sign of the Verdet constant was changed accordingly. During the above calculation, Gaussian error propagation as well as the unit conversions

$$\begin{aligned} 1° &= 60', \\ 1\,\mathrm{A} &= \frac{4\pi}{1000}\,\mathrm{Oe\,m} = \frac{4\pi}{10}\,\mathrm{Oe\,cm} \end{aligned} \tag{23}$$

were used.

2.3.3 Determination of the Verdet Constant using the Intensity Maxima

Analogously to the last section we attempt to determine the Verdet constant using the measurements of the angles $\alpha_{\text{hell}}^{\text{min}}, \alpha_{\text{hell}}^{\text{max}}$ of the bright setting of equal illumination. The calculated angles of equal illumination are listed alongside their respective measurements in table 6. Defining

$$\alpha_{\text{hell}} \coloneqq \frac{\alpha_{\text{hell}}^{\text{max}} - \alpha_{\text{hell}}^{\text{min}}}{2} \tag{24}$$

and carrying out a linear regression of the form

$$\alpha_{\text{hell}} = \gimel \cdot I + \eth, \tag{25}$$

one retains the regression parameters

$$\begin{aligned} \gimel &= (-4.43 \pm 0.25) \, \frac{\circ}{\text{A}} \\ \text{and} \quad \eth &= (83.8 \pm 0.7)^\circ. \end{aligned} \tag{26}$$

The data pairs $(I, \alpha_{\text{hell}})$ are shown with the linear regression in fig. 5. In addition, a line with slope $\aleph$ from the linear regression from section 2.3.2 is delineated as well, acting as a comparison. Using the parameters of regression, one can calculate the Verdet constant V:

$$\begin{aligned} V &= -\frac{\gimel}{2554.85} \\ &= (1.73 \pm 0.09) \times 10^{-3} \, \frac{\circ}{\text{A}} \\ &= (8.3 \pm 0.4) \, \frac{'}{\text{Oe cm}} \end{aligned} \tag{27}$$

2.3.4 Determination of the Polarimeter Shift 2ε

Lastly, the angle ε between the polarizer and the analyzer in the half-shade polarimeter is to be determined. As our meausurements of β and γ correspond to the minima in fig. 3, one is able to extract 2ε by subtracting the two angles from one another and taking the mean value $\overline{2\varepsilon}$. As the error on β and γ is $s_\beta = s_\gamma = 2^\circ$, the error on each value of 2ε can be calculated using Gaussian error propagation:

$$2\varepsilon = \gamma - \beta \implies s_{2\varepsilon} = \sqrt{s_\gamma^2 + s_\beta^2} = 3^\circ \tag{28}$$

Taking the mean of the 2ε's yields

$$\overline{2\varepsilon} = (12.2 \pm 0.6)^\circ, \tag{29}$$

where the error on the mean was calculated using

$$s_{\overline{2\varepsilon}} = \frac{s_{2\varepsilon}}{N}, \tag{30}$$

where N is the amount of values of 2ε. The data used to compute the mean is shown in table 7.

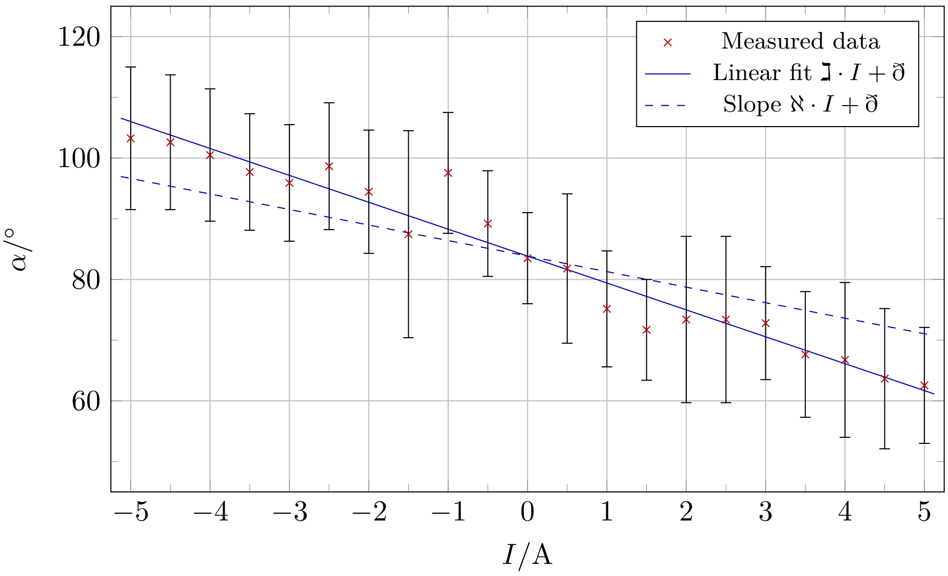


Figure 5: Plot of the polarizing angle, with respect of the current using the brighter regions.

3 Pockels Effect

3.1 Physical Basis

For crystals the dielectric constant ϵ isn't really a constant, but rather depends on the electric field applied to the crystal. Due to the dependence of the refractive index on the dielectric constant, also the refractive index depends on the electric field. The correlation between the dielectric displacement D and electric field E can be described as

$$D = aE + bE^2 + cE^3 + ..., \text{ where } a, b, c, \ldots \in \mathbb{R} \tag{31}$$

and with that, ϵ can be described as

$$\epsilon = \frac{\partial D}{\partial E} = a + 2bE + 3cE^2 + \ldots . \tag{32}$$

The linear term is decisive for the Pockels effect, which is the one observed in this experiment. Terms of higher-order can be neglected. For the experiment a pockels cell consisting of four ADP crystals (ammonium dihydrogen phosphate $NH_4H_2PO_4$) is used. The ADP crystals are optical single axis crystals without a center of symmetry. Two reasons exist for using multiple crystals: If the light which is linearly polarized parallel to the x_1-axis of the first crystal gets into the crystal, the light splits up in an ordinary and an extraordinary ray. To compensate for this effect, other crystals are placed behind the first one which are rotated relatively. The rotated crystals also compensate the natural birefringence. It is important that the polarity of the

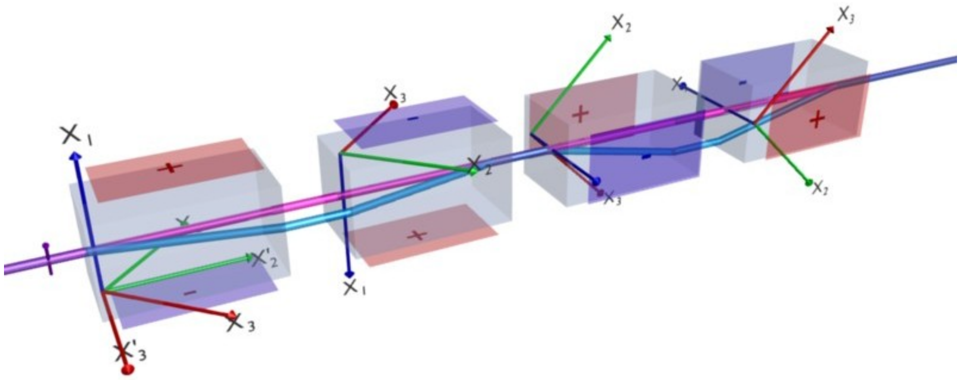


Figure 6: Alignment of the crystals in the Pockels cell[2, p. 3].

voltage for the second pair of crystals is reversed, otherwise the pockels effect would also be compensated. In fig. 6 is a sketched illustration of the arrangement of the used crystals. If a voltage is applied along the x_1-axis of an ADP crystal, the index ellipsoid can be described as

$$\frac{x_1^2}{n_1^2} + 2r_{41}x_2Ex_3 + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1\,, \tag{33}$$

with the electrooptical coefficient r_{41}. Now if the coordinate system gets rotated by 45° around the x_1-axis, the coordinates transform to:

$$x_2 = \frac{1}{\sqrt{2}}(x_2' + x_3') \qquad\qquad x_3 = \frac{1}{\sqrt{2}}(x_2' - x_3') \tag{34}$$

with the new coordinates and the definition of

$$n_x^2 = \frac{1}{2}\left(\frac{1}{n_1^2} + \frac{1}{n_3^2}\right), \tag{35}$$

the index ellipsoid can be written as

$$\frac{x_1^2}{n_1^2} + \frac{{x'_2}^2}{n_x^2}(1 + r_{41}En_x^2) + \frac{{x'}_3^2}{n_x^2}(1 - r_{41}En_x^2) + x_2'x_3'\left(\frac{1}{n_1^2} - \frac{1}{n_3^2}\right) = 1\,. \tag{36}$$

The use of a Taylor series to the third order in n_x yields the refractive indices of the new coordinates:

$$n_{x_2'} = \frac{n_x}{\sqrt{1 + r_{41}En_x^2}} \approx n_x + \frac{1}{2}r_{41}En_x^3 \tag{37}$$

$$n_{x_3'} = \frac{n_x}{\sqrt{1 - r_{41}En_x^2}} \approx n_x - \frac{1}{2}r_{41}En_x^3\,. \tag{38}$$

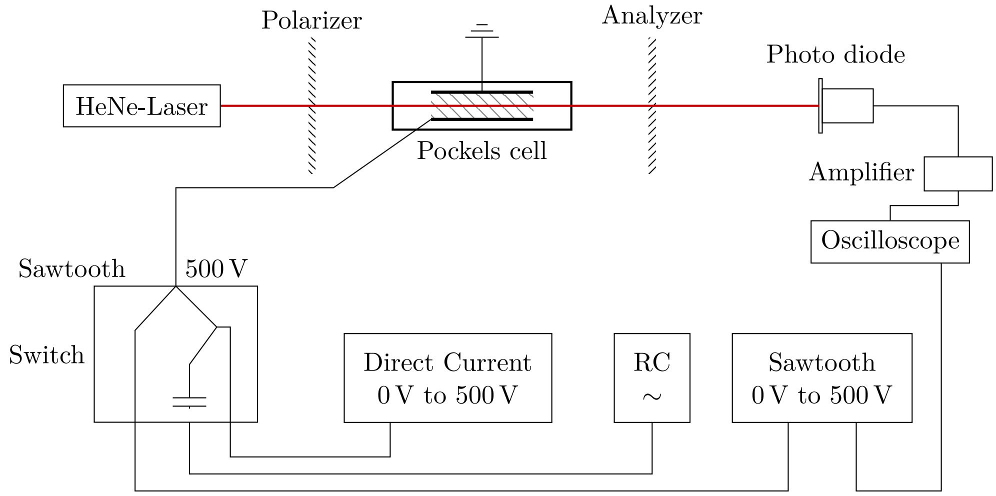


Figure 7: A sketched visualization of the setup for the Pockels-effect.

The phase shift $\Delta\phi$ caused by birefringence can be calculated as

$$\Delta\phi = k(n_1 - n_{x'_2})l' \tag{39}$$

with the wave number k and a path length traversed by the light of l'. Substituting $n_{x'_2}$ using eq. (37) and neglecting the natural birefringence $n_1 - n_x$, the phase shift can be written as

$$\Delta\phi = \frac{\pi}{\lambda} r_{41} E n_x^3 l \,. \tag{40}$$

Since the pockels cell consists of four ADP crystals, the length l' is four times the length l of one crystal. As the electric field is mostly uniform, E can be expressed through the voltage U applied to the crystal, $E = \frac{U}{d}$, where d is the thickness of a crystal. At a certain voltage the phase shift after the pockels cell is exactly π; this certain voltage is called half-wavelength voltage and is denoted by $U_{\lambda/2}$. Overall, eq. (40) can be solved for r_{41}:

$$r_{41} = \frac{\lambda d}{4 l U_{\lambda/2}} \left[\frac{1}{2}\left(\frac{1}{n_1^2} + \frac{1}{n_3^2}\right)\right]^{\frac{3}{2}} . \tag{41}$$

3.2 Setup and Procedure

3.2.1 Setup

In fig. 7 the setup for the Pockels experiment is delineated. A He-Ne-Laser attached to an optical bench is used to create a ray of monochromatic light. As linearly polarized light is needed for the experiment, a polarizer is mounted behind the laser. Next, the pockels cell described in section 3.1 is attached to the optical bench. Behind the cell is an analyzer in order to measure the polarization of the light after passing through the cell. The analyzer is set to be 90° relative to the polarizer. A photo diode is mounted at the end of the optical bench to measure the intensity of the light.

The voltage applied to the Pockels cell can be changed with different voltage generators, where one is able to switch between the different voltage types quickly. The voltage applied to the Pockels cell and the signal of the photo diode can be visualized with an oscilloscope.

3.2.2 Procedure

Firstly the laser was turned on. The components of the optical bench have been adjusted so that the reflected part of the laser beam is adjusted to be reflected back into the beam path of the laser. Afterwards the sawtooth voltage was applied to the Pockels cell and the oscilloscope was turned on. To minimize measurement errors, the analyzer was set so that the signal on the oscilloscope was maximum. The oscilloscope was connected to a computer where the measurement data could be saved. In addition, the data was displayed graphically and with cursors two different positions of the graphs could be marked. Eight different sets of data and screenshoots of the corresponding graphs, on which the maxima and minima of the diode signal are marked, were saved. The sawtooth voltage was changed to a direct current voltage modulated with a sine voltage. First, a positive voltage was applied and increased until the diode signal had just doubled the frequency of the sine signal. This voltage and the frequency of the sine signal have been noted. Then, a negative voltage was applied and the measurement was carried out just as before. Because the signal of the diode fluctuated strongly, it was difficult to find the right voltage, especially with the negative voltage. Therefore the signal of the sawtooth voltage got be dumped before getting to the oscilloscope, this dumping factor must also be measured. To do so, the sine voltage got plugged into the oscilloscope two times, once with the signal getting damped and the other directly. Both measurements were saved for later analysis.

3.3 Analysis

3.3.1 Damping factor

To determine the damping factor a sine fit of the form

$$U_{\mathrm{diode}} = \aleph + \beth \cdot \sin(\gimel t + \daleth) \tag{42}$$

was applied to the damped and the raw sine signal, where $\aleph, \beth, \gimel$ and $\daleth$ are parameters of regression. The fit was made using the method `curve_fit` from the Python module `scipy.optimize`. The values of the fit parameters can be viewed in table 2.

$\aleph$/V	$\beth$/V	$\gimel/\frac{1}{s}$	$\daleth$
0.070 ± 0.005	5.705 ± 0.007	3234.2 ± 0.4	-0.213 ± 0.002
$-0.000\,95 \pm 0.000\,05$	$0.053\,80 \pm 0.000\,07$	3234.2 ± 0.5	-0.745 ± 0.003

Table 2: Fit parameters of the sine fit for the damped and undamped measurements of the sine signal.

The damping factor is the quotient of the undamped and damped amplitude:

$$\kappa = \frac{\beth_{\text{undamped}}}{\beth_{\text{damped}}} = 106.04 \tag{43}$$

with Gaussian error propagation one gets an error of 0.12.

3.3.2 Sawtooth voltage

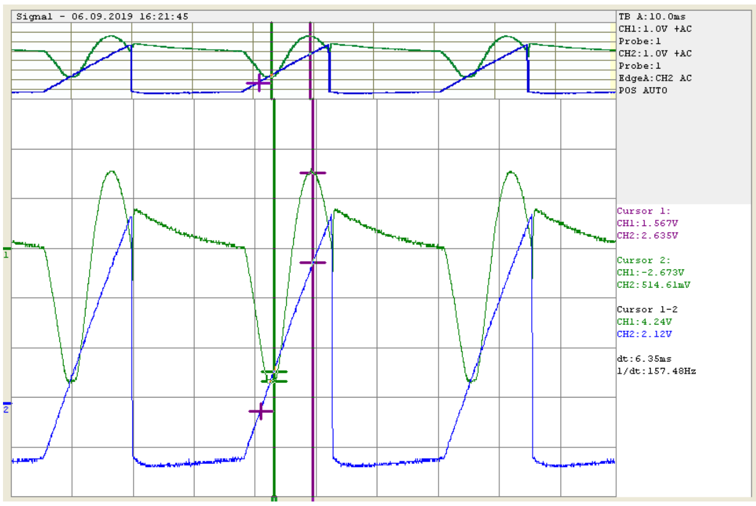


Figure 8: Picture from the analysis program of one measurement.

As shown in fig. 8, the maxima and minima of the diode signal were determined by using the cursors. The voltage difference of the sawtooth voltage which is the dampened halve wave voltage could then be retrieved from the program. The voltages obtained in this way can be viewed in table 3. The errors were estimated at 0.2 V, which was based on accuracy of the cursors when trying to determine the maxima and minima. The plots of the remaining measurements can be found in the appendix (cf. figs. 11 to 18). The mean of the values were calculated to be $\overline{U} = (2.3 \pm 0.3)\,\mathrm{V}$. In order to obtain the half wave voltage, the damping factor was multiplied to $\overline{U}$:

$$U_{\lambda/2}^{\text{sawtooth}} = \kappa \cdot \overline{U} = 240\,\mathrm{V} \tag{44}$$

$U_{\text{damped}}/\text{V}$	$U_{\text{damped}}/\text{V}$
2.1 ± 0.2	2.2 ± 0.2
2.5 ± 0.2	2.1 ± 0.2
2.6 ± 0.2	2.1 ± 0.2
2.1 ± 0.2	2.3 ± 0.2

Table 3: The damped voltages, which were read directily from the computer

with Gaussian error propagation one arrives at an error of $s_{U^{\text{sawtooth}}_{\lambda/2}} = 30\,\text{V}$. To calculate the electrooptical coefficient using eq. (41) the following values are required which have been taken from the instructions [2]:

$$\begin{aligned} n_1 &= 1.522 & n_3 &= 1.477 \\ d &= 2.4\,\text{mm} & l &= 20\,\text{mm} \\ \lambda &= 632.8\,\text{nm}. \end{aligned} \tag{45}$$

This leads to the following value for the electrooptical coefficient:

$$r_{41}^{\text{sawtooth}} = (24 \pm 3)\,\frac{\text{pm}}{\text{V}}. \tag{46}$$

In addition, using the method `curve_fit` from the Python module `scipy.optimize`, a sine fit of form

$$U_{\text{diode}} = \aleph + \beth \cdot \sin(\gimel t + \daleth) \tag{47}$$

was applied to the diode data, where $\aleph, \beth, \gimel$ and $\daleth$ are parameters of regression. With the help of the fits the positions of the maxima and minima of the diode signal were determined. Also, using the method `curve_fit` from the Python module `scipy.optimize`, a linear fit of the form

$$U_{\text{sawtooth}} = \eth + \digamma \cdot t, \tag{48}$$

where $\eth$ and $\digamma$ are parameters of regression, was applied to the data of the sawtooth voltage. The fits of one of the measurement series is shown in fig. 9and all the values of the fit parameters can be viewed in table 8 and table 9. In table 10 the times and voltages calculated with the fits can be viewed.

The values of the sawtooth voltage at the time of the maxima and minima of the diode signal were determined. The difference of the values results in the damped half wave voltage, which can be seen in table 4.

The mean of these voltages was calculated:

$$\overline{U}_{\text{damped, fit}} = (2.20 \pm 0.05)\,\text{V}. \tag{49}$$

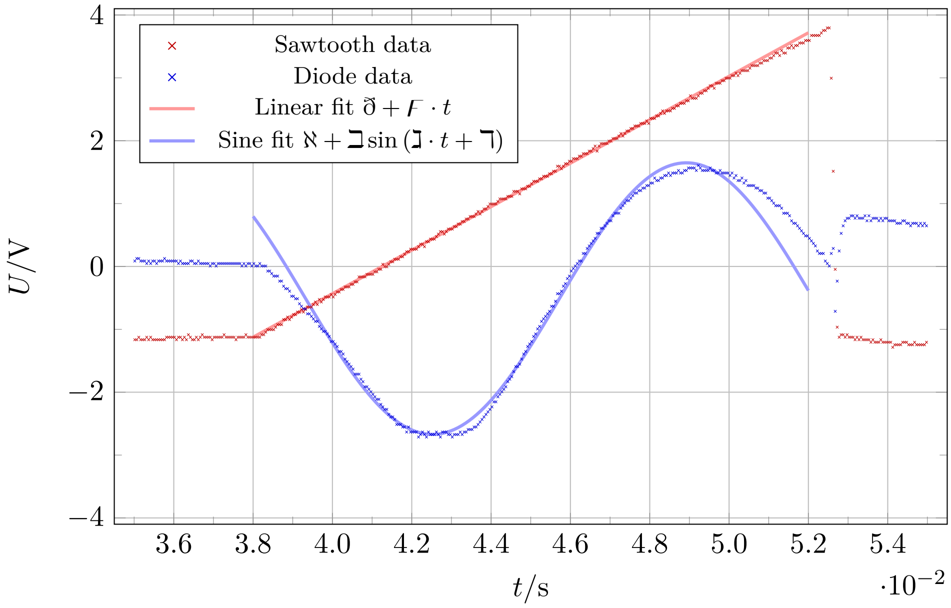


Figure 9: One example of the sine fit and linear fit applied to the data.

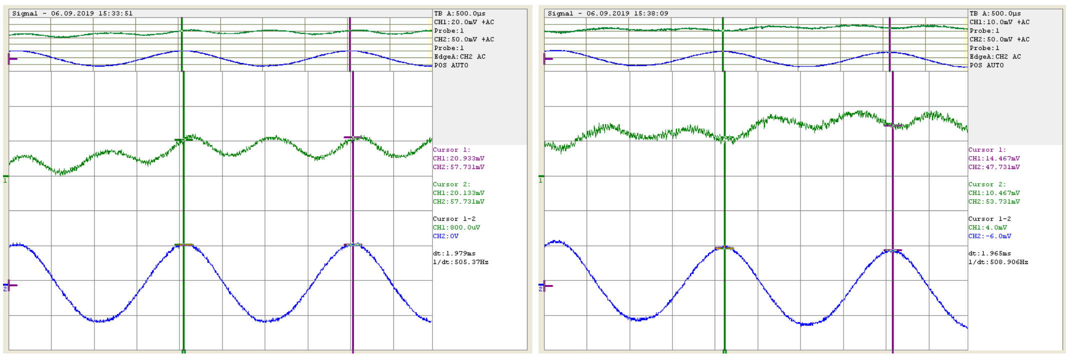


Figure 10: On the left a picture of the setting with positive voltage applied, at which the diode signal had doubled the frequency. On the right the one with negative voltage applied.

$U_{\text{damped}}^{\text{fit}}/\text{V}$	$U_{\text{damped}}^{\text{fit}}/\text{V}$
2.18 ± 0.13	2.20 ± 0.13
2.18 ± 0.14	2.19 ± 0.12
2.19 ± 0.15	2.21 ± 0.12
2.22 ± 0.12	2.20 ± 0.14

Table 4: The damped voltages determined by the fits.

If the damping factor is multiplied to this voltage, the half wave voltage $U_{\lambda/2,\mathrm{fit}} = (233 \pm 5)\,\mathrm{V}$ can be obtained. With eq. (41) and eq. (45) the value for the electro optical coefficient can be calculated again:

$$r_{41}^{\mathrm{fit}} = (24.2 \pm 0.5)\,\frac{\mathrm{pm}}{\mathrm{V}}. \tag{50}$$

3.3.3 Sine modulated direct current voltage

The voltage at which the diode signal had doubled frequency of the sine signal amounts for the positive direct current voltage:

$$U_{+} = (191.5 \pm 2.0)\,\mathrm{V}. \tag{51}$$

While the negative voltage was applied, the corresponding voltage was measured as

$$U_{-} = (-51.9 \pm 3.0)\,\mathrm{V}. \tag{52}$$

The error on the negative voltage was estimated greater because we had more difficulty to determine the voltage where the frequency doubled. A picture of both signals can be seen in fig. 10. The half wave voltage can be calculated as:

$$U_{\lambda/2,\mathrm{DC}} = U_{+} - U_{-} = 243.4\,\mathrm{V}. \tag{53}$$

With Gaussian error propagation one arrives at an uncertainty of $s_{U_{\lambda/2,\mathrm{DC}}} = 4\,\mathrm{V}$. Like before, the electrooptical coefficient can be calculated using eq. (41) and eq. (45):

$$r_{41}^{\mathrm{DC}} = (23.1 \pm 0.3)\,\frac{\mathrm{pm}}{\mathrm{V}}. \tag{54}$$

4 Discussion

4.1 Faraday Effect

Although the calculation of the magnetic field in order to compute the Verdet constant isn't as analytically plain as the one of an approximation (14), the use of the latter results in a discrepancy of 20.8 % which is too high to be reasonably justifiable and therefore lead us to use the more exact result.

By using a half-shade polarimeter and attempting to equally illuminate both regions, the Verdet constant V of a heavy-flint rod was determined to be

$$\begin{aligned} V &= (4.781 \pm 0.019) \times 10^{-2}\,\frac{'}{\mathrm{Oe\,cm}} \\ \text{and} \quad V &= (8.3 \pm 0.4)\,\frac{'}{\mathrm{Oe\,cm}} \end{aligned} \tag{55}$$

by using intensity minima and maxima, respectively. Both values don't seem to fit together very well, the Verdet constant from the maxima lying within 196σ of the one from the minima and V from the minimum measurement lying within 8.8σ of the maximum. As already described in section 2.2.2, the minimum measurement is to be preferred due to the nature of the human eye. Furthermore, if one considers fig. 4 where both slopes corresponding to the Verdet constants listed above are delineated, the error don't seem to suggest inconsistent measurements[1] as the slope obtained from the minimum measurements intersects with the majority of data points. Even by adapting a method where we tried to measure the "bounding angles" of equal illumination instead of the angle itself, we could not seem to obtain a reasonable result: As the method used to obtain the constant does not seem to be as reliable and consistent as the other measurement and the error does not seem to diminish this condition adequately, leading us to discard the Verdet constant of the intensity maximum measurement.

However, the Verdet constant retained from the intensity minimum measurement compares well with the manufacturer specification of $5 \times 10^{-2} \frac{'}{\mathrm{Oe\,cm}}$. Admittedly, the specification lies within a 12σ range within the minimum measurement, but, as the manufacturer only specifies one significant digit without indication of an error, the discrepancy is rather marginal[2]. This leads us to conclude that our measurement (at least the one of the intensity minimum) corroborates with the manufacturer's specification.

It should be mentioned that we adjusted the sign of the Verdet constant in accordance with the specification of the manufacturer. As the direction of current directly influences the direction of the magnetic field which is responsible for the sign, this condition can be explained by a simple polarity issue and should be of no influence to the magnitude of our result.

Lastly, the polarimeter shift was determined to be

$$\overline{2\varepsilon} = (12.2 \pm 0.6)^\circ. \tag{56}$$

The quality of the result is difficult to compare, as no reference value for the half-shade polarimeter used in this experiment is known to us. It should be mentioned, however, that the method of measurement can be quite prone to errors, as one has to adjust one region of the half-shade polarimeter to be completely black. The fact that – due to the nature of the polarimeter – it is not possible to obtain complete darkness in both regions coupled with the nature of the human eye to compare regions of darkness with one another leads us to note that the result may very well be afflicted by a systematic error.

[1] At least not on the scale of 196σ

[2] In fact, if one where to round our value to one significant digit, it coincides exactly with the specification.

4.2 Pockels Effect

At first the damping factor was determined. Therefore the damped and undamped sine signal was compared and the damping factor was calculated as

$$\kappa = (106.04 \pm 0.12)$$

Next, the electro-optical coefficient was calculated. To do so, the half wave voltage was determined using two different methods, once with a sawtooth voltage applied to the Pockels cell and second with a direct current voltage modulated with a sine voltage. With the first method, the half-wave voltage was obtained directly using the graphical depiction by the computer, but also got determined by fitting the data. The values for the electro-optical coefficient obtained with the different methods can be viewed in table 5 alongside the manufacturer's specification. Using the sawtooth

Method of Measurement	$r_{41} / \frac{\mathrm{pm}}{\mathrm{V}}$
sawtooth, graphical	$24 \quad \pm 3$
sawtooth, fit	24.2 ± 0.5
modulated DC	23.1 ± 0.3
manufacturers declaration	23.4

Table 5: Results of the Pockels part.

method and the graphical determination of the half-wave voltage, the manufacturer's specification is in an 1σ environment of the measured value. Compared to the other values, the relative error is quite large which may be due to the flattened extrema (cf. figs. 11 and 12 for example) of the data which resulted in an imprecise identification of the extrema. The manufacturer's specification is just in an 2σ environment of the value obtained by using the fits. This is on the one hand due to the quite small relative error given by the fit; on the other hand it could also be due to the fact that the diode signal does not behave like a perfect sine. Furthermore, the boundary effect could have reduced the accuracy of the fit. For the modulated direct current voltage method the manufacturer's specification is in an 1σ environment of our result. The error of the result comes through our approximation of how accurate the voltage at which the frequency had doubled could be identified. This was approved not just by checking the frequency of the diode signal, but also the amplitude; therefore the amplitude is minimal when the frequency has doubled. It should also be said that the electro-optical coefficient depends on the temperature of the crystal which we were neither able to measure nor to control; the value of the manufacturer's specification refers to a crystal at the temperature of 21 °C. Therefore, a deviation from the manufacturer's specification – albeit minimal – could be reasoned by a difference in temperature.

A Appendix

A.1 Figures and Tables

I/A	$\alpha_{\mathrm{dunkel}}^{\mathrm{min}}/^\circ$	$\alpha_{\mathrm{dunkel}}^{\mathrm{max}}/^\circ$	$\alpha_{\mathrm{dunkel}}/^\circ$	$\alpha_{\mathrm{hell}}^{\mathrm{min}}/^\circ$	$\alpha_{\mathrm{hell}}^{\mathrm{max}}/^\circ$	$\alpha_{\mathrm{hell}}/^\circ$
-5.0 ± 0.1	12.8	13.3	13.05 ± 0.25	91.5	115.0	103 ± 12
-4.5 ± 0.1	11.8	12.1	11.95 ± 0.15	91.5	113.7	103 ± 11
-4.0 ± 0.1	10.6	11.0	10.80 ± 0.20	89.6	111.4	100 ± 11
-3.5 ± 0.1	9.2	9.7	9.45 ± 0.25	88.1	107.3	98 ± 10
-3.0 ± 0.1	8.1	8.5	8.30 ± 0.20	86.3	105.5	96 ± 10
-2.5 ± 0.1	6.6	7.1	6.85 ± 0.25	88.2	109.1	99 ± 10
-2.0 ± 0.1	5.2	5.6	5.40 ± 0.20	84.3	104.6	94 ± 10
-1.5 ± 0.1	3.9	4.6	4.3 ± 0.4	70.4	104.5	87 ± 17
-1.0 ± 0.1	2.4	3.1	2.8 ± 0.4	87.6	107.5	98 ± 10
-0.5 ± 0.1	1.2	1.8	1.5 ± 0.3	80.5	97.9	89 ± 9
0.0 ± 0.1	0.5	1.0	0.75 ± 0.25	76.0	91.0	84 ± 8
0.5 ± 0.1	-1.0	-0.7	-0.85 ± 0.15	69.5	94.1	82 ± 12
1.0 ± 0.1	-2.1	-1.8	-1.95 ± 0.15	65.6	84.7	75 ± 10
1.5 ± 0.1	-3.5	-3.2	-3.35 ± 0.15	63.4	80.0	72 ± 8
2.0 ± 0.1	-4.8	-4.3	-4.55 ± 0.25	59.7	87.1	73 ± 14
2.5 ± 0.1	-6.1	-5.8	-5.95 ± 0.15	59.7	87.1	73 ± 14
3.0 ± 0.1	-7.6	-6.9	-7.3 ± 0.4	63.5	82.1	73 ± 9
3.5 ± 0.1	-9.1	-8.6	-8.85 ± 0.25	57.3	78.0	68 ± 10
4.0 ± 0.1	-9.9	-9.5	-9.70 ± 0.20	54.0	79.5	67 ± 13
4.5 ± 0.1	-11.5	-10.9	-11.2 ± 0.3	52.1	75.2	64 ± 12
5.0 ± 0.1	-12.5	-12.1	-12.30 ± 0.20	53.0	72.1	63 ± 10

Table 6: Measurements of angle bounds with calculated angles $\alpha_{\mathrm{dunkel}}, \alpha_{\mathrm{hell}}$ of equal illumination with respect to the coil current I.

I/A	$\beta/^\circ$	$\gamma/^\circ$	$2\varepsilon/^\circ$
-5.0 ± 0.1	6.5 ± 2.0	18.6 ± 2.0	12 ± 3
-4.5 ± 0.1	5.5 ± 2.0	17.0 ± 2.0	12 ± 3
-4.0 ± 0.1	4.3 ± 2.0	16.1 ± 2.0	12 ± 3
-3.5 ± 0.1	2.9 ± 2.0	15.9 ± 2.0	13 ± 3
-3.0 ± 0.1	1.9 ± 2.0	14.6 ± 2.0	13 ± 3
-2.5 ± 0.1	0.2 ± 2.0	13.1 ± 2.0	13 ± 3
-2.0 ± 0.1	0.0 ± 2.0	12.0 ± 2.0	12 ± 3
-1.5 ± 0.1	-2.1 ± 2.0	11.6 ± 2.0	14 ± 3
-1.0 ± 0.1	-2.9 ± 2.0	8.8 ± 2.0	12 ± 3
-0.5 ± 0.1	-4.6 ± 2.0	7.1 ± 2.0	12 ± 3
0.0 ± 0.1	-5.7 ± 2.0	6.5 ± 2.0	12 ± 3
0.5 ± 0.1	-6.6 ± 2.0	5.0 ± 2.0	12 ± 3
1.0 ± 0.1	-7.0 ± 2.0	3.5 ± 2.0	10 ± 3
1.5 ± 0.1	-8.8 ± 2.0	2.4 ± 2.0	11 ± 3
2.0 ± 0.1	-11.1 ± 2.0	1.8 ± 2.0	13 ± 3
2.5 ± 0.1	-11.1 ± 2.0	1.8 ± 2.0	13 ± 3
3.0 ± 0.1	-13.1 ± 2.0	-1.0 ± 2.0	12 ± 3
3.5 ± 0.1	-15.6 ± 2.0	-3.1 ± 2.0	12 ± 3
4.0 ± 0.1	-16.5 ± 2.0	-3.1 ± 2.0	13 ± 3
4.5 ± 0.1	-18.2 ± 2.0	-6.9 ± 2.0	11 ± 3
5.0 ± 0.1	-19.5 ± 2.0	-8.0 ± 2.0	12 ± 3

Table 7: Measurements of the angles β and γ corresponding to the point where the inner/outer region where completely dark with respect to the current I. The difference between the two angles 2ε is also shown.

$\aleph/\mathrm{V}$	$\beth/\mathrm{V}$	$\gimel/\frac{1}{\mathrm{s}}$	$\daleth$
-0.751 ± 0.011	2.363 ± 0.010	516.0 ± 2.0	-1.91 ± 0.08
-0.771 ± 0.011	2.326 ± 0.011	515.0 ± 2.0	-1.88 ± 0.09
-0.764 ± 0.012	2.351 ± 0.011	513.0 ± 2.0	-1.80 ± 0.09
-0.511 ± 0.007	2.160 ± 0.008	491.0 ± 2.0	-3.60 ± 0.08
-0.479 ± 0.007	2.188 ± 0.009	493.0 ± 2.0	-3.66 ± 0.09
-0.493 ± 0.007	2.158 ± 0.008	495.0 ± 2.0	-3.78 ± 0.08
-0.498 ± 0.007	2.182 ± 0.008	490.0 ± 2.0	-3.56 ± 0.08
-0.504 ± 0.007	2.186 ± 0.008	493.0 ± 2.0	-9.99 ± 0.08

Table 8: Fit parameters of the sine fit for the eight different measurements

$\eth$/V	$F/\frac{\mathrm{V}}{\mathrm{s}}$
-11.71 ± 0.04	357.8 ± 0.9
-11.66 ± 0.04	357.7 ± 0.9
-11.69 ± 0.04	357.8 ± 0.9
-14.267 ± 0.028	345.9 ± 0.6
-14.187 ± 0.029	344.7 ± 0.6
-14.25 ± 0.03	345.5 ± 0.7
-14.192 ± 0.027	344.2 ± 0.6
-14.211 ± 0.027	344.5 ± 0.6

Table 9: Fit parameters of the linear fit for the eight different measurements

t_{min}/s	t_{max}/s	U_{min}/V	U_{max}/V
$0.037\,19 \pm 0.000\,23$	$0.043\,27 \pm 0.000\,25$	1.59 ± 0.09	3.77 ± 0.10
$0.037\,17 \pm 0.000\,24$	$0.043\,26 \pm 0.000\,26$	1.64 ± 0.09	3.82 ± 0.10
$0.037\,16 \pm 0.000\,25$	$0.043\,28 \pm 0.000\,27$	1.61 ± 0.10	3.80 ± 0.11
$0.042\,55 \pm 0.000\,23$	$0.048\,96 \pm 0.000\,25$	0.45 ± 0.08	2.67 ± 0.09
$0.042\,48 \pm 0.000\,24$	$0.048\,85 \pm 0.000\,26$	0.45 ± 0.09	2.65 ± 0.09
$0.042\,56 \pm 0.000\,23$	$0.048\,91 \pm 0.000\,24$	0.45 ± 0.08	2.65 ± 0.09
$0.042\,54 \pm 0.000\,22$	$0.048\,95 \pm 0.000\,24$	0.45 ± 0.08	2.66 ± 0.09
$0.055\,34 \pm 0.000\,26$	$0.061\,71 \pm 0.000\,27$	4.86 ± 0.09	7.05 ± 0.10

Table 10: The maxima and minima times and the associated sawtooth voltages.

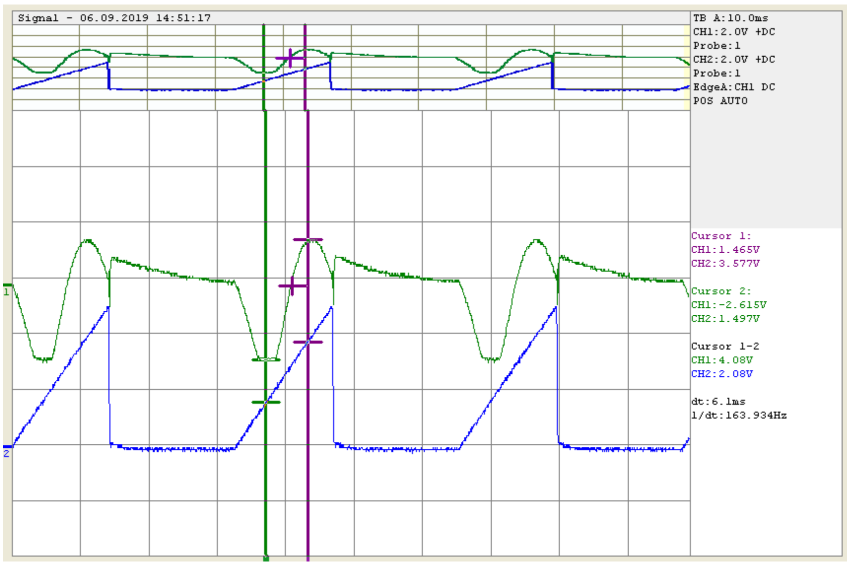


Figure 11: Picture of first measurement of the sawtooth voltage.

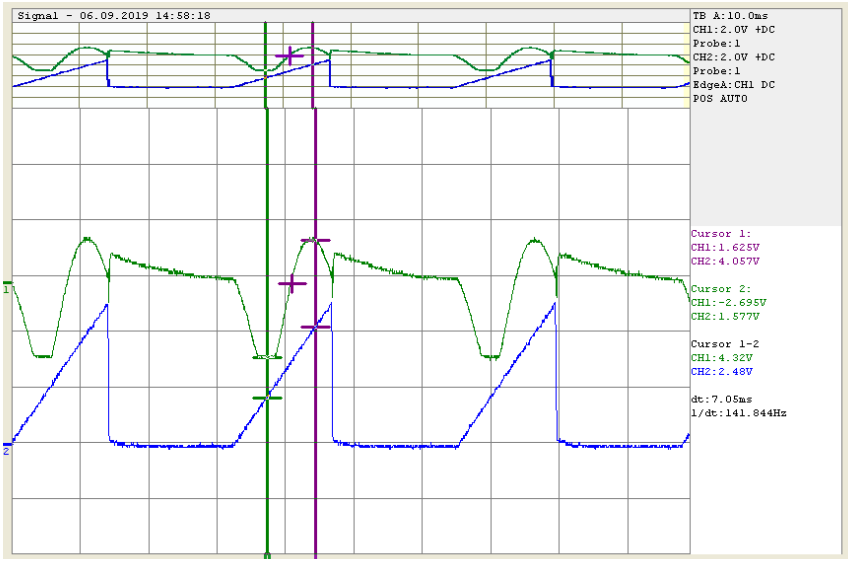


Figure 12: Picture of second measurement of the sawtooth voltage.

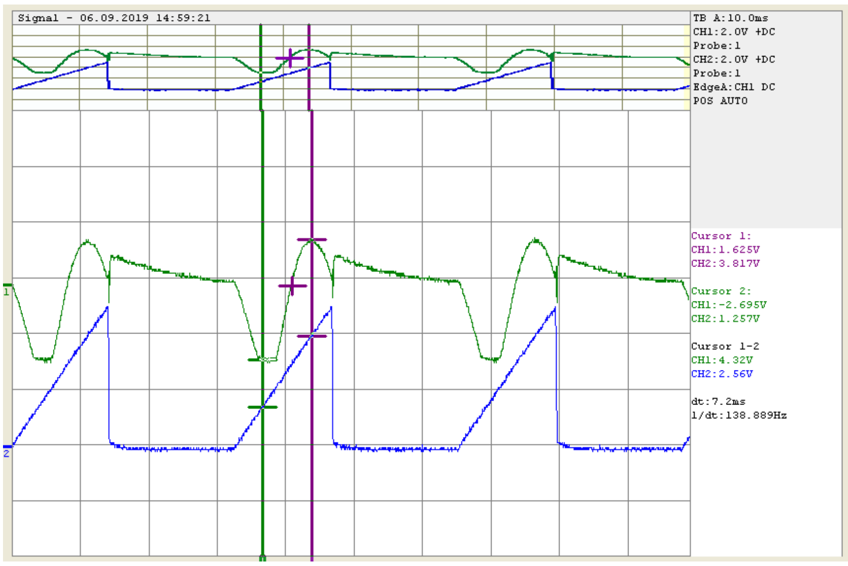


Figure 13: Picture of the third measurement of the sawtooth voltage.

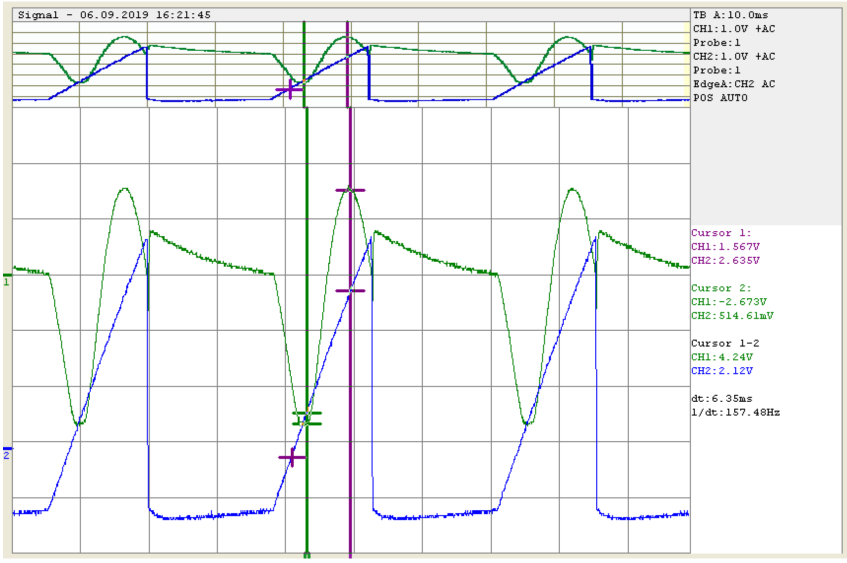


Figure 14: Picture of the fourth measurement of the sawtooth voltage.

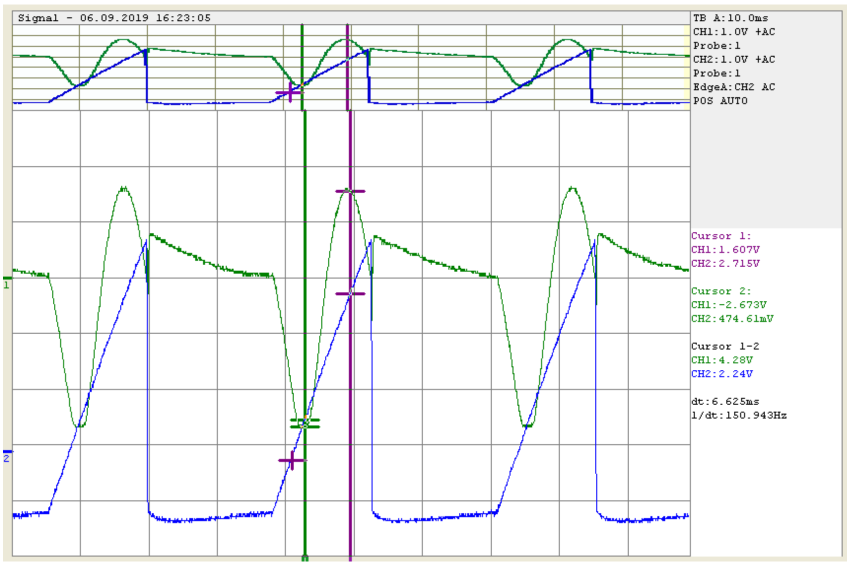


Figure 15: Picture of the fith measurement of the sawtooth voltage.

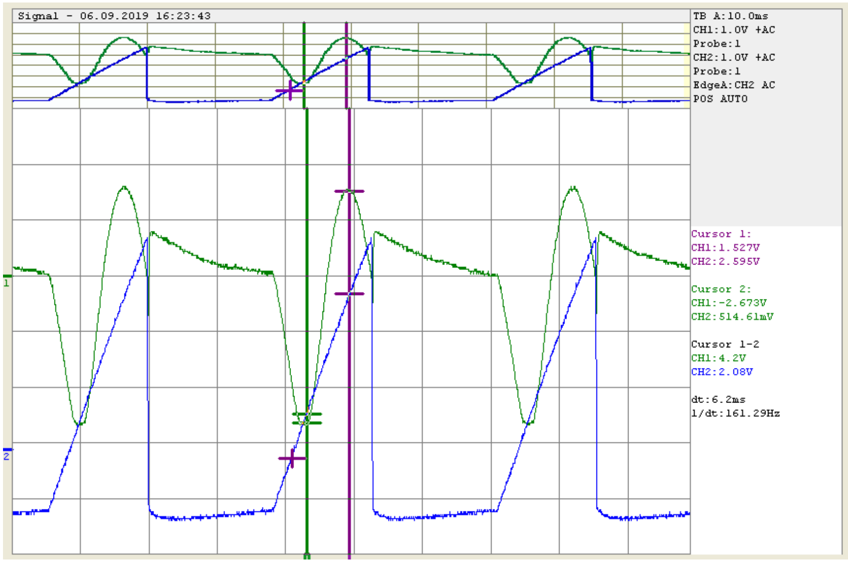


Figure 16: Picture of the sixth measurement of the sawtooth voltage.

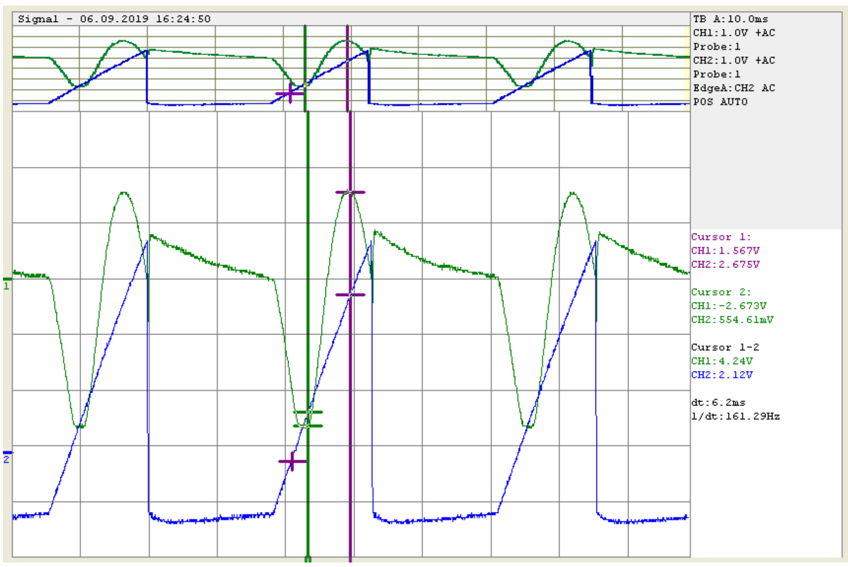


Figure 17: Picture of the seventh measurement of the sawtooth voltage.

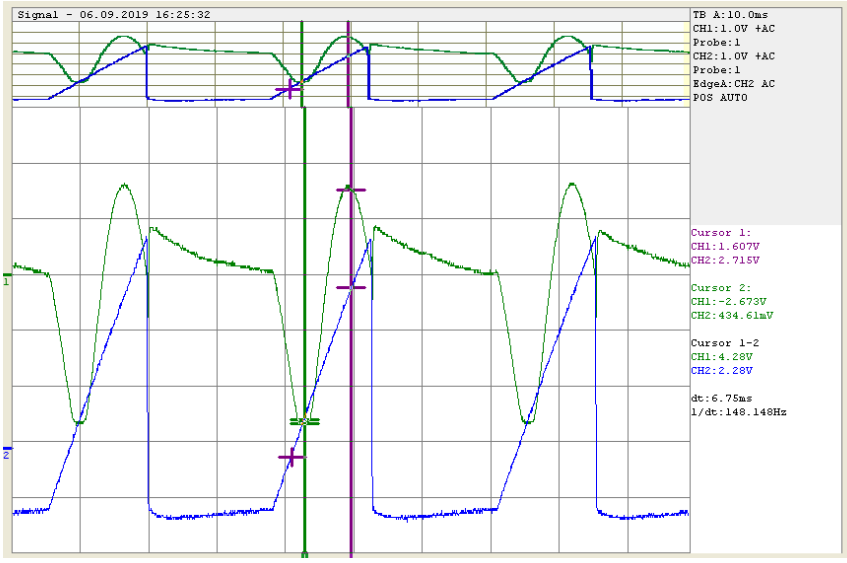


Figure 18: Picture of the eighth measurement of the sawtooth voltage.

A.2 Lab notes

FarPock

06.05.2019

Pockels effect

1. Sägezahnmethode
   (Alle Daten ~~…~~ abgespeichert) .bmp, .csv auf USB-Stick

2. Modulierte Gleichspannung

~~Negative~~ Positive Spannung $U_+ = 191{,}5$ $f = 0{,}510$ KHz $s_{U_+} = 2V$

Negative Spannung $U_- = 571{,}9$ $f = 0{,}510$ KHz $s_{U_-} = 3V$

Dämpfungsbestimmung: Gespeichert.

Faraday effect

I/A	$\alpha_{Dunkel}/°$	$\alpha_{Hell}/°$	$\beta/°$	$\gamma/°$
0,00	0,8	84,9	-5,0	7,0
0,50	-0,9	84,0	-8,5	5,4
1,00	-2,0	78,0	-10,0	4,0
1,50	-4,3			

$\beta \hat{=}$ Mitte dunkel
$\gamma \hat{=}$ außen dunkel

Abbruch wegen Methodik

I/A	$\alpha^{min}_{Dunkel}/°$	$\alpha^{max}_{Dunkel}/°$	$\alpha^{min}_{Hell}/°$	$\alpha^{max}_{Hell}/°$	$\beta/°$	$\gamma/°$
0,00	0,5	1,0	76,0	91,0	-5,7	6,5
0,50	-1,0	-0,7	69,5	94,1	-6,6	5,0
1,00	-1,8 ⟷	-2,1	65,6	84,7	-7,0	3,5
1,50	-3,2 ⟷	-3,5	63,4	80,0	-8,8	2,4
2,00	-4,3 ⟷	-4,8	59,7	87,1	-11,1	1,8
2,50	-6,1	-5,8	59,3	85,0	-12,2	-0,5
3,00	-7,6	-6,9	63,5	82,1	-13,1	-1,0
3,50	-9,1	-8,6	57,3	78,0	-15,6	-3,1
4,00	-9,9	-9,5	54,0	79,5	-16,5	-3,1
4,50	-11,5	-10,9	52,1	75,2	-18,2	-6,9
5,00	-12,5	-12,1	53,0	72,1	-19,5	-8,0

I/A	$\alpha^{min}_{Dunkel}/°$	$\alpha^{max}_{Dunkel}/°$	$\alpha^{min}_{Hell}/°$	$\alpha^{max}_{Hell}/°$	$\beta/°$	$\gamma/°$
-0,50	1,2	1,8	80,5	97,9	-4,6	7,1
-1,00	2,4	3,1	87,6	107,5	-2,9	8,8
-1,50	3,9	4,6	70,4	104,5	-2,1	11,6
-2,00	5,2	5,6	84,3	104,6	0,0	12,0
-2,50	6,6	7,1	88,2	109,1	0,2	13,3
-3,00	8,1	8,5	86,3	105,5	1,9	14,6
-3,50	9,2	9,7	88,1	107,3	2,9	15,9
-4,00	10,6	11,0	89,6	111,4	4,3	16,1
-4,50	11,8	12,1	91,5	113,7	5,5	17,0
-5,00	12,8	13,3	91,5	115,0	6,5	18,6
Fehler:						
±0,01	±0,1°	±0,1°	± 5°	±5°	±2°	±2°

Bibliography

[1] HERRMANN, Bernd: *Elektrooptischer-Effekt und Faraday-Effekt.* 1977

[2] T. WILBOIS, M. K.: *Versuchsanleitungen Fortgeschrittenen Praktikum Teil I – Faraday- und Pockelseffekt.* 2012