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Faraday- and Pockels-Effect

Group 13:
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Abstract

In the Faraday-Pockels-experiment, two major optical effects are evaluated. The Pockels-effects as a linear electro-optical effect and the Faraday-effect as a magneto-optical effect. In the first part the Pockels-effect is used to find the half-wave-voltage of the used ADP-crystal by applying two different measuring techniques. From the half-wave-voltage then the electro-optical coefficient can be calculated. The second part focuses on the Faraday-effect. Its main goal is to find the Verdet-constant of the used flint glass. In addition the characteristic angle of the half shade polarimeter 2ϵ is measured.

In the part about the Pockels-effect, with both methods an electro-optical coefficient could be determined, with $r_{41} = (2.334 \pm 0.014) \times 10^{-11}\,\mathrm{m\,V^{-1}}$ being very close to the actual value and $r_{41} = (2.246 \pm 0.003) \times 10^{-11}\,\mathrm{m\,V^{-1}}$ showing a high incompatibility. For the second part both measurements could be conducted successfully with a Verdet-constant of $V = (1.023 \pm 0.014) \times 10^{-3}\,{}^{\circ}\,\mathrm{A^{-1}}$ being extremely close to the value provided by the manufacturer. For the characteristic angle, $2\epsilon = (13.6 \pm 0.4)^{\circ}$ is determined.

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1 Introduction

The Faraday-Pockels-experiment investigates two major optical effects with high relevance in research. It is splitted into two parts with the first part focusing on the linear electro-optical Pockels-effect and the second part covering the magneto-optical Faraday-effect. For the first part a Pockels-cell containing four ADP-crystals is used. The main goal is to find the half-wave-voltage and thereby the electro-optical coefficient of the crystal. Therefore two different methods explained in the theory part are applied. In the second part a flint glass in a coil is used to perform measurements of the Faraday-effect. After determining the magnetic field inside the coil, the Verdet-constant and the characteristic angle for the half shade polarimeter can be found.

Both Pockels- and Faraday-effect are extremely important in optical physics. Whereas the Faraday-effect is mainly used to produce optical isolators, an application of the Pockels-effect is the Pockels-cell used to create controlled phase shifts.

2 Theory

The following sections introducing the theory and methodology necessary for the experiment are mainly based on the experiment description, which is provided by the advanced physics lab team [1] and the diploma thesis by Bernd Herrmann [2]. In the appendix, all the variables and symbols used in this protocol and especially in the theory part are assembled in Table 2.

2.1 Basic theory for electro- and magneto-optics

An important characteristic of light is its polarisation. Generally three different kinds of polarisation are distinguished. If the electric field, which is always perpendicular to the direction of propagation, only oscillates in one direction, its polarisation is linear. Light is called circular-polarised if the electric field in x- and y-direction oscillate with a phase shift of $\pi/2$ and the same amplitude. All other cases of polarised light are called elliptically polarised. In the experiment, a polarisation filter is used to get linear-polarised light that can be used to observe several optical effects [2].

An important effect that has to be considered in the experiment is the birefringence. In anisotropic materials the refractive index and therefore the velocity of light changes depending on direction and polarisation of the used light. Therefore the beam can split into an ordinary and an extraordinary beam. An important tool to describe the dependence of refractive indices of the direction and the polarisation of light is the index ellipsoid [2]:

$$\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1. \tag{1}$$

x_i describes the axis of the coordinate system and n_i the respective refractive index.

2.2 Pockels-Effect

2.2.1 Basics about the Pockels-Effect

The Pockels-effect is a linear electro-optical effect. That means, when applying an electric field to a specific material, its optical behaviour is changed due to variations in the refractive indices. The Pockels-effect is based on the fact, that the dielectric constant $\epsilon = \frac{\partial D}{\partial E}$ shows a small dependence on the electric field [2]. By only considering the first term of the Taylor-approximation, the refractive index n changes linearly with the applied electric field.

The index ellipsoid for the material gets deformed by considering the Pockels-effect. In the given case of only applying an electric field in one of the directions, one can get the following description of the ellipsoid [1]:

$$\frac{x_1^2}{n_1^2} + 2r_{41}x_2E_1x_3 + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1. \tag{2}$$

For the used crystal, which is cut in a 45°-Y-angle, we can perform a coordinate transformation by rotating by 45° around the x_1-axis and afterwards introducing a new refractive index [1]:

$$x_2 = \frac{x_2' + x_3'}{\sqrt{2}}, \tag{3}$$

$$x_3 = \frac{x_2' - x_3'}{\sqrt{2}}, \tag{4}$$

$$\frac{1}{n_x^2} = \frac{1}{2}\left(\frac{1}{n_1^2} + \frac{1}{n_3^2}\right). \tag{5}$$

Plugging the transformations into the index ellipsoid one can get two expressions for the refractive indices for light travelling along the x_2'- or x_3'-axis [1]. The two expressions then can be simplified with a Taylor-approximation:

$$n_{x_2'} = \frac{n_x}{\sqrt{1 + r_{41}En_x^2}} \approx n_x + \frac{1}{2}r_{41}E_1n_x^3, \tag{6}$$

$$n_{x_3'} = \frac{n_x}{\sqrt{1 - r_{41}En_x^2}} \approx n_x - \frac{1}{2}r_{41}E_1n_x^3. \tag{7}$$

For a crystal of length l one can therefore get the following phase shift ωt between two beams of different polarisation with the wavelength λ:

$$\omega t = \frac{2\pi(n_1 - n_{x_2'})l}{\lambda}. \tag{8}$$

2.2.2 Realisation of a Pockels Cell in the given setup

To correct the birefringence caused by the 45°-Y-Cut, after the first ADP-crystal another crystal with reversed polarity of the electric field is added. The second crystal recombines the beams which have been splitted into ordinary and extraordinary beam by the first crystal. In addition the natural birefringence also has to be compensated by two extra crystals [1]. The complete arrangement of the crystals can be found in Figure 1:

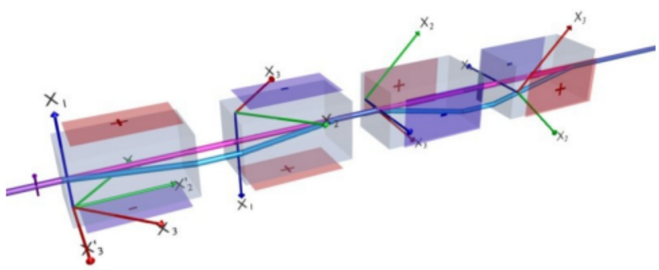


Fig. 1: The graphic portrays the arrangement of the four ADP-crystals in the Pockels-cell. In addition the polarity of the electric field is shown and the optical path of the light is plotted. The graphic is taken from [1], p. 3.

Taking into account the path of the beam through all four of the crystals one can get the following expression for the phase shift [1]:

$$\omega t = \frac{4\pi}{\lambda} r_{41} E_1 n_x^3 l, \tag{9}$$

$$r_{41} = \frac{\lambda d}{4 l U_{\lambda/2}} \sqrt{\frac{1}{2}\left(\frac{1}{n_1^2} + \frac{1}{n_3^2}\right)}^{3}, \tag{10}$$

with r_{41} being the electro-optical coefficient, E the electric field, λ the wavelength, d the thickness and l the length of one of the crystals. The half-wave-voltage $U_{\lambda/2}$ can later be determined in the experiment.

2.2.3 First Method to find the half-wave-voltage

In a first step a sawtooth voltage is applied to the Pockels-cell to find its half-wave-voltage $U_{\lambda/2}$. By using the sawtooth, the voltage and therefore the electric field also passes the point with maximal amplification and maximal erasure of the light. Therefore the resulting signal measured by a photo-diode has a sine-wave-form with one period. The difference in the voltages between the minimal point and the maximal point thereby is the half-wave-voltage $U_{\lambda/2}$ [2].

2.2.4 Second Method to find the half-wave-voltage

As a second method to find the half-wave-voltage $U_{\lambda/2}$, a sine-voltage is used that is added to a constant voltage that can be modified. The half-wave-voltage can be found by adjusting this constant current until a sine-wave with doubled frequency can be observed. Because this effect only happens for the voltage of minimal and maximal intensity, those two voltages can be found easily and then be converted into a half-wave-voltage.

The effect of frequency doubling can be explained with the following Figure 2 taken from [2]:

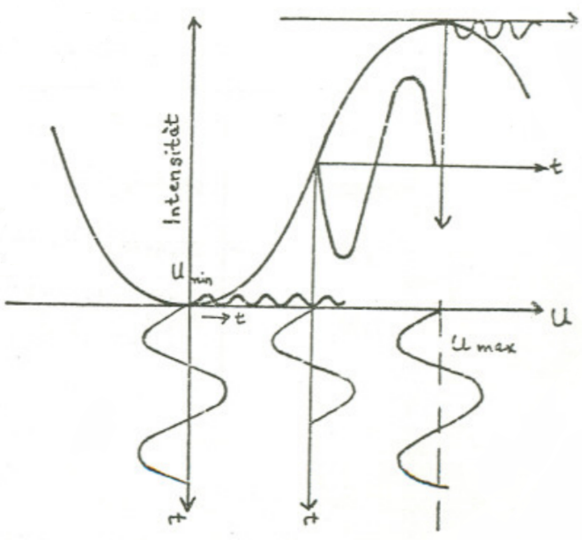


Fig. 2: As an explanatory graphic for the frequency doubling, that only occurs for the minimum and the maximum voltage, this graphic is taken from [2], p. 39. On the y-axis the intensity is plotted against the voltage U on the x-axis. In addition the applied sine-voltage and the resulting intensity detected by the diode are portrayed for three different places in the graphic.

For a voltage between the voltage for the minimum and the maximum intensity, a sine-modulation also results in the intensity signal increasing and decreasing with the modulation signal. In Figure 2 this is portrayed by the central plot. If we now look at the voltage for minimal intensity for example, the situation is completely different: because the intensity increases with higher and lower voltage, a modulation leads to hills in the intensity signal for minima as well as maxima of the modulation. This leads to a signal with doubled frequency as shown in the left plot of Figure 2. Analogously we can find the doubling of the frequency for the voltage of maximal intensity.

2.3 Faraday-Effect

2.3.1 Basics about the Faraday-Effect

In contrast to the Pockels-effect, the Faraday effect is a magneto-optical effect. This means that it occurs when applying a magnetic field to a medium. When linear polarised light travels through a Faraday-cell the polarisation-plane rotates. The angle of rotation α depends on the magnetic field H, the length of the cell l and the Verdet-constant V which depends on the material [2]:

$$\alpha = lHV. \tag{11}$$

The Faraday-effect can be explained with a semi-classical description. Linear polarised light can also be seen as a combination of two circular polarised beams that rotate in different directions. When light travels through a medium, its electrons are affected by the electric field of the light beam. With an outer magnetic field added to the system, the electrons additionally feel a Lorentz-force that either slows them down or speeds them up. Therefore one of the circular polarised beams has a higher velocity than the other one which leads to a phase shift between the beams. When recombining the two beams, the polarisation plane is rotated in comparison to the original plane.

2.3.2 Finding the Magnetic field inside the coil

To find the Verdet-constant V, the current I and the angle α are measured and then Equation 11 is used. To get a formula depending on I and α, another expression for the product of magnetic field H and length l can be found by first finding an expression for $H(z)$ in the middle of the coil using the Biot-Savart-Law and then integrating over the length of the medium inside the coil. The expression for $H(z)$ can also be found in [2]:

$$\mathrm{d}H = \frac{\mathrm{d}I}{2}\frac{x^2}{(r^2+l^2)^{3/2}}, \tag{12}$$

$$\Rightarrow H(z) = \frac{NI}{2L(r_2-r_1)}\left[(L-z)\ln\left(\frac{r_2+\sqrt{(L-z)^2+r_2^2}}{r_1+\sqrt{(L-z)^2+r_1^2}}\right)+z\ln\left(\frac{r_2+\sqrt{z^2+r_2^2}}{r_1+\sqrt{z^2+r_1^2}}\right)\right]. \tag{13}$$

N is the number of windings, I the applied current, L the length of the coil and r_2 and r_1 the outer and inner radius of the coil. For $l \cdot H$ one can get [2]:

$$lH = \int_{\frac{L-l}{2}}^{\frac{L+l}{2}} H(z)\mathrm{d}z. \tag{14}$$

By later solving this integral, a relation between Verdet-constant V, the current I and the angle α can be found.

3 Setup and measurements

3.1 Pockels-Effect

For the first part of the experiment, a HeNe-laser with wavelength $\lambda = 632.8\,\mathrm{nm}$ sends out light which is then polarised by a fixed polariser. After passing the Pockels-cell containing the four ADP-crystals as described before, a fixed analyser is used to get a shift in the intensity of the signal for different phase differences. The light then hits a photo diode that sends a signal to an amplifier and afterwards to an oscilloscope. Before performing the measurements, it is checked whether the beam hits all the components in the centre.

Three generators produce different signals (sawtooth, sine and constant voltage) and with a switch the signal can be changed between the sawtooth signal and the added sine and constant signal. The signal then is applied to the Pockels-cell and can also be observed on the oscilloscope. In a block diagram all the different electronics and the optical path of the light are portrayed in Figure 3.

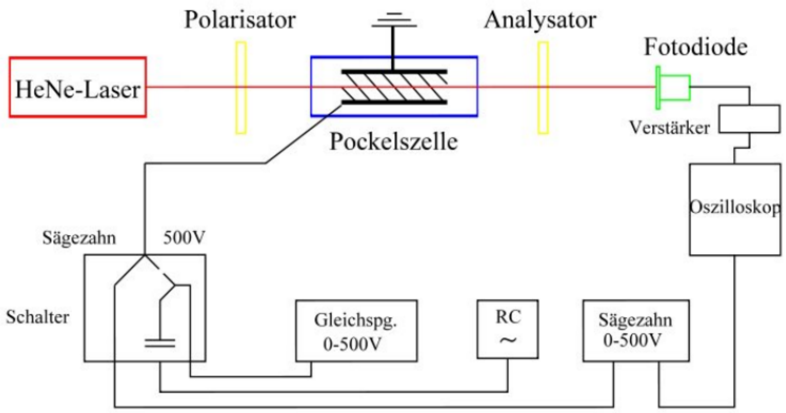


Fig. 3: Block diagram of the optical path and the used electronics for the measurements performed with the Pockels-cell. The HeNe-Laser sends light through a polariser, the Pockels-cell and an analyser before hitting the photo diode. On the bottom the three generators and the switch are portrayed. The graphic is taken from [1].

In the first part three measurements are performed. First the damping factor of the electronics is found by comparing the amplitude of the signal produced by the sine-generator with the amplitude of the same signal after passing the electronics. In a second measurement the first method to find the half-wave-voltage described in subsubsection 2.2.3 is performed. Therefore the sawtooth generator is connected to the Pockels-cell and ten different measurements are taken. The last measurement of this part is using the second method described in subsubsection 2.2.4. Hence a sine-voltage is added to a constant voltage which is varied until a frequency doubling can be observed. Again ten measurements are taken to get more precise measurements.

3.2 Faraday-Effect

For the second part of the experiment, a Na-vapor-lamp is used, because the observation is done with the eyes. The light then travels through a half shade polarimeter which has the property of producing slightly different polarisations for two areas – one in the middle and one at the sides. With an analyser behind the Faraday-cell, this can be used to get a good measurement of the angle of polarisation, because for the eye it is easier to find the angles where two areas have the same brightness then finding the angle with the darkest or brightest light. The Faraday-cell is made of a coil, that provides the magnetic field when a current is applied, surrounding a rod made of flint glass. The cell is cooled by a water supply to minimise the effects of temperature changes. The light can be observed by a telescope.

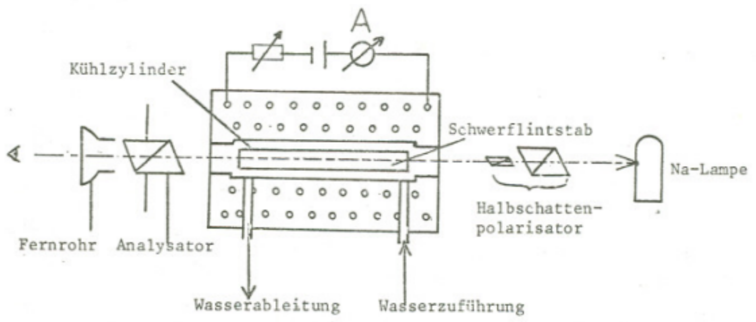


Fig. 4: Setup for the measurements of the Faraday-effect. A Na-vapor-lamp sends light through a half shade polarimeter into the coil with the flint glass. Afterwards it passes an analysor and can be observed with a telescope. The graphic is taken from [2].

For this part of the experiment two measurements are taken. In the first measurement, for different currents applied to the coil, the angle is taken for which the two areas of the half shade polarimeter have the same brightness. The angles then can be used to find the Verdet-constant for the flint glass. In a second step the characteristic angle 2ϵ for the half shade polarimeter is measured by recording the angles for which the inner and outer area are darkest.

4 Data analysis and discussion of uncertainties

4.1 Pockels-Effect

4.1.1 Finding the Damping-factor of the electronics

As a preparation for the actual measurements, the damping-factor D of the electronics has to be determined, so the following measurements can be corrected. The signal is generated by a sine generator with a frequency of $f = 999\,\mathrm{Hz}$. In Figure 5 the damped and non-damped signal are both displayed.

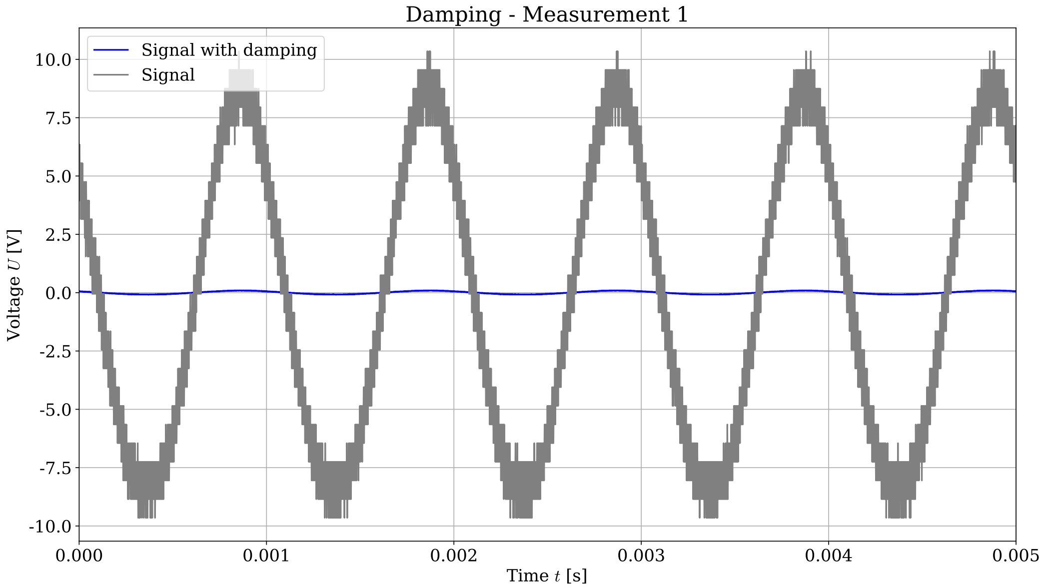


Fig. 5: In this graphic the first measurement of the damping is visible. The damped signal is portrayed in blue and the non-damped signal in grey.

To find the damping-factor the following formula can be used and its uncertainty can be found with Gaussian propagation of uncertainty [3]:

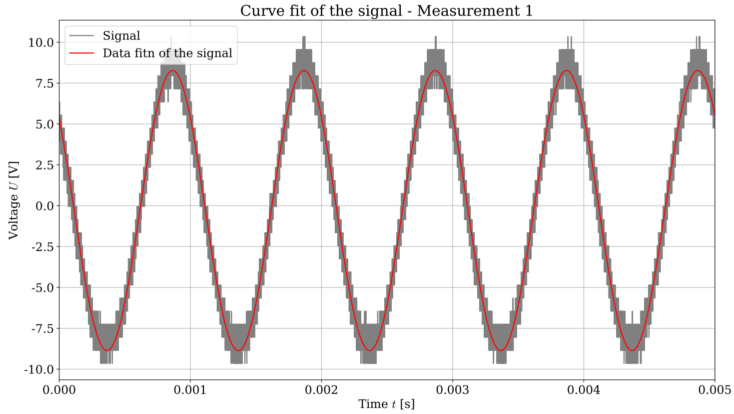
$$D = \frac{A_1}{A_2}, \tag{15}$$

$$\Delta D = \sqrt{\left(\frac{\Delta A_1}{A_2}\right)^2 + \left(-\frac{A_1 \Delta A_2}{A_2^2}\right)^2}, \tag{16}$$

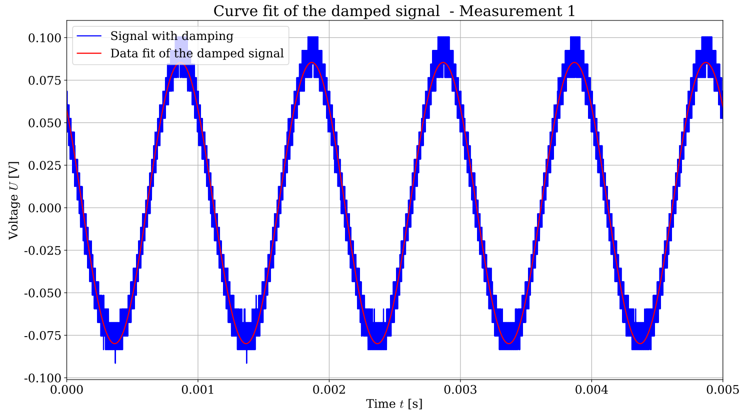
with A_1 and A_2 being the amplitudes of the damped and non-damped signal. To find the respective amplitude of both signals, a data fit with `scipy.optimize.curve_fit` is performed with Equation 17:

$$f(x) = A_{1/2} \sin(\omega(x + b)) + c. \tag{17}$$

The data fit for the non-damped signal can be found in Figure 6a and the one for the damped signal is visualised in Figure 6b.



(a) Curve fit of the signal



(b) Curve fit of the damped signal

Fig. 6: In the first graphic one can see the curve fit in red of the non-damped signal in grey. In the second plot the curve fit in red of the damped signal in blue is displayed.

The resulting fit-parameters $A_{1/2}$ for the amplitudes are:

$$A_1 = (8.57 \pm 0.04)\,\mathrm{V},$$
$$A_2 = (0.0827 \pm 0.0003)\,\mathrm{V}.$$

The other fit parameters can be ignored due to not being relevant for the damping factor. By plugging in those values in Equation 15 and Equation 16 the resulting damping-factor is:

$$D_1 = 103.6 \pm 0.6.$$

To receive more precise knowledge of the damping-factor, four additional measurements were taken and analysed analogously. The plots of the other measurements can be found in the appendix in Figure 12 to Figure 23. Their fit parameters for the amplitude and their resulting damping factors can be found in Table 3 in the appendix. As a final value for the damping-factor the mean of the measurements with the standard deviation of the mean as its uncertainty [3] is calculated:

$$D = 103.72 \pm 0.09.$$

4.1.2 Method 1: using a sawtooth-voltage

The first method to find the half-wave voltage of the Pockels-cell, which is later used to calculate the electro-optical coefficient r_{41}, is by applying a sawtooth-voltage. In this case the ratio between the rising voltage U of the sawtooth and the rising time T is equal to the ratio of the half-wave voltage $U_{\lambda/2}$ and the time τ between the minimum and maximum of the measured sine-signal:

$$\frac{U_{\lambda/2}}{\tau} = \frac{U}{T}. \tag{18}$$

Due to the damping of the electronics, the voltage U has to be multiplied with the damping-factor D which leads to:

$$U_{\lambda/2} = \frac{\tau}{T} U D, \tag{19}$$

$$\Delta U_{\lambda/2} = \sqrt{\left(\frac{\Delta\tau}{T} U D\right)^2 + \left(-\frac{\tau \Delta T}{T^2} U D\right)^2 + \left(\frac{\tau}{T} D \Delta U\right)^2 + \left(\frac{\tau}{T} U \Delta D\right)^2}. \tag{20}$$

The values for the rising voltage U and the rising time T can be directly found with the given software, which is exemplary shown in Figure 7. The uncertainties are approximated by analysing the effect of changing the values slightly by hand.

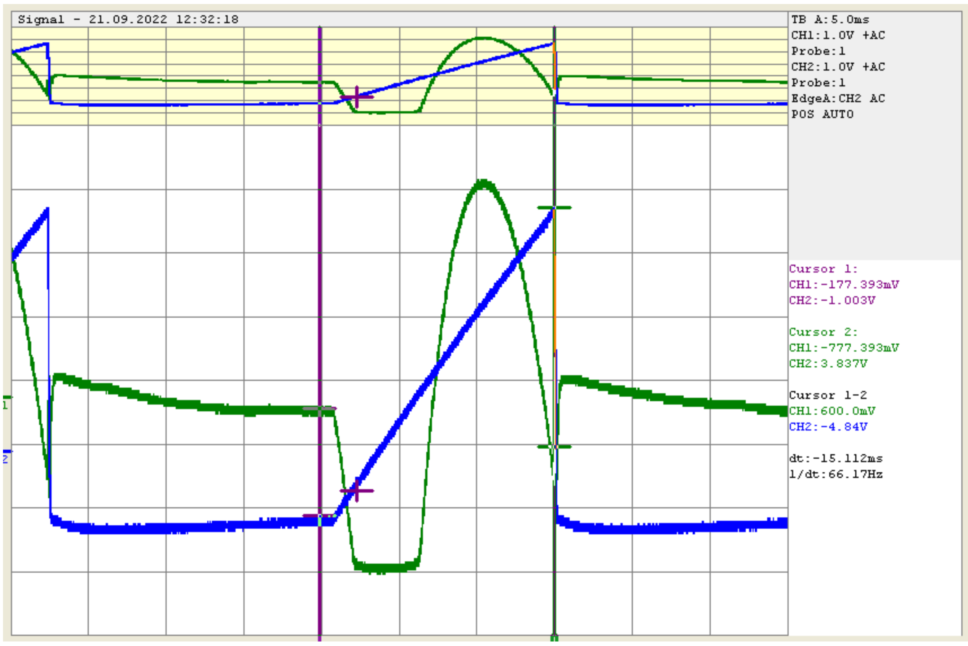


Fig. 7: In this graphic the sawtooth-voltage in blue and the sine-signal in green are both displayed in the reading software. With the purple cursor the beginning of the sawtooth and with the green cursor the end of the sawtooth is marked. In this case the sawtooth-voltage is measured in Channel 2 (CH2) and the voltages can be found on the right hand side for both cursors. The rising time is displayed as dt and can also be found on the right side.

For the first measurement the following values for T and U can be found:

$$U = (8.57 \pm 0.04)\,\text{V},$$
$$T = (14.3 \pm 0.3)\,\text{ms}.$$

All the other values for T and U with the respective uncertainties can be found in Table 4 in the appendix.

To find the time τ between the minimum and maximum of the measured sine-signal, a data fit is again performed with `scipy.optimize.curve_fit`. The formula which is fitted to the sine-signal is given as:

$$f(x) = a \sin\left(\pi \frac{x+b}{\tau}\right) + c. \tag{21}$$

The π inside of the sine is needed, because only half a period – from minimum to maximum – of the sine-wave is needed to find the half-wave voltage. A plot of the sawtooth-voltage, the sine-signal, its fit and the limits for the used fit values can be found in Figure 8.

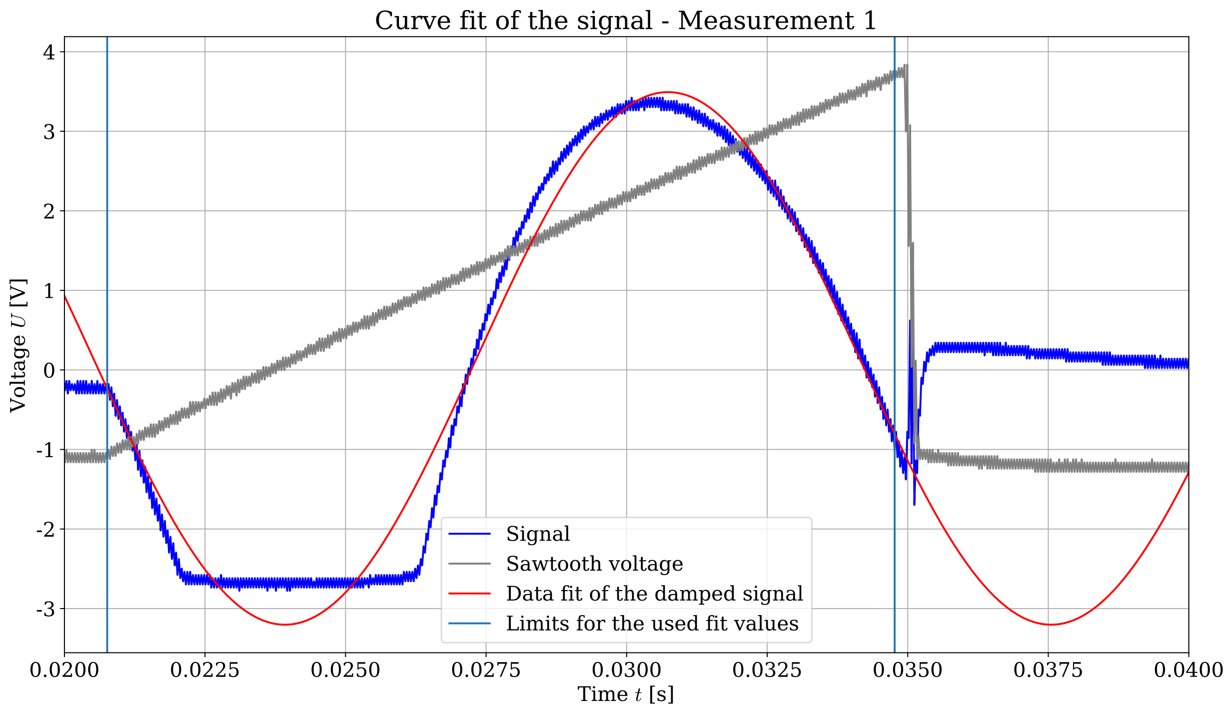


Fig. 8: In this plot the sawtooth-voltage in grey, the sine-signal in blue and its data fit in red are displayed for the first measurement. There are also two vertical lines to show the limits of the used values for the data fit.

The fit parameter τ is given as:

$$\tau = (6.81 \pm 0.02)\,\mathrm{ms}.$$

Again all the other fit-parameters are irrelevant for this measurement and thus can be ignored. With Equation 19 and Equation 20 for the half-wave voltage one gets:

$$U_{\lambda/2} = (247 \pm 15)\,\mathrm{V}.$$

Again this measurement has been taken multiple times and the plots with the data fits can be found in the appendix in Figure 24 to Figure 32 and the fit parameter τ, the measurements for

U and T and the respective half-wave voltage for each measurement can also be found in the appendix in Table 4.
Once again, the mean and the standard deviation of the mean [3] are determined considering all measurements. The resulting value for the half-wave voltage is:

$$U_{\lambda/2} = (241.5 \pm 1.5)\,\mathrm{V}.$$

Finally, the electro-optical coefficient r_{41} can be calculated with the following formula, which is introduced in the theory part:

$$r_{41} = \frac{\lambda d}{4 l U_{\lambda/2}} \sqrt{\frac{1}{2}\left(\frac{1}{n_1^2} + \frac{1}{n_3^2}\right)}^{3}, \tag{22}$$

$$\Delta r_{41} = r_{41} \frac{\Delta U_{\lambda/2}}{U_{\lambda/2}}. \tag{23}$$

The needed constants for the formula are [1]:

$$\begin{aligned} n_1 &= 1.522, \\ n_3 &= 1.477, \\ d &\approx 2.4\,\mathrm{mm}, \\ l &\approx 20\,\mathrm{mm}, \\ \lambda &= 632.8\,\mathrm{nm}. \end{aligned}$$

The resulting value for the electro-optical coefficient r_{41} is:

$$r_{41} = (2.334 \pm 0.014) \times 10^{-11}\,\mathrm{m\,V^{-1}}.$$

This value will be compared to the value in literature in the discussion part.

4.1.3 Method 2: using a sine-voltage

Similarly to the sawtooth-method, with the sine-method the half-wave voltage $U_{\lambda/2}$ must be found before calculating the electro-optical coefficient r_{41}. In this case the half-wave voltage is determined by finding the difference in the voltages at which the measured sine-signal has a doubled frequency which corresponds with a maximum or minimum in intensity:

$$U_{\lambda/2} = U_{\mathrm{max}} - U_{\mathrm{min}}, \tag{24}$$

$$\Delta U_{\lambda/2} = \sqrt{(\Delta U_{\mathrm{max}})^2 + (-\Delta U_{\mathrm{min}})^2}. \tag{25}$$

An exemplary measurement of the maximum can be found in Figure 9 and a measurement of the minimum can be found in Figure 10.

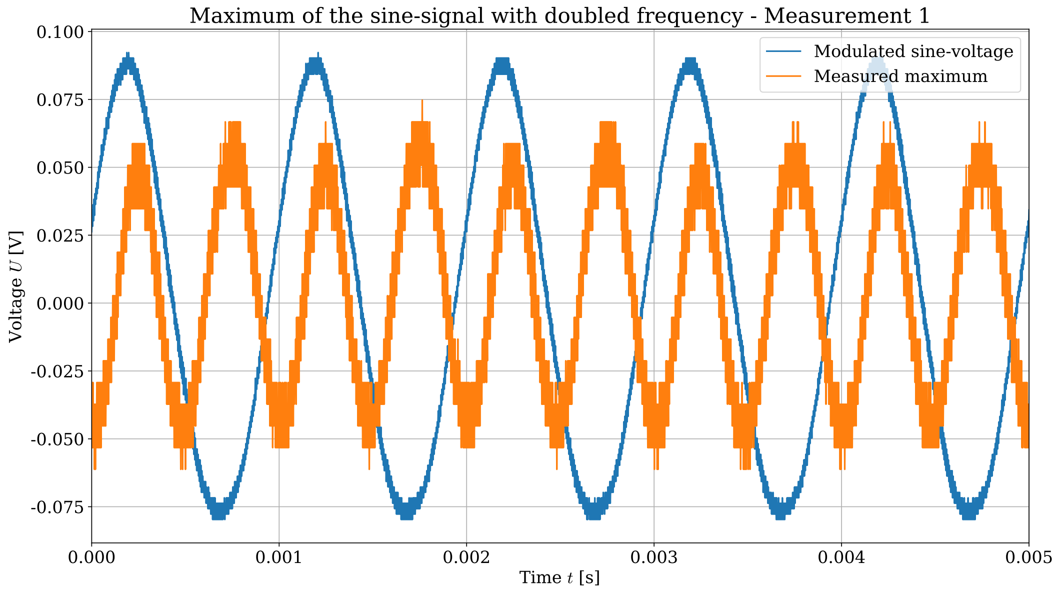


Fig. 9: In this plot the modulated sine-voltage in blue and the maximum of the sine-signal with doubled frequency in orange are displayed for the first measurement.

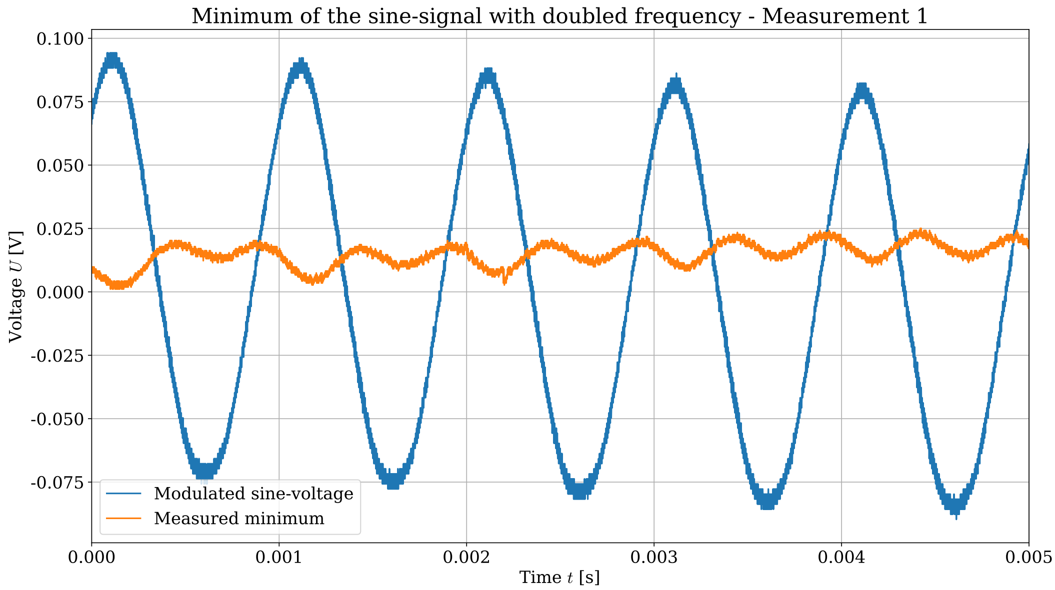


Fig. 10: In this plot the modulated sine-voltage in blue and the minimum of the sine-signal with doubled frequency in orange are displayed for the first measurement.

Again this measurement has been taken multiple times in order to achieve more precise measurements. The plots can be found in the appendix in Figure 33 to Figure 50.
The resulting voltages for the maximum and minimum for each measurement and the calculated

half-wave voltage are listed in Table 1. The uncertainty for the voltage is approximated by testing the range of the voltage in which a doubled frequency can be observed on the oscilloscope.

Tab. 1: In this table the voltage $U_{\min}$ of the minimal signal with doubled frequency, the voltage $U_{\max}$ of the maximal signal with doubled frequency and the resulting half-wave voltage $U_{\lambda/2}$ are listed for each measurement with the second method.

Measurement	Voltage $U_{\min}$	Voltage $U_{\max}$	Half-wave voltage $U_{\lambda/2}$
1	(-114 ± 4) V	(138 ± 2) V	(252 ± 4) V
2	(-115 ± 4) V	(138 ± 2) V	(254 ± 4) V
3	(-113 ± 4) V	(137 ± 2) V	(250 ± 4) V
4	(-116 ± 4) V	(136 ± 2) V	(252 ± 4) V
5	(-115 ± 4) V	(135 ± 2) V	(250 ± 4) V
6	(-116 ± 4) V	(135 ± 2) V	(251 ± 4) V
7	(-115 ± 4) V	(135 ± 2) V	(251 ± 4) V
8	(-115 ± 4) V	(135 ± 2) V	(250 ± 4) V
9	(-116 ± 4) V	(135 ± 2) V	(251 ± 4) V
10	(-115 ± 4) V	(135 ± 2) V	(250 ± 4) V

As mentioned in the other parts of the analysis, the mean of the half-wave voltage $U_{\lambda/2}$ is calculated:

$$U_{\lambda/2} = (251.0 \pm 0.4)\,\mathrm{V}.$$

Analogously to the previous part, the electro-optical coefficient r_{41} is calculated with Equation 22 and Equation 23:

$$r_{41} = (2.246 \pm 0.003) \times 10^{-11}\,\mathrm{m\,V^{-1}}.$$

This value will later be compared with the value in literature.

4.2 Faraday-Effect

4.2.1 Calculating the magnetic field

As a first step the magnetic field inside the coil should be calculated to get a relation between current I, angle α and Verdet-constant V. Therefore the following integral, which has been introduced in the theory part, must be solved:

$$lH = \int_{\frac{L-l}{2}}^{\frac{L+l}{2}} H(z)\mathrm{d}z. \tag{26}$$

This integral can be solved using `Wolfram Mathematica`. With $l = 15.0\,\mathrm{cm}$, $L = 17.5\,\mathrm{cm}$, $r_1 = 1.0\,\mathrm{cm}$, $r_2 = 7.5\,\mathrm{cm}$ and $N = 3600$ we get:

$$lH = 2554.848I, \tag{27}$$
$$\Rightarrow \alpha = 2554.848VI. \tag{28}$$

By using an idealised coil with a constant magnetic field the equation provides a different value:

$$H_{\mathrm{id}} = \frac{NI}{L}, \tag{29}$$
$$\Rightarrow lH_{\mathrm{id}} = \frac{lNI}{L} = 3085.714I. \tag{30}$$

Both values are used to find the Verdet-constant and both resulting values will later be compared.

4.2.2 Finding the Verdet-Constant

Before taking measurements for the Faraday effect, the thesis that darker colours are easier to separate for the eye is verified. With the half shade polarimeter there are two different angles for which the areas are equally bright. For the brighter one no difference between the two areas can be measured by eye in a range of approximately 40°. In contrast, for the darker one, a difference of 0.5° can already lead to two areas being separable. Hence a measurement with the darker spot used to find the correct angle seams to be more sensible.

To find the Verdet constant V, Equation 28 is used by measuring the angle α for different currents I and then performing a linear regression:

$$\alpha = 2554.848VI, \tag{31}$$
$$\Rightarrow \alpha = aI + b \quad \text{with} \quad a = 2554.848V \quad \text{and} \quad b = 0. \tag{32}$$

The angle is thereby measured by taking the angle where both of the areas of the half shade polarimeter are equally dark. The linear regression is performed with `scipy.optimize.curve_fit` and can be found in Figure 11.

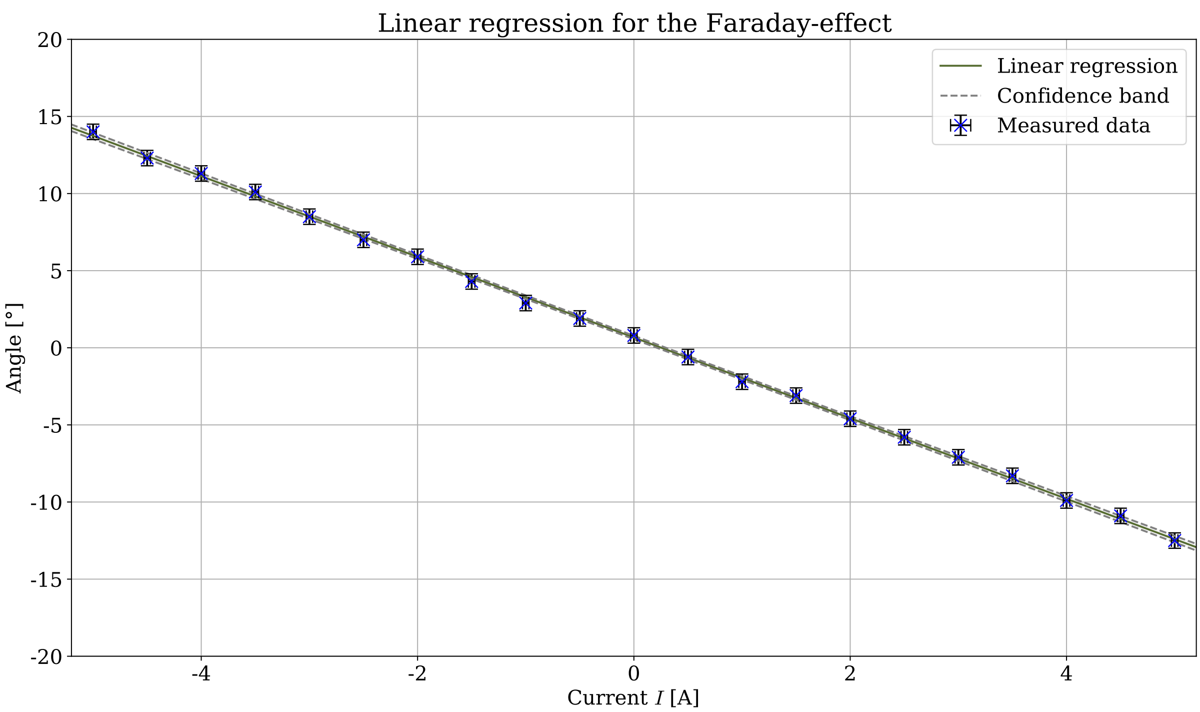


Fig. 11: In this plot the measured angle in ° is plotted against the applied current I in A. Additionally a linear regression with a confidence band is displayed.

For the slope a and the intercept b we get the following values with `scipy.optimize.curve_fit`:

$$a = (-2.61 \pm 0.04)\,^\circ\,\mathrm{A}^{-1},$$
$$b = (0.67 \pm 0.11)^\circ.$$

The intercept b only depends on the initial setting of the angles and thus can be ignored. From the slope a we can get the Verdet-constant. The uncertainty is calculated by Gaussian propagation of uncertainties [3]:

$$V = -\frac{a}{2554.848}, \tag{33}$$

$$\Delta V = \frac{\Delta a}{2554.848}. \tag{34}$$

The minus comes from the fact, that the Verdet-constant is defined positive, but our slope is negative. By plugging in the values we get the following Verdet-constant:

$$V = (1.023 \pm 0.014) \times 10^{-3}\,^\circ\,\mathrm{A}^{-1}.$$

Using the factor 3085.714 instead of 2554.848 that is derived from the simplified calculation of the magnetic field, one gets the following Verdet-constant:

$$V = (0.847 \pm 0.011) \times 10^{-3}\,^\circ\,\mathrm{A}^{-1}.$$

Both values will be compared to the value presented by the manufacturer of the flint glass in the discussion part.

4.2.3 Finding the characteristic angle of the half shade polarimeter

In a last step the characteristic angle 2ϵ of the half shade polarimeter can be measured. Therefore two measurements are taken: In the first one, the angle is found where the inner area is darkest and in the second one the angle for the outer area being the darkest is found. The difference between the two angles leads to the characteristic angle 2ϵ. To improve the measurement five different measurements are taken and afterwards averaged. All the measurements can be found in Table 5 in the appendix. As an uncertainty, the standard deviation of the mean is taken [3]:

$$2\epsilon = (13.6 \pm 0.4)^\circ.$$

5 Summary and discussion of the results

5.1 Comparison with expectation

A comparison between the measured data and the values in literature can be done by using a t-value. The t-value is calculated by dividing the difference between measured and real value with the uncertainty of the measured value. A t-value smaller than two corresponds with a good measurement.

The value in literature for the electro-optical coefficient $r_{41,\mathrm{Lit}}$ is taken from the experiment description provided by the advanced physics lab team [1]:

$$r_{41} = 23.4\,\mathrm{pm\,V^{-1}}.$$

One should also consider, that this value is given for a temperature of 21 °C due to the electro-optical coefficient being dependent on the temperature [4]. For the measurement of r_{41} with the sawtooth-method, which has a relative error of 0.6%, we get the following t-value:

$$t = 0.40.$$

This shows a good compatibility between the measurement and the value in literature.
For the r_{41} measurement with the sine-method, which has a relative error of 0.15%, we get a t-value of:

$$t = 28.93.$$

Reasons for the big discrepancy between the measurement and the value in literature will later be discussed.

The calculated value for the Verdet-constant V from the Faraday-effect can also be compared by using a t-value. For the Verdet-constant of the flint glass only a manufacturers specification is known [1]:

$$V_{\mathrm{manu}} = 1.047 \times 10^{-3}\,{}^\circ\,\mathrm{A}^{-1}.$$

The resulting t-value for the Verdet-constant with the properly calculated magnetic field is

$$t = 1.68,$$

whereas the t-value for the Verdet-constant with the idealised magnetic field is given as:

$$t = 17.11.$$

The deviation from the manufacturers specification is much higher when using the idealised magnetic field for the calculations, which will later be discussed.

5.2 Summary of the results

For the Pockels-effect the first step was to calculate the damping factor of the used electronics. The following factor could be found by comparing the amplitudes of a sine-voltage before and after the damping:

$$D = 103.72 \pm 0.09.$$

Afterwards by using the sawtooth voltage a first value for the half-wave-voltage and therefore for the electro-optical coefficient could be found:

$$U_{\lambda/2} = (241.5 \pm 1.5)\,\mathrm{V},$$
$$r_{41} = (2.334 \pm 0.014) \times 10^{-11}\,\mathrm{m\,V^{-1}}.$$

With a t-value of $t = 0.40$ and a relative error of 0.6% this measurement was very successful. In comparison, with the other method the following values could be taken:

$$U_{\lambda/2} = (251.0 \pm 0.4)\,\mathrm{V},$$
$$r_{41} = (2.246 \pm 0.003) \times 10^{-11}\,\mathrm{m\,V^{-1}}.$$

In this case a t-value of 28.93 shows a very high incompatibility in comparison with the literature. With a even smaller relative error of 0.15% the uncertainty is definitely too small.

In the second part the Verdet-constant for the flint class could be found with the following values being calculated:

$$V = (1.023 \pm 0.014) \times 10^{-3}\,{}^\circ\,\mathrm{A}^{-1},$$
$$V = (0.847 \pm 0.011) \times 10^{-3}\,{}^\circ\,\mathrm{A}^{-1}.$$

The first value, which was calculated using the integral to get the right magnetic field inside the coil has a good compatibility ($t = 1.68$) with the value given by the manufacturer. As expected, the second value which only uses an approximation for the magnetic field has a much higher t-value of 17.11. For the characteristic angle of the half shade polarimeter the following value was found:

$$2\epsilon = (13.6 \pm 0.4)^\circ.$$

5.3 Discussion of results and uncertainties

A first discussion point is the fact, that the signal for negative constant currents during the performance of the Pockels-measurements was much more unstable than the signal for positive constant currents. This can have two main reasons. The first one can come from the electronics that have a problem dealing with negative voltages. It could be possible that the oscilloscope or other parts of the electronic somehow have higher fluctuation when negative voltages are applied. A second more reasonable cause of the fluctuation comes from an anisotropy in the crystals. Theoretically the stability of the signal should not depend on the polarity of the electric field. It still is possible that during production or later when doing the experiment somehow pollution and imperfections in the crystal occur. These imperfections would lead to different behaviour for different polarities and could explain the effect of higher fluctuations for negative voltage.

The most important discussion point regarding the Pockels-effect is the high incompatibility of the second electro-optical coefficient, measured by adjusting the voltage until a doubling in frequency could be detected. With a t-value of $t = 28.93$ the value is not close enough to the theoretical value and its uncertainty is underestimated too. In the following, the main factors for the high t-value are discussed.

A first important point is the uncertainty only being derived by the standard deviation of the mean. Therefore statistical uncertainties should cancel out, but systematic uncertainties still can affect the measurement. It would have been sensible to also taking into account the high uncertainty of $2 - 4$ V by finding the right voltage for frequency-doubling. Because of high fluctuation this effect has a deep impact on the uncertainty of the measurement. By also taking into account this uncertainty, the t-value could be reduced to a value of $t = 5.94$ which is much better. Still this uncertainty could be underestimated, especially for the measurements with the negative voltage, where extremely high fluctuations were visible.

Because an underestimation of the uncertainty still does not solve the problem of the value being too far away from the theoretical value, another factor could be the electronics that did not show the exact value of the voltage. Also for completely closing the rotary button for the voltage, a voltage unequal to zero was shown. Therefore it could be possible that the used voltage for the analysis could be off from the real value. Another even more important point is the dependence of the electro-optical coefficient of the temperature. It can not be guarantied that the temperature was always constant at 21 °C which is the temperature for which the theoretical value was taken. As described in [4], the electro-optical coefficient can vary with the temperature. Since no measurements of the temperature were performed, this fact could not be included in the analysis of the experiment.

A general disturbance in the measurement was, that the first oscilloscope broke in the middle of the sine-voltage-measurement. This also might have lead to a falsification of a few data points.

The first discussion point for the measurement of the Faraday-effect is the fact, that the measured angle is set by eye. Estimating by eye when both areas have roughly the same intensity in order to set the angle, leads to great uncertainties in the measuring process. This argument can be emphasised by the observation, that depending on the person who measured the position of the two areas being equally dark, different angles where measured. An alternative and more precise approach to find the angle is to use photo-diodes to measure the actual intensity in both areas and then setting the angle for which the photo-diodes show the same intensity.

Another interesting discussion point is the comparison of the values for the Verdet-constant. One of the values was calculated by solving the integral for the magnetic field whereas the other one was found by only using an approximation. As expected, only the value calculated by using the integral shows a good agreeableness with the theoretical value by having a t-value smaller than two. The other value in comparison is not compatible at all, which shows that only taking an approximation for the magnetic field is not suitable for more complex coils.

References

[1] T. Wilbois, M. Köhli: *Faraday- und Pockelseffekt - Versuchsanleitung Fortgeschrittenen Praktikum Teil 1*, (Freiburg im Breisgau: 2012)

[2] Bernd Hermann: *Elektrooptischer Effekt und Faraday-Effekt - Zulassungsarbeit zur wissenschaftlichen Prüfung für das Lehramt an Gymnasien*, (Freiburg im Breisgau: 1977)

[3] Dr. Christof Bartels, Dr. Lukas Bruder, Dr. Thomas Pfohl: *Datenanalyse Teil A - Skript zur Vorlesung am 06.09.2021*, (Freiburg im Breisgau: 2021/22)

[4] Volker Wirth: *Temperaturabhängige elektrooptische und elektrostriktive Untersuchungen an kristallen mit ferroischen Phasenumwandlungen*, Dissertation, p. 65f (1999)

[5] Dr. Christof Bartels, Dr. Lukas Bruder, Dr. Thomas Pfohl: *Datenanalyse Teil B - Skript zur Vorlesung am 28.02.2022*, (Freiburg im Breisgau: 2021/22)

7 Attachment

7.1 Tables and graphics

7.1.1 Table with the used symbols in the protocol

Tab. 2: Table of the used symbols for the parameters used in the protocol.

Symbol	Parameter
x_i	coordinate axis
n_i	refractive index
E	electrical field
D	electric displacement field
ϵ	dielectric constant
r_{41}	electro-optical coefficient
l	crytal/cell length
d	crystal thickness
ωt	phase shift
λ	wavelength of light
$U_{\lambda/2}$	half-wave voltage
α	angle of rotation
H	magnetic field
V	Verdet-constant
L	coil length
r_1, r_2	inner/outer coil radius
I	current

7.1.2 Tables with measured data

Tab. 3: In this table the amplitude A_1 of the signal, the damped amplitude A_2 of the signal and the resulting damping-factor D are listed for each measurement of the damping.

Measurement	Signal amplitude A_1	Damped amplitude A_2	Damping-factor D
1	(8.57 ± 0.04) V	(0.0827 ± 0.0003) V	103.6 ± 0.6
2	(8.55 ± 0.04) V	(0.0824 ± 0.0003) V	103.8 ± 0.6
3	(8.55 ± 0.04) V	(0.0823 ± 0.0003) V	103.9 ± 0.6
4	(8.54 ± 0.04) V	(0.0822 ± 0.0003) V	103.9 ± 0.6
5	(8.53 ± 0.04) V	(0.0824 ± 0.0003) V	103.4 ± 0.6

Tab. 4: In this table the rising voltage U and the rising time T of the sawtooth-voltage, the fit parameter τ for the time between the minimum and maximum of the sine-signal and the resulting half-wave voltage $U_{\lambda/2}$ are listed for each measurement with the sawtooth-voltage.

Measurement	Rising voltage U	Rising time T	τ	$U_{\lambda/2}$
1	(8.57 ± 0.04) V	(14.3 ± 0.3) ms	(6.81 ± 0.02) ms	(247 ± 15) V
2	(8.57 ± 0.04) V	(14.1 ± 0.3) ms	(6.80 ± 0.02) ms	(242 ± 15) V
3	(8.57 ± 0.04) V	(14.3 ± 0.3) ms	(6.81 ± 0.02) ms	(239 ± 15) V
4	(8.57 ± 0.04) V	(14.5 ± 0.3) ms	(6.79 ± 0.02) ms	(236 ± 15) V
5	(8.57 ± 0.04) V	(14.3 ± 0.3) ms	(6.81 ± 0.02) ms	(240 ± 15) V
6	(8.57 ± 0.04) V	(14.2 ± 0.3) ms	(6.80 ± 0.02) ms	(246 ± 15) V
7	(8.57 ± 0.04) V	(14.4 ± 0.3) ms	(6.79 ± 0.02) ms	(245 ± 15) V
8	(8.57 ± 0.04) V	(14.4 ± 0.3) ms	(6.79 ± 0.02) ms	(245 ± 15) V
9	(8.57 ± 0.04) V	(14.2 ± 0.3) ms	(6.79 ± 0.02) ms	(244 ± 15) V
10	(8.57 ± 0.04) V	(14.2 ± 0.3) ms	(6.78 ± 0.02) ms	(232 ± 15) V

Tab. 5: Conclusion of all the measurements performed to find the characteristic angle 2ϵ of the half shade polarimeter. In addition to the number of the measurement and the angle 2ϵ, the current used for the measurement is presented. For all the angles the uncertainty is $\Delta 2\epsilon = 1.4°$.

Measurement	Current I	Angle 2ϵ
1	-5.0 A	12.0°
2	-2.5 A	14.4°
3	0.0 A	13.4°
4	2.5 A	14.3°
5	5.0 A	13.9°

7.1.3 Measurements of the damping-factor

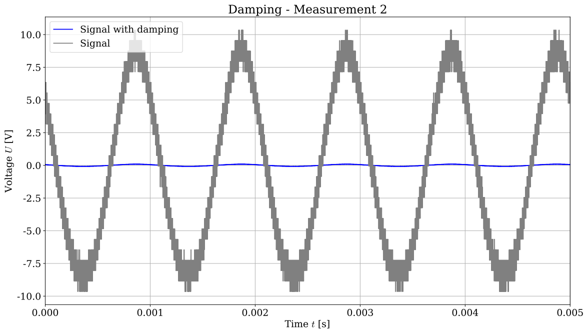


Fig. 12: Second measurement of the damping factor. Damped signal in blue, normal signal in grey.

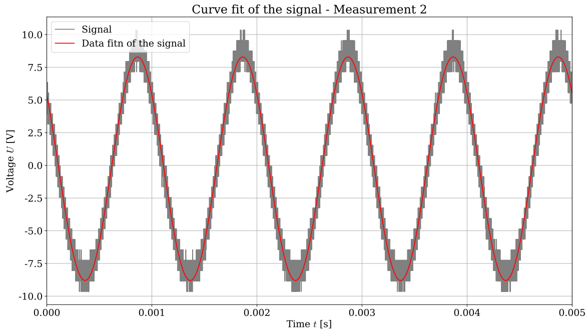


Fig. 13: A data fit of a sine wave in red for the non-damped signal in grey from the second measurement.

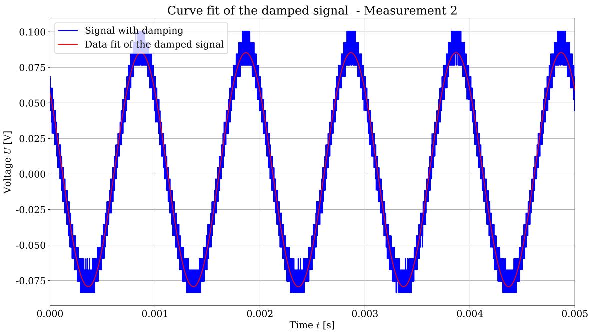


Fig. 14: A data fit of a sine wave in red for the damped signal in blue from the second measurement.

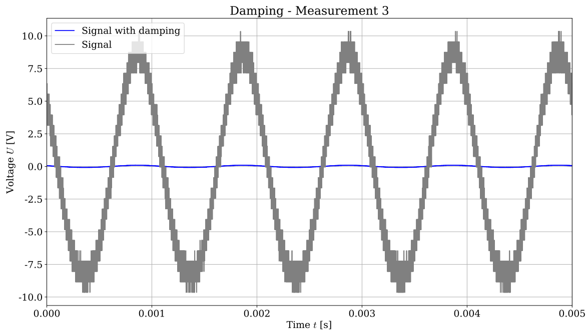


Fig. 15: Third measurement of the damping factor. Damped signal in blue, normal signal in grey.

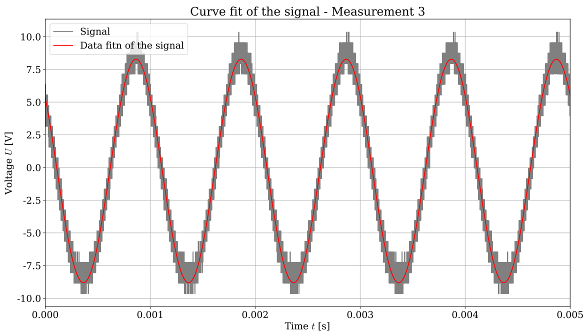


Fig. 16: A data fit of a sine wave in red for the non-damped signal in grey from the third measurement.

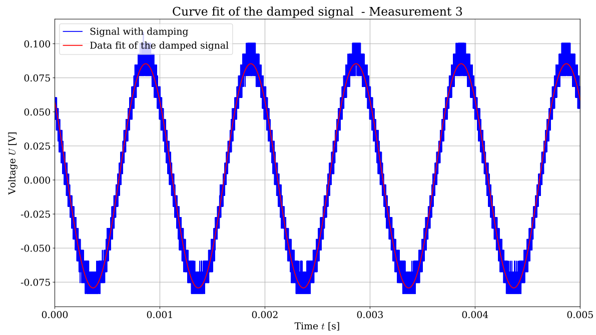


Fig. 17: A data fit of a sine wave in red for the damped signal in blue from the third measurement.

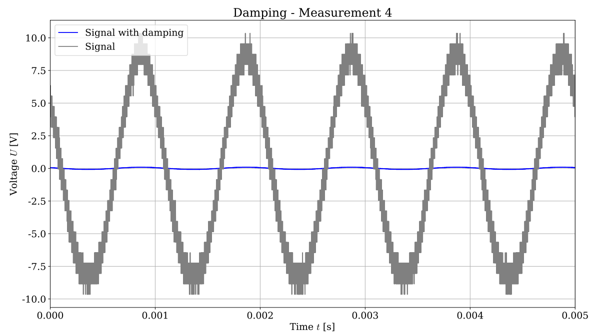


Fig. 18: Fourth measurement of the damping factor. Damped signal in blue, normal signal in grey.

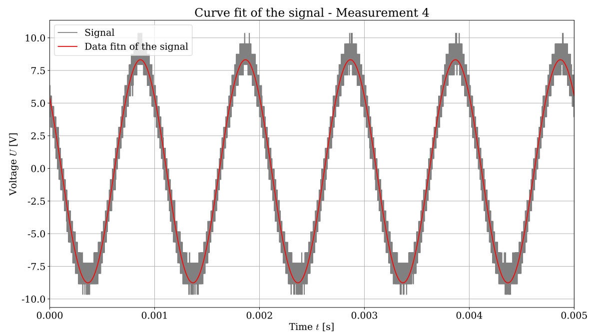


Fig. 19: A data fit of a sine wave in red for the non-damped signal in grey from the fourth measurement.

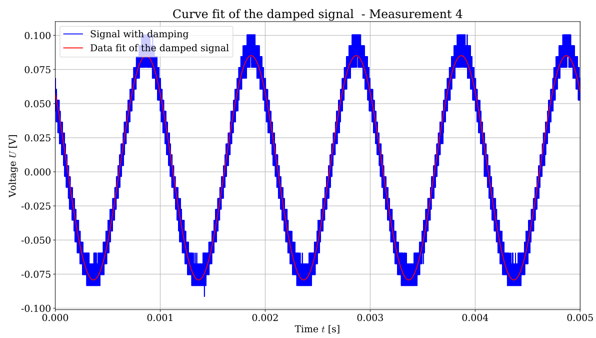


Fig. 20: A data fit of a sine wave in red for the damped signal in blue from the fourth measurement.

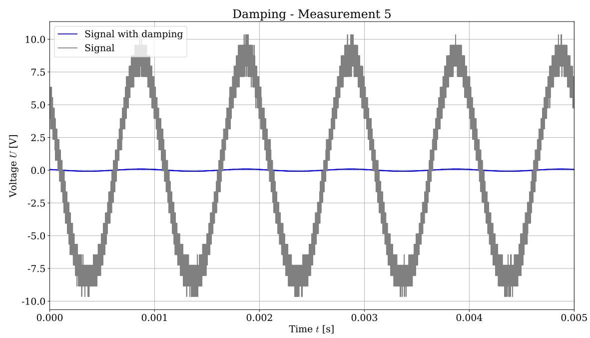


Fig. 21: Fifth measurement of the damping factor. Damped signal in blue, normal signal in grey.

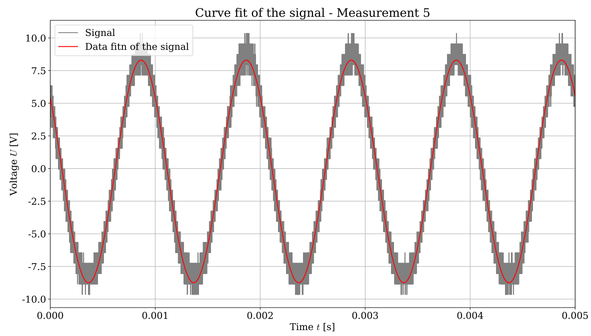


Fig. 22: A data fit of a sine wave in red for the non-damped signal in grey from the fifth measurement.

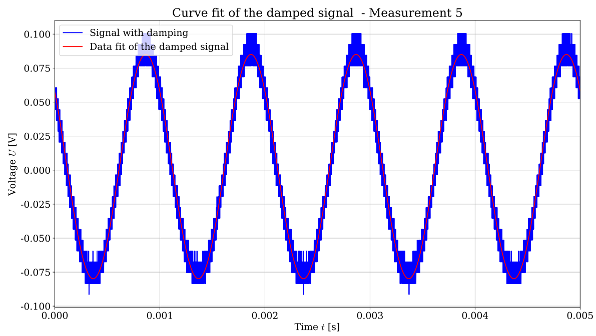


Fig. 23: A data fit of a sine wave in red for the damped signal in blue from the fifth measurement.

7.1.4 Measurement of the Pockels-effect with the sawtooth-method

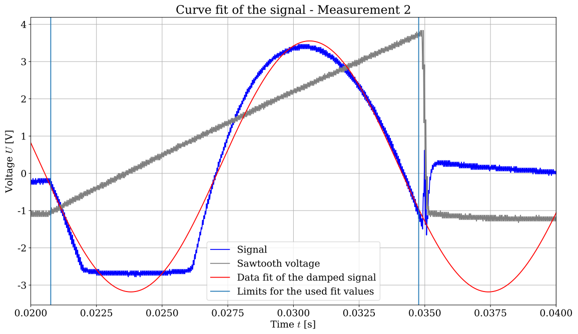


Fig. 24: Signal with a data fit of the second sawtooth measurement. Signal in blue, data fit in red.

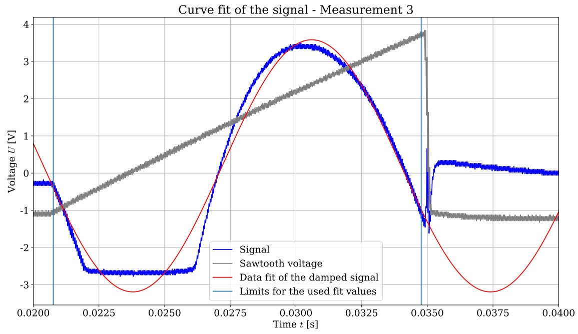


Fig. 25: Signal with a data fit of the third sawtooth measurement. Signal in blue, data fit in red.

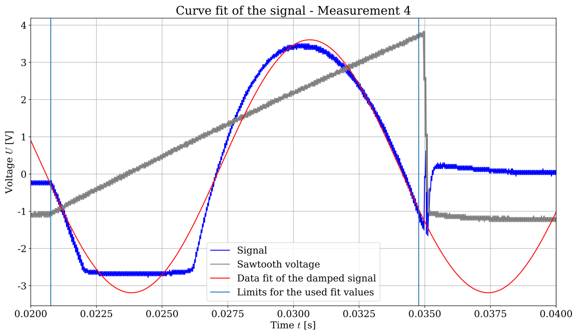


Fig. 26: Signal with a data fit of the fourth sawtooth measurement. Signal in blue, data fit in red.

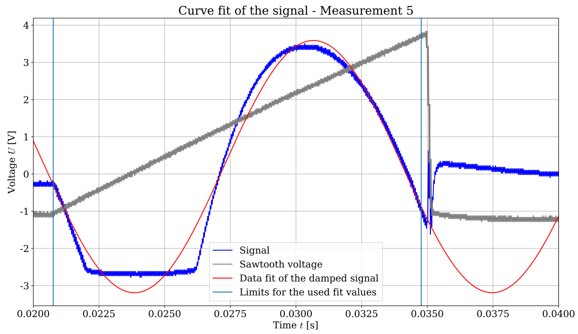


Fig. 27: Signal with a data fit of the fifth sawtooth measurement. Signal in blue, data fit in red.

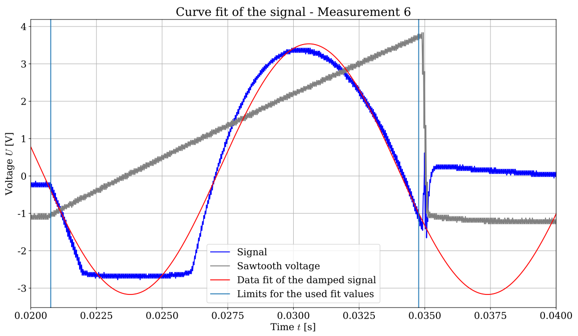


Fig. 28: Signal with a data fit of the sixth sawtooth measurement. Signal in blue, data fit in red.

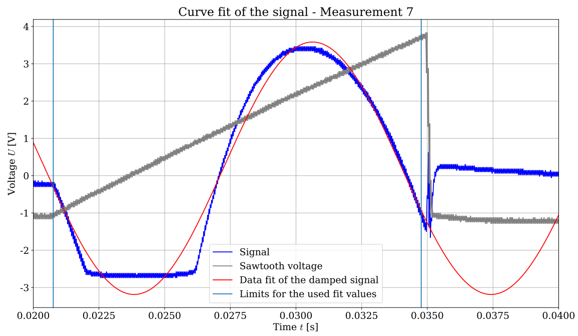


Fig. 29: Signal with a data fit of the seventh sawtooth measurement. Signal in blue, data fit in red.

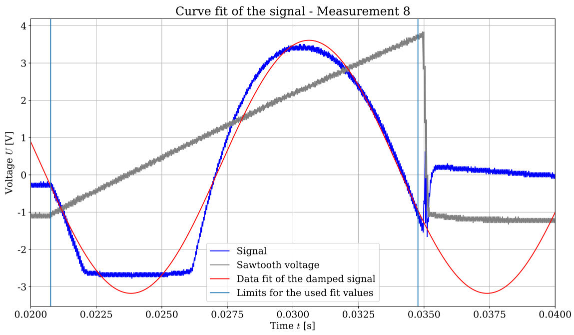


Fig. 30: Signal with a data fit of the eigth sawtooth measurement. Signal in blue, data fit in red.

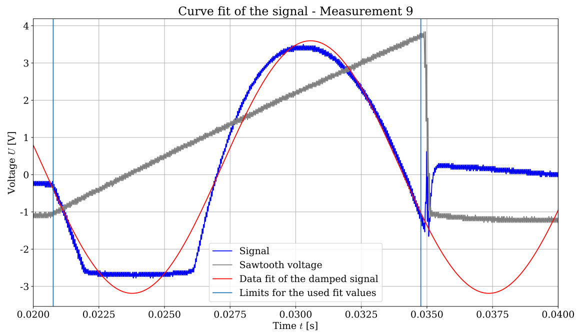


Fig. 31: Signal with a data fit of the ninth sawtooth measurement. Signal in blue, data fit in red.

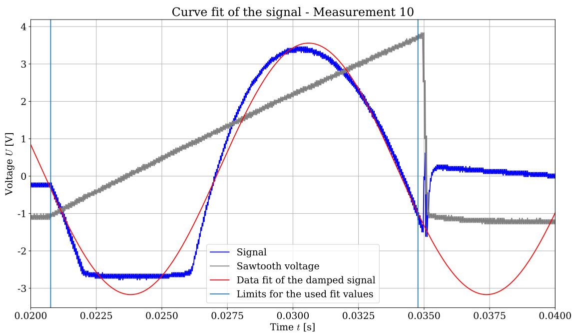


Fig. 32: Signal with a data fit of the tenth sawtooth measurement. Signal in blue, data fit in red.

7.1.5 Measurement of the Pockels-effect with the sine-method

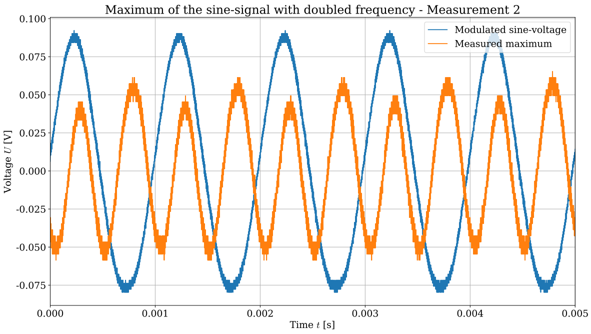


Fig. 33: Maximum of the sine-signal with doubled frequency in measurement 2. Signal in orange, sine-voltage in blue.

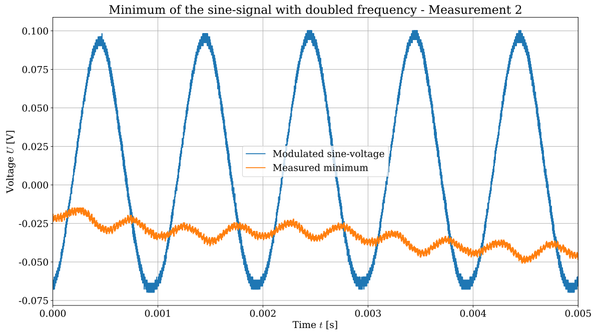


Fig. 34: Minimum of the sine-signal with doubled frequency in measurement 2. Signal in orange, sine-voltage in blue.

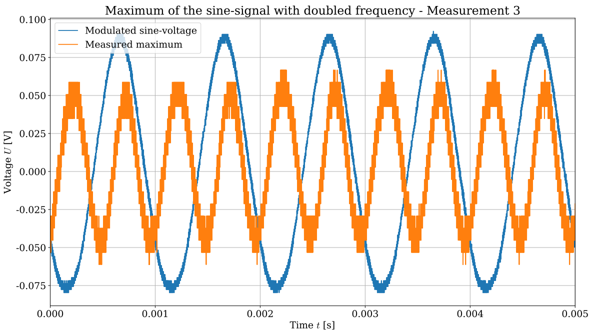


Fig. 35: Maximum of the sine-signal with doubled frequency in measurement 3. Signal in orange, sine-voltage in blue.

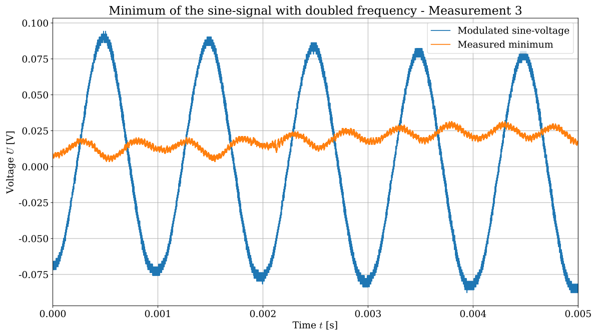


Fig. 36: Minimum of the sine-signal with doubled frequency in measurement 3. Signal in orange, sine-voltage in blue.

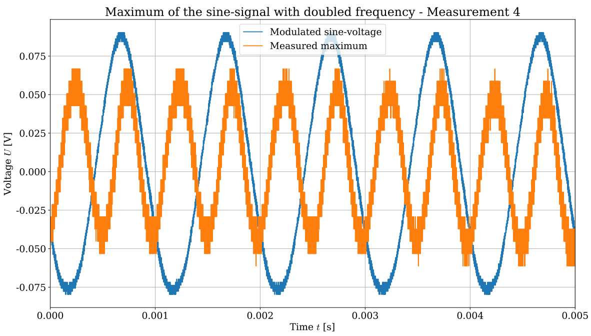


Fig. 37: Maximum of the sine-signal with doubled frequency in measurement 4. Signal in orange, sine-voltage in blue.

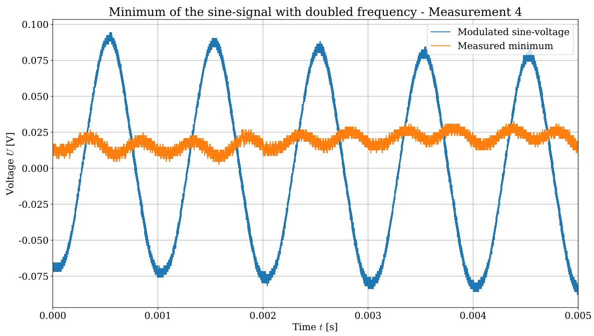


Fig. 38: Minimum of the sine-signal with doubled frequency in measurement 4. Signal in orange, sine-voltage in blue.

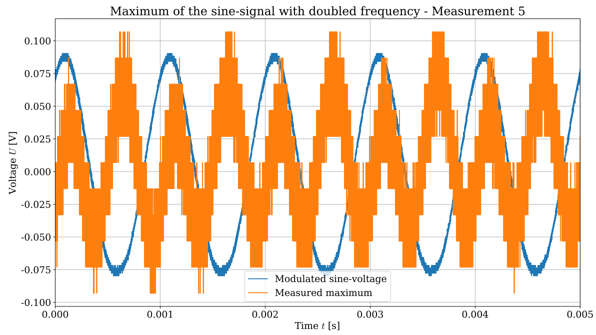


Fig. 39: Maximum of the sine-signal with doubled frequency in measurement 5. Signal in orange, sine-voltage in blue.

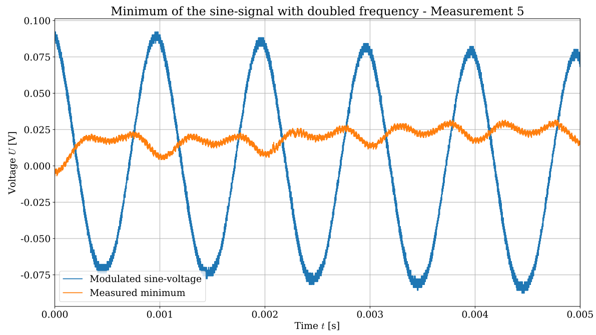


Fig. 40: Minimum of the sine-signal with doubled frequency in measurement 5. Signal in orange, sine-voltage in blue.

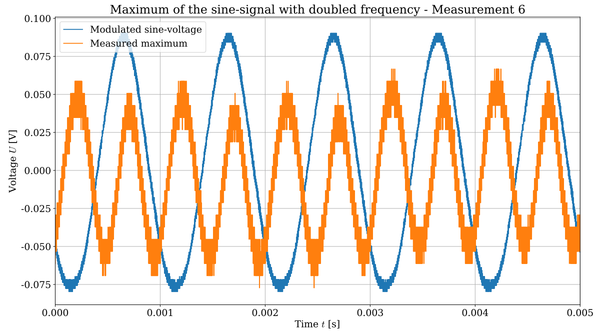


Fig. 41: Maximum of the sine-signal with doubled frequency in measurement 6. Signal in orange, sine-voltage in blue.

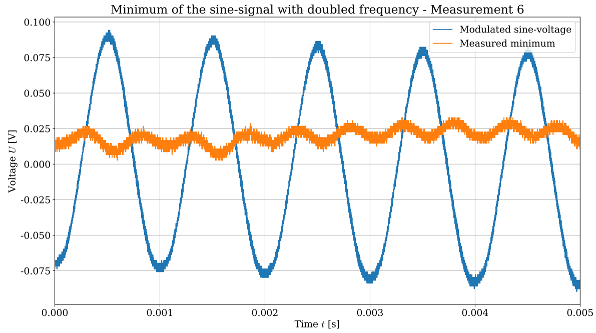


Fig. 42: Minimum of the sine-signal with doubled frequency in measurement 6. Signal in orange, sine-voltage in blue.

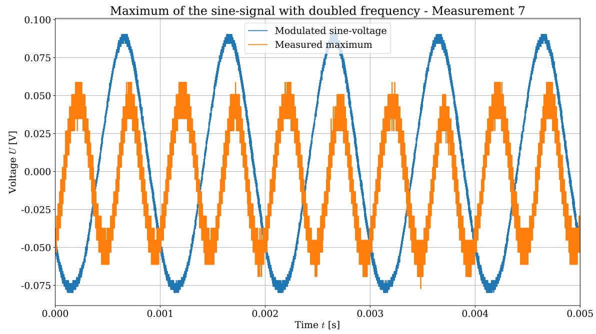


Fig. 43: Maximum of the sine-signal with doubled frequency in measurement 7. Signal in orange, sine-voltage in blue.

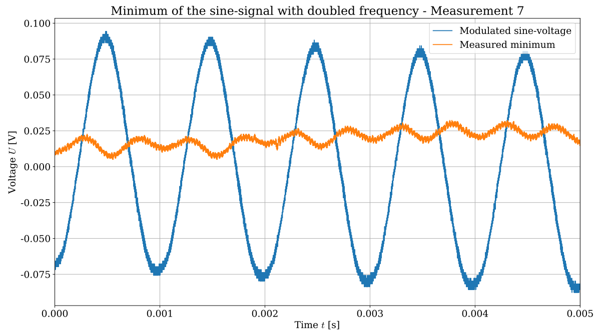


Fig. 44: Minimum of the sine-signal with doubled frequency in measurement 7. Signal in orange, sine-voltage in blue.

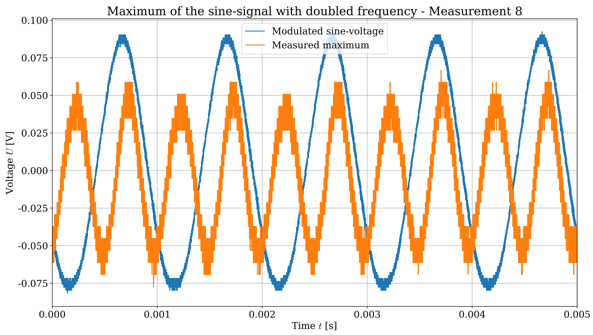


Fig. 45: Maximum of the sine-signal with doubled frequency in measurement 8. Signal in orange, sine-voltage in blue.

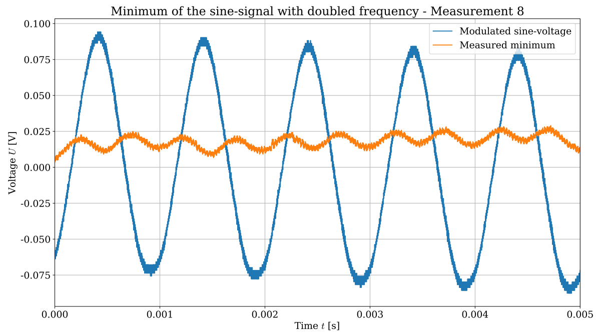


Fig. 46: Minimum of the sine-signal with doubled frequency in measurement 8. Signal in orange, sine-voltage in blue.

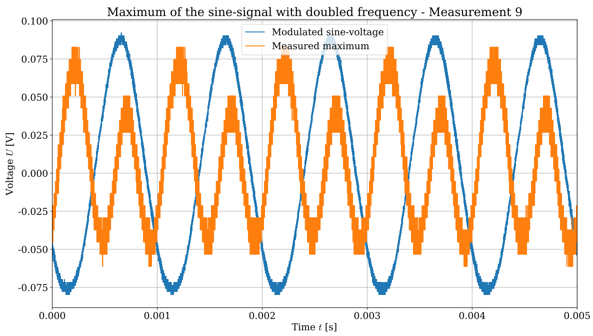


Fig. 47: Maximum of the sine-signal with doubled frequency in measurement 9. Signal in orange, sine-voltage in blue.

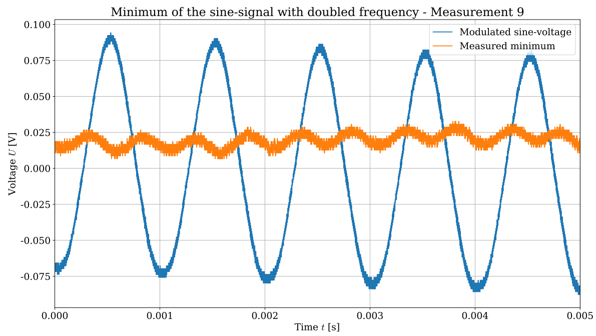


Fig. 48: Minimum of the sine-signal with doubled frequency in measurement 9. Signal in orange, sine-voltage in blue.

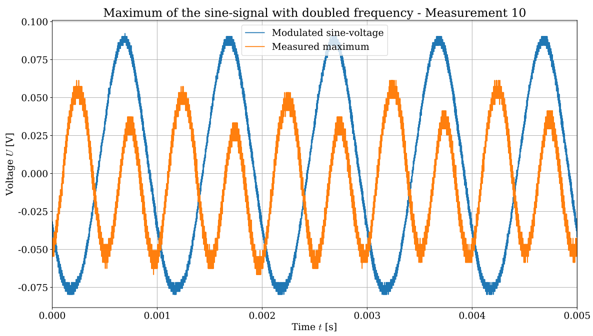


Fig. 49: Maximum of the sine-signal with doubled frequency in measurement 10. Signal in orange, sine-voltage in blue.

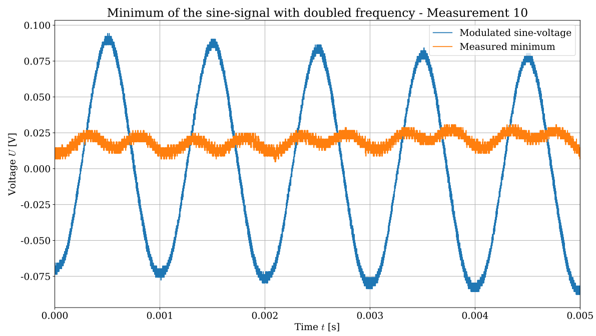


Fig. 50: Minimum of the sine-signal with doubled frequency in measurement 10. Signal in orange, sine-voltage in blue.

7.2 Lab notes

Faraday - Pockels - Effect

1. Amplification / Damping - Factor

=> 3 measurements (csv => PC)

=> Amplitude

CH A : damped

CH B : signal

2.

=> min max of Sine (csv => PC)

=>

$\Delta t =$

measurement	U_{min}	U_{max}	$t_{max} - t_{min}$
1	-1,163 V	3,837 V	14,325 ms
2	-1,003 V	3,837 V	14,1 ms
3	-1,043 V	3,797 V	14,372 ms
4	-1,003 V	3,837 V	14,462 ms
5	-1,003 V	3,837 V	14,25 ms
6	-1,163 V	3,797 V	14,237 ms
7	-1,163 V	3,837 V	14,387 ms
8	-1,163 V	3,837 V	14,387 ms
9	-1,123 V	3,797 V	14,237 ms
10	-1,003 V	3,677 V	14,175 ms

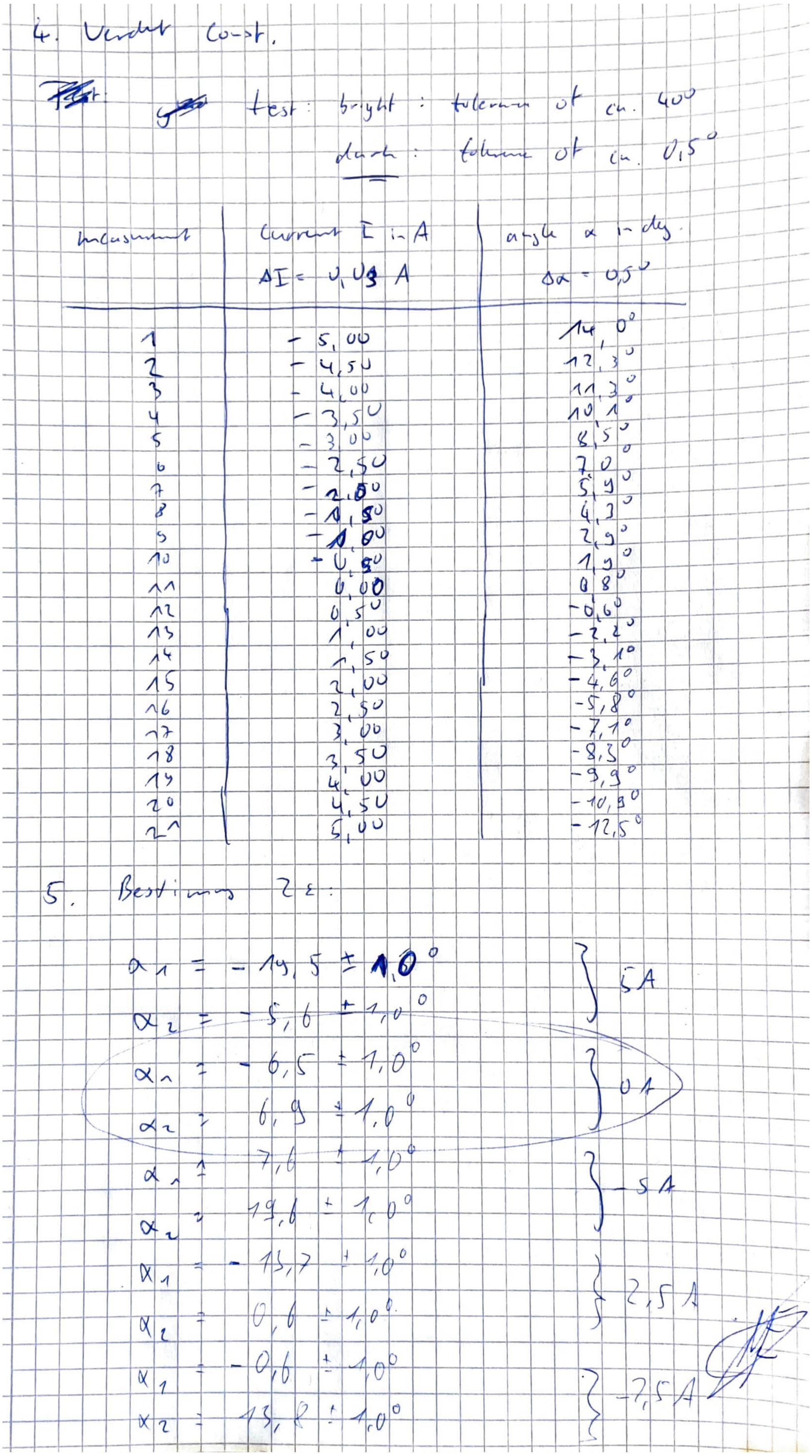
$\Delta U = 0,2\,V$ $\sigma(\Delta t) = 0,3\,ms$

3. modulated Voltage

measurement	U_1	U_2	~~$U_{...2}$~~
1	-114,2 V	137,6	
2	-115,4 V	138,2	
3	-113,3 V	136,9	
4	-115,7 V	136,2	
5	-115,0 V	134,8	
6	-116,0 V	135,1	
7	-115,4 V	135,3	
8	-115,1 V	135,0	
9	-115,5 V	135,1	
10	-115,3 V	134,9	

$\Delta U_1 = 4\,V$ $\Delta U_2 = 2\,V$

Fig. 51: Lab notes - page 1



4. Verdet Const.

~~Test~~: test: bright: tolerance of ca. 40°

dark: tolerance of ca. 0,5°

measurement	Current I in A, $\Delta I = 0,03$ A	angle α in deg, $\Delta\alpha = 0,5°$
1	-5,00	14,0°
2	-4,50	12,3°
3	-4,00	11,3°
4	-3,50	10,1°
5	-3,00	8,5°
6	-2,50	7,0°
7	-2,00	5,9°
8	-1,50	4,3°
9	-1,00	2,9°
10	-0,50	1,9°
11	0,00	0,8°
12	0,50	-0,6°
13	1,00	-2,2°
14	1,50	-3,1°
15	2,00	-4,6°
16	2,50	-5,8°
17	3,00	-7,1°
18	3,50	-8,3°
19	4,00	-9,9°
20	4,50	-10,9°
21	5,00	-12,5°

5. Bestimmung ZE:

$$\left.\begin{array}{l}\alpha_1 = -14,5 \pm 1,0° \\ \alpha_2 = -5,6 \pm 1,0°\end{array}\right\} 5A$$

$$\left.\begin{array}{l}\alpha_1 = -6,5 \pm 1,0° \\ \alpha_2 = 6,9 \pm 1,0°\end{array}\right\} 0A$$

$$\left.\begin{array}{l}\alpha_1 = 7,6 \pm 1,0° \\ \alpha_2 = 19,6 \pm 1,0°\end{array}\right\} -5A$$

$$\left.\begin{array}{l}\alpha_1 = -13,7 \pm 1,0° \\ \alpha_2 = 0,6 \pm 1,0°\end{array}\right\} 2,5A$$

$$\left.\begin{array}{l}\alpha_1 = -0,6 \pm 1,0° \\ \alpha_2 = 13,8 \pm 1,0°\end{array}\right\} -2,5A$$

Fig. 52: Lab notes - page 2

7.3 Python-Code

FarPock

September 23, 2022

[ ]:
```
import numpy as np
import matplotlib.pyplot as m
import glob as glob
from scipy.optimize import curve_fit
from scipy.optimize import fsolve
from scipy.signal import peak_widths
from scipy.special import wofz
from scipy.special import jv
from sklearn.utils import resample
import matplotlib.ticker as ticker
SMALL_SIZE = 16
MEDIUM_SIZE = 20
BIGGER_SIZE = 20
m.rc('font', size=SMALL_SIZE) # controls default text sizes
m.rc('axes', titlesize=MEDIUM_SIZE) # fontsize of the axes title
m.rc('axes', labelsize=SMALL_SIZE) # fontsize of the x and y labels
m.rc('xtick', labelsize=SMALL_SIZE) # fontsize of the tick labels
m.rc('ytick', labelsize=SMALL_SIZE) # fontsize of the tick labels
m.rc('legend', fontsize=SMALL_SIZE) # legend fontsize
m.rc('figure', titlesize=BIGGER_SIZE) # fontsize of the figure title
import matplotlib as mpl
import matplotlib.font_manager as font_manager
mpl.rcParams['font.family']='serif'
cmfont = font_manager.FontProperties(fname=mpl.get_data_path() + '/fonts/ttf/
 ↪,→cmr10.ttf')
#mpl.rcParams['font.serif']=cmfont.get_name()
mpl.rcParams['mathtext.fontset']='cm'
mpl.rcParams['axes.unicode_minus']=False

import matplotlib.pyplot as plt
import numpy as np

def mittel(l: list) -> float:
    return sum(l)/len(l)

def staw(daten: list) -> float:
    mittelw = mittel(daten)
```

1

Fig. 53: Python-Code, Page 1

```
    standard = 0
    for mess in daten:
        standard += (mess-mittelw)**2
    standard = standard/(len(daten)-1)
    standard = standard**(1/2)
    return standard

def stawm(daten: list) -> float:
    return staw(daten)/(len(daten)**(1/2))

def f1(x):
        return x
def f2(x):
        return 1################

def linregnice(x, y, dely, ohne=False):
    if ohne:
        Vyo = np.identity(len(y))
        Mo = []
        for i in x:
            l1o = [f1(i), f2(i)]################
            Mo.append(l1o)
        Mo = np.array(Mo)
        Vpo = np.linalg.inv(np.matmul(np.transpose(Mo), np.matmul(np.linalg.
↪inv(Vyo),Mo)))
        po = np.matmul(Vpo,np.matmul(np.transpose(Mo), np.matmul(np.linalg.
↪inv(Vyo),y)))
        s2 = 1/(len(y)-len(po))*np.sum([(i-po[0]*f1(j)-po[1]*f2(j))**2 for i,j␣
↪in zip(y,x)])###############
        Vy = np.diag(len(y)*[s2])
    M = []
    for i in x:
        l1 = [f1(i), f2(i)]################
        M.append(l1)
    M = np.array(M)
    if ohne==False:
        dely2 = [i**2 for i in dely]
        Vy = np.diag(dely2)
    Vp = np.linalg.inv(np.matmul(np.transpose(M), np.matmul(np.linalg.
↪inv(Vy),M)))
    p = np.matmul(Vp,np.matmul(np.transpose(M), np.matmul(np.linalg.inv(Vy),y)))
    return [p,Vp]

def Modell(xbereich, p):
    return [p[0]*f1(i)+p[1]*f2(i) for i in xbereich]

def Konfidenz(xbereich, p, Vp):
```

2

Fig. 54: Python-Code, Page 2

```
    des = [np.sqrt(np.matmul(np.transpose(np.array([f1(i), f2(i)])),np.
 ↪matmul(Vp,np.array([f1(i), f2(i)])))) for i in xbereich]
    return [[p[0]*f1(i)+p[1]*f2(i)+des[j] for i,j in␣
 ↪zip(xbereich,range(len(xbereich)))],[p[0]*f1(i)+p[1]*f2(i)-des[j] for i,j in␣
 ↪zip(xbereich,range(len(xbereich)))]]

def amittel(x, del_x):
    w = [1/i**2 for i in del_x]
    return sum([i*j for i,j in zip(w,x)])/sum(w)

def astawm(del_x):
    w = [1/i**2 for i in del_x]
    return 1/np.sqrt(sum(w))

def empkor(x,y):
    sx = staw(x)
    sy = staw(y)
    sxy = 1/(len(x)-1)*sum([(i-mittel(x))*(j-mittel(y)) for i,j in zip(x,y)])
    return sxy/(sx*sy)

def chiq(x,y,dely):
    return sum([(j-h(i))**2/delj**2 for i,j,delj in zip(x,y,dely)])

hquer=6.62607015e-34/(2*np.pi) #Js
```

```
[ ]: #fitfunktion
def f(x,a,b,c):
    return a*np.sin(2*np.pi*999*(x+b))+c
```

Dämpfung

```
[ ]: def auslesen(Datei,i,Plot=True):
    with open(Datei, "r") as doc:
        string = doc.read()
        Zeilen = string.split("\n")
        Zeilen.pop(0)
        Zeilen.pop(-1)
        Zeit = []
        ChannelA = []
        ChannelB = []
        for z in Zeilen:
            eintrag = z.split(",")
            Zeit.append(float(eintrag[0]))
            ChannelA.append(float(eintrag[1]))
            ChannelB.append(float(eintrag[2]))

    p1 = curve_fit(f,Zeit,ChannelA)[0]
```

3

Fig. 55: Python-Code, Page 3

x x

```python
    Vp1 = curve_fit(f,Zeit,ChannelA)[1]

    p2 = curve_fit(f,Zeit,ChannelB)[0]
    Vp2 = curve_fit(f,Zeit,ChannelB)[1]

    De=p2[0]/p1[0]
    delDe= np.sqrt(Vp2[0][0]/p1[0]**2+p2[0]**2*Vp1[0][0]/p1[0]**4)


    if Plot==True:

        plt.subplots(figsize=(16,9))

        plt.plot(Zeit,ChannelA,label='Signal with damping', color='blue') #Channal A ist gedämpft
        plt.plot(Zeit,ChannelB,label='Signal', color='grey')
        plt.legend()
        plt.xlim(0,0.005)
        plt.grid()
        plt.title("Damping - Measurement "+str(i)+"")
        plt.xlabel("Time $t$ [s]")
        plt.ylabel("Voltage $U$ [V]")
        plt.savefig("MessungD"+str(i)+".pdf")
        plt.show()

        plt.subplots(figsize=(16,9))

        x = np.linspace(-0.005,0.01,5000)

        plt.plot(Zeit,ChannelB,label='Signal', color='grey')

        plt.plot(x, [f(i,p2[0],p2[1],p2[2]) for i in x], label="Data fitn of the signal", color='red')

        plt.legend()
        plt.grid()
        plt.xlim(0,0.005)
        plt.title("Curve fit of the signal - Measurement "+str(i)+"")
        plt.xlabel("Time $t$ [s]")
        plt.ylabel("Voltage $U$ [V]")
        plt.savefig("linregSig"+str(i)+".pdf")
        plt.show()

        plt.subplots(figsize=(16,9))

        x = np.linspace(-0.005,0.01,5000)
```

4

Fig. 56: Python-Code, Page 4

```
        plt.plot(Zeit,ChannelA,label='Signal with damping', color='blue')␣
↪#Channal A ist gedämpft

        plt.plot(x, [f(i,p1[0],p1[1],p1[2]) for i in x], label="Data fit of the␣
↪damped signal", color = 'red')

        plt.legend()
        plt.grid()
        plt.xlim(0,0.005)
        plt.title("Curve fit of the damped signal  - Measurement "+str(i)+"")
        plt.xlabel("Time $t$ [s]")
        plt.ylabel("Voltage $U$ [V]")
        plt.savefig("linregDamp"+str(i)+".pdf")
        plt.show()

        print(De,delDe,p2[0],np.sqrt(Vp2[0][0]),p1[0],np.sqrt(Vp1[0][0]))

    return De,delDe
```

[ ]:
```
for i in range(1,6):
    auslesen('Damping '+str(i)+'_HM1508.csv',i)
```

[ ]:
```
D=[]
delD=[]

for i in range(1,6):
    D.append(auslesen('Damping '+str(i)+'_HM1508.csv',i,False)[0])
    delD.append(auslesen('Damping '+str(i)+'_HM1508.csv',i,False)[1])

Dmittel=mittel(D)
delDmittel=stawm(D)
Dmittel,delDmittel
```

Sägezahn

[ ]:
```
def Sägezahn(Datei,i,Plot=True):
    with open(Datei, "r") as doc:
        string = doc.read()
        Zeilen = string.split("\n")
        Zeilen.pop(0)
        Zeilen.pop(-1)
        Zeit = []
        ChannelA = []
        ChannelB = []
        for z in Zeilen:
            eintrag = z.split(",")
            Zeit.append(float(eintrag[0]))
```

5

Fig. 57: Python-Code, Page 5

```
            ChannelA.append(float(eintrag[1]))
            ChannelB.append(float(eintrag[2]))

        def f(x,a,b,c,tau):
            return a*np.sin(np.pi*(x+b)/tau)+c

        p1 = curve_fit(f,Zeit[830:1390],ChannelA[830:1390])[0]
        Vp1 = curve_fit(f,Zeit[830:1390],ChannelA[830:1390])[1]

        if Plot==True:
            plt.subplots(figsize=(16,9))

            x = np.linspace(0,0.05,5000)

            plt.plot(Zeit,ChannelA,label='Signal', color='blue') #Channel A ist␣
↪Sinus
            plt.plot(Zeit,ChannelB,label='Sawtooth voltage', color='grey')

            plt.plot(x, [f(i,p1[0],p1[1],p1[2],p1[3]) for i in x], label="Data␣
↪fit of the damped signal", color = 'red')
            plt.axvline(Zeit[830],0,1,label='Limits for the used fit values')
            plt.axvline(Zeit[1390],0,1)

            plt.legend()
            plt.grid()
            plt.xlim(0.02,0.04)
            plt.title('Curve fit of the signal - Measurement '+str(i)+'')
            plt.xlabel("Time $t$ [s]")
            plt.ylabel("Voltage $U$ [V]")
            plt.savefig("Sägezahn"+str(i)+".pdf")
            plt.show()

            print(p1,Vp1)

    return p1[3],np.sqrt(Vp1[3][3])
```

[ ]:
```
for i in range(1,11):
    Sägezahn('Mes '+str(i)+'_HM1508.csv',i)
```

[ ]:
```
tau=[]
deltau=[]

for i in range(1,11):
    tau.append(Sägezahn('Mes '+str(i)+'_HM1508.csv',i,False)[0])
    deltau.append(Sägezahn('Mes '+str(i)+'_HM1508.csv',i,False)[1])

tau,deltau #in s
```

6

Fig. 58: Python-Code, Page 6

```
[ ]: Umin =[1.163,1.003,1.043,1.003,1.003,1.163,1.163,1.163,1.123,1.003]
     Umax =[3.837,3.837,3.797,3.837,3.837,3.797,3.837,3.837,3.797,3.677]
     DeltaU=[i+j for i,j in zip(Umin,Umax)]
     delDeltaU=[np.sqrt(2)*0.2]*len(DeltaU)
     DeltaT=[i*10**(-3) for i in[14.325,14.1,14.312,14.462,14.25,14.237,14.387,14.
      ↪387,14.237,14.175]]
     delDeltaT=[0.0003]*len(DeltaT)
```

```
[ ]: U=[i/j*k*Dmittel for i,j,k in zip(tau,DeltaT,DeltaU)]
     delU=[np.sqrt((deli/j*k*Dmittel)**2+(delj*i/j**2*k*Dmittel)**2+(i/
      ↪j*delk*Dmittel)**2+(i/j*k*delDmittel)**2) for i,j,k,deli,delj,delk in␣
      ↪zip(tau,DeltaT,DeltaU,deltau,delDeltaT,delDeltaU)]
     U,delU
```

```
[ ]: mittelU=mittel(U)
     delmittelU=stawm(U)
     delmittelUsyst=np.sqrt(sum([i**2 for i in delU]))/len(delU)
     mittelU,delmittelU,delmittelUsyst
```

```
[ ]: rlit=23.4e-12 #m/V
     n1=1.522
     n3=1.477
     lam=632.8e-9 #nm
     d=2.4e-3 #mm
     l=20e-3 #mm

     r=lam*d/(mittelU*l*4)*np.sqrt(0.5*(1/n1**2+1/n3**2))**3
     delr=r*delmittelU/mittelU
     delrsyst=r*delmittelUsyst/mittelU
     rrel=delr/r

     r,delr,delrsyst,rrel
```

```
[ ]: tr=(rlit-r)/delr
     trsyst=(rlit-r)/(delr+delrsyst)
     tr,trsyst
```

Modulierter Sinus

```
[ ]: def Sinusmoda(Datei,i):
         with open(Datei, "r") as doc:
             string = doc.read()
             Zeilen = string.split("\n")
             Zeilen.pop(0)
             Zeilen.pop(-1)
             Zeit = []
             ChannelA = []
             ChannelB = []
```


Fig. 59: Python-Code, Page 7

```
        for z in Zeilen:
            eintrag = z.split(",")
            Zeit.append(float(eintrag[0]))
            ChannelA.append(float(eintrag[1]))
            ChannelB.append(float(eintrag[2]))

        plt.subplots(figsize=(16,9))

        plt.plot(Zeit,ChannelA,label='Modulated sine-voltage') #Channel A ist␣
↪Sinus
        plt.plot(Zeit,ChannelB,label='Measured minimum')

        plt.legend()
        plt.grid()
        plt.xlim(0,0.005)
        plt.title('Minimum of the sine-signal with doubled frequency -␣
↪Measurement '+str(i)+'')
        plt.xlabel("Time $t$ [s]")
        plt.ylabel("Voltage $U$ [V]")
        plt.savefig("Minimum"+str(i)+".pdf")
        plt.show()
```

```
[ ]: for i in range(1,11):
         Sinusmoda('u'+str(i)+'a_HM1508.csv',i)
```

```
[ ]: def Sinusmodb(Datei,i):
         with open(Datei, "r") as doc:
             string = doc.read()
             Zeilen = string.split("\n")
             Zeilen.pop(0)
             Zeilen.pop(-1)
             Zeit = []
             ChannelA = []
             ChannelB = []
             for z in Zeilen:
                 eintrag = z.split(",")
                 Zeit.append(float(eintrag[0]))
                 ChannelA.append(float(eintrag[1]))
                 ChannelB.append(float(eintrag[2]))

             plt.subplots(figsize=(16,9))

             plt.plot(Zeit,ChannelA,label='Modulated sine-voltage') #Channel A ist␣
     ↪Sinus
             plt.plot(Zeit,ChannelB,label='Measured maximum')

             plt.legend()
```

8

Fig. 60: Python-Code, Page 8

```
        plt.grid()
        plt.xlim(0,0.005)
        plt.title('Maximum of the sine-signal with doubled frequency -␣
↪Measurement '+str(i)+'')
        plt.xlabel("Time $t$ [s]")
        plt.ylabel("Voltage $U$ [V]")
        plt.savefig("Maximum"+str(i)+".pdf")
        plt.show()
```

```
[ ]: for i in range(1,11):
         Sinusmodb('u'+str(i)+'b_HM1508.csv',i)
```

```
[ ]: Umin2 =[114.2,115.4,113.3,115.7,115.0,116.0,115.4,115.1,115.5,115.3]
     Umax2 =[137.6,138.2,136.9,136.2,134.8,135.1,135.3,135.0,135.1,134.9]
     DeltaU2=[i+j for i,j in zip(Umin2,Umax2)]
     delDeltaU2=[np.sqrt(4**2+2**2)]*len(DeltaU2)

     DeltaU2,delDeltaU2
```

```
[ ]: mittelU2=mittel(DeltaU2)
     delmittelU2=stawm(DeltaU2)
     delmittelU2syst=np.sqrt(sum([i**2 for i in delDeltaU2]))/len(delDeltaU2)
     mittelU2,delmittelU2,delmittelU2syst
```

```
[ ]: r2=lam*d/(mittelU2*l*4)*np.sqrt(0.5*(1/n1**2+1/n3**2))**3
     delr2=r2*delmittelU2/mittelU2
     delr2syst=r2*delmittelU2syst/mittelU2

     r2rel=delr2/r2

     r2,delr2,r2rel
```

```
[ ]: tr2=(rlit-r2)/delr2
     tr2syst=(rlit-r2)/(delr2+delr2syst)
     tr2,tr2syst
```

Faraday

```
[ ]: current = [-5,-4.5,-4,-3.5,-3,-2.5,-2,-1.5,-1,-0.5,0,0.5,1,1.5,2,2.5,3,3.5,4,4.
     ↪5,5]
     delcurrent=[0.03]*len(current)
     angle = [14,12.3,11.3,10.1,8.5,7.0,5.9,4.3,2.9,1.9,0.8,-0.6,-2.2,-3.1,-4.6,-5.
     ↪8,-7.1,-8.3,-9.9,-10.9,-12.5]
     delangle= [0.5]*len(angle)
```

```
[ ]: def f(x,a,b):
             return a*x+b
```

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Fig. 61: Python-Code, Page 9

```python
p1 = curve_fit(f,current,angle,sigma=delangle,absolute_sigma=True)[0]
Vp1 = curve_fit(f,current,angle,sigma=delangle,absolute_sigma=True)[1]


plt.subplots(figsize=(16,9))

x = np.linspace(-10,18.3,5000)
mod1 = Modell(x,p1)
konf1 = Konfidenz(x,p1,Vp1)

plt.plot(x, mod1, label="Linear regression", color="darkolivegreen")
plt.plot(x, konf1[0],linestyle="dashed", color="grey", label="Confidence band")
plt.plot(x, konf1[1],linestyle="dashed", color="grey")
plt.fill_between(x,konf1[0],konf1[1],color="whitesmoke")

plt.errorbar(current,angle, xerr=delcurrent ,yerr=delangle, ecolor="black",␣
 ↪marker="x", ls="", color="blue", markersize=10, capsize=5, label="Measured␣
 ↪data")

plt.legend()
plt.grid()
plt.xlim(-5.2,5.2)
plt.ylim(-20,20)
plt.title("Linear regression for the Faraday-effect")
plt.xlabel("Current $I$ [A]")
plt.ylabel("Angle [°]")
plt.savefig("linregFaraday.pdf")
plt.show()

Steig=p1[0]
delSteig=np.sqrt(Vp1[0][0])

chiq=sum([(j-(p1[0]*i+p1[1]))**2/delj**2 for i,j,delj in␣
 ↪zip(current,angle,delangle)])

p1, Vp1, chiq
```

Verdet mit richtigem Integral

[ ]:
```python
Hl=2554.848

Verdet=-Steig/Hl
delVerdet=delSteig/Hl

Verdet,delVerdet #in degree per Ampere
```

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Fig. 62: Python-Code, Page 10

```
[ ]: umoersted=79.5775 #von A zu Oe
     umwinkel=60 #von deg zu min

     Verdetlit=0.05/umoersted*100/umwinkel #in degree per Ampere
     Verdetlit
```

```
[ ]: tVerdet=(Verdetlit-Verdet)/delVerdet
     tVerdet
```

Verdet mit idealisierter Spule

```
[ ]: Hlideal=3085.714

     Verdetideal=-Steig/Hlideal
     delVerdetideal=delSteig/Hlideal

     Verdetideal,delVerdetideal #in degree per Ampere
```

```
[ ]: tVerdet=(Verdetlit-Verdetideal)/delVerdetideal
     tVerdet
```

2 ϵ-Messung

```
[ ]: a1=[-19.5,-6.5,7.6,-13.7,-0.6]
     a2=[-5.6,6.9,19.6,0.6,13.8]

     epsi=[abs(i-j) for i,j in zip(a1,a2)]
     delepsi=[np.sqrt(2)]*len(epsi)
     mittelepsi=mittel(epsi)
     delmittelepsi=stawm(epsi)

     epsi,delepsi,mittelepsi,delmittelepsi
```

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Fig. 63: Python-Code, Page 11