Contents

1. Tasks

The task of this experiment is to measure the mean lifetime τ for the excited state 6s6p ($^3P_1$) of mercury. To do so we want to measure the Hanle signal at different polarisation angles of the used light dependent on the pressure/temperature to examine the coherence narrowing effect. To do so, we need to perform the following steps:

- Calibrate the magnetic field of the Helmholtz coil pairs to shield the apparatus off of other electrical fields.
- Measure at rising temperatures, the width of the Hanle curve with 0° and 90° polarisation.
- To do so plot the Hanle signal against the applied magnetic field.
- Measure the Hanle signal at 45° as a dispersion signal and evaluate the measurement qualitatively.

2. Theoretical basics

In this chapter most of the equations, pictures and explanations are based on [VeFP]. If not, they are labelled with their corresponding reference.

The mean lifetime τ of the excited 6s6p ($^3P_1$) state of a mercury atom can be measured via the Hanle effect. To do so one needs linear polarised light of a high frequency Hg-lamp, which is send into a resonance cell, with mercury steam inside of it. The Hg atoms get excited by the light hitting them. After the time τ has passed, the atoms fall back into their ground state and send photons out, which is called resonance fluorescence . If we measure the intensity perpendicular to the input direction we will measure very little. If we apply a magnetic field parallel to the input direction of the light, we observe, that with a stronger magnetic filed we will measure a higher intensity. This was first conclusively described by Mr. Hanle in 1924, without the then recently discovered Zeeman effect. The Hanle effect can be explained semi-classical and quantum mechanical.

2.1. Semi-classical explanation

A Hg-atom which has absorbed a photon can be described as a oscillating dipole. It oscillates in the polarisation direction and therefore sends out dipole radiation, until the radiation dampening brings it to a hold. Knowing the dipole emission characteristic, no radiation is emitted in the oscillation axis direction. If we apply a magnetic field parallel to the direction of the light the atom starts to precess with the Larmor frequency

$$\omega_L = \frac{g_J \mu_B}{\hbar} B.$$

The precess trajectory is shown in Figure 1.

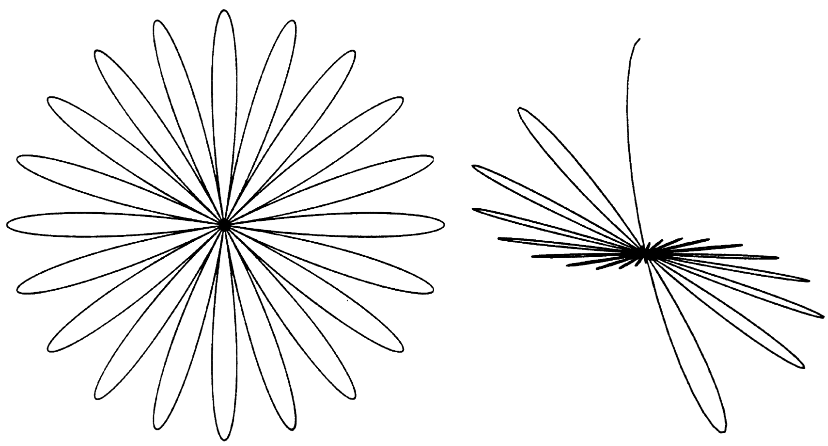

Figure 1: Precess trajectory of a oscillator with weak (left) and strong (right) dampening.

Quantitatively the distribution of the dipole radiation follows $\sin^2 \theta$. If we replace θ with $\omega_L t$ and describe the dampening with $e^{-t/\tau}$, we get

$$I = C \int_0^\infty \sin^2(\omega_L t) e^{-\frac{t}{\tau}} dt,$$

where C is a proportional factor. Evaluated we get a inverse Lorentz distribution.
If the polarisation is not parallel to the observing axis, but shifted off by $\pi/2$ (which we call 0°

polarisation) we get

$$I = C\int_0^\infty \sin^2(\omega_L t + \frac{\pi}{2})e^{-\frac{t}{\tau}}dt = C\int_0^\infty \cos^2(\omega_L t)e^{-\frac{t}{\tau}}dt.$$

Which is a normal Lorentz curve (also shown in Figure 2).

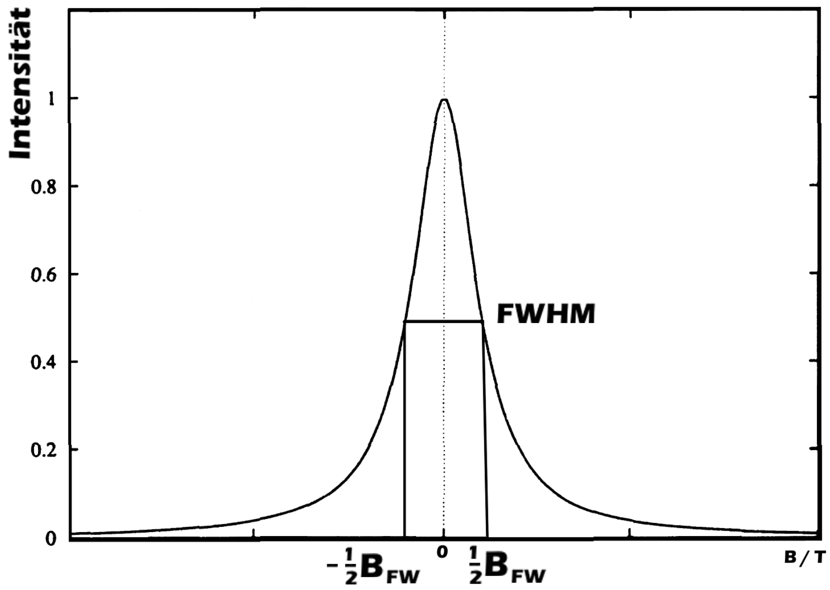

Figure 2: Lorentz curve with 0° polarisation.

For the full width at half maximum ($FWHM$) we get

$$FWHM = \frac{\hbar}{g_J \mu_B \tau},$$

where g_J is the Landè factor and μ_B the Bohr magneton. With the $FWHM$ being B_{FW} for τ we get

$$\tau = \frac{\hbar}{g_J \mu_B B_{FW}}.$$

2.2. Quantum mechanical explanation

The Hanle effect is a special case of the so called level-crossing.
If we have a fine structure splitting up into different Zeeman-levels without applying an external magnetic field, we can make the levels cross, if we apply a magnetic field, meaning that at a certain magnetic field two levels have the same energy. At that point we can observe a largely higher intensity than before.
The special case for the Hanle effect is, that this happens at $B = 0$. The shape of this intensity distribution can be derived quantum mechanical. With certain conditions, which are fulfilled in this experiment and a normal HG-lamp we can use the Breit equation, an equation that describes the rates of absorption and re-emission of photons in our mercury cell, with polarisation f and g.
A system in its ground state a, two crossing states b & c, we can split the rate, with $\Delta\nu(b,c)$ as the frequency difference between the two excited states:

$$R(f,g) = C\left(R_0 + \frac{A}{1-2\pi i\tau\Delta\nu(b,c)} + \frac{A*}{1-2\pi i\tau\Delta\nu(b,c)}\right).$$

For a constant polarisation the rate is proportional to the measured intensity. Therefore for an imaginary A we get a dispersion curve, and for a real one a Lorentz curve.

Having the $^3\mathrm{P}_1$ state, we can have $m_J = -1, 0, 1$. Only the $m_J = \pm 1$ parts are observed. Therefore we get with a real A

$$\Delta\nu = \frac{2 g_J \nu_B}{h} B.$$

For the $FWHM$ of the Lorentz curve we get

$$\Delta\nu = \frac{1}{\pi\tau} = \frac{2 g_J \nu_B}{h} B,$$

and therefore

$$\tau = \frac{\hbar}{g_J \mu_B B_{FW}},$$

which is the same result as the semi-classical.

2.3. Coherence narrowing

The measured mean lifespan of our excited state is dependent on the pressure of our gas, and therefore the temperature in the resonance cell. The cause of this effect is the so-called coherence narrowing. The resonance radiation emitted by an atom is absorbed by another atom. This atom then starts to precess like the first atom and the sending out resonance radiation with the same energy, phase relation and spatial orientation as the absorbed one. Therefore the measured mean lifespan is longer than it actually is. With a higher temperature/pressure this time gets even longer, because it is more likely that the radiation will get absorbed by more atoms before being measured.
To compensate this effect we need to measure at different temperatures and then extrapolate to $T = 0\,\mathrm{K}$, meaning $p = 0\,\mathrm{Pa}$ to get the real value, because we then would have only one atom in our resonance cell.

3. Setup and procedure

Setup

The experiment setup is shown in Figure 3. A high-frequency gas discharge in our mercury steam lamp gives us the ultraviolet light we need. Through a setup of lenses it gets parallelized and focused, with an interference filter its wavelength selected and with a polarizer polarised. It then travels to our mercury probe, which is surrounded by three Helmholtz pair coils to shield it from electro-magnetic fields around it and cooled by a Peltier cooler, which is outside of our magnets and connected to our mercury via heat pipes. Also outside of the magnetic coil pairs there is a photomultiplier perpendicular to the travel axis of our light. All this is powered and connected to our rack shown in Figure 4.

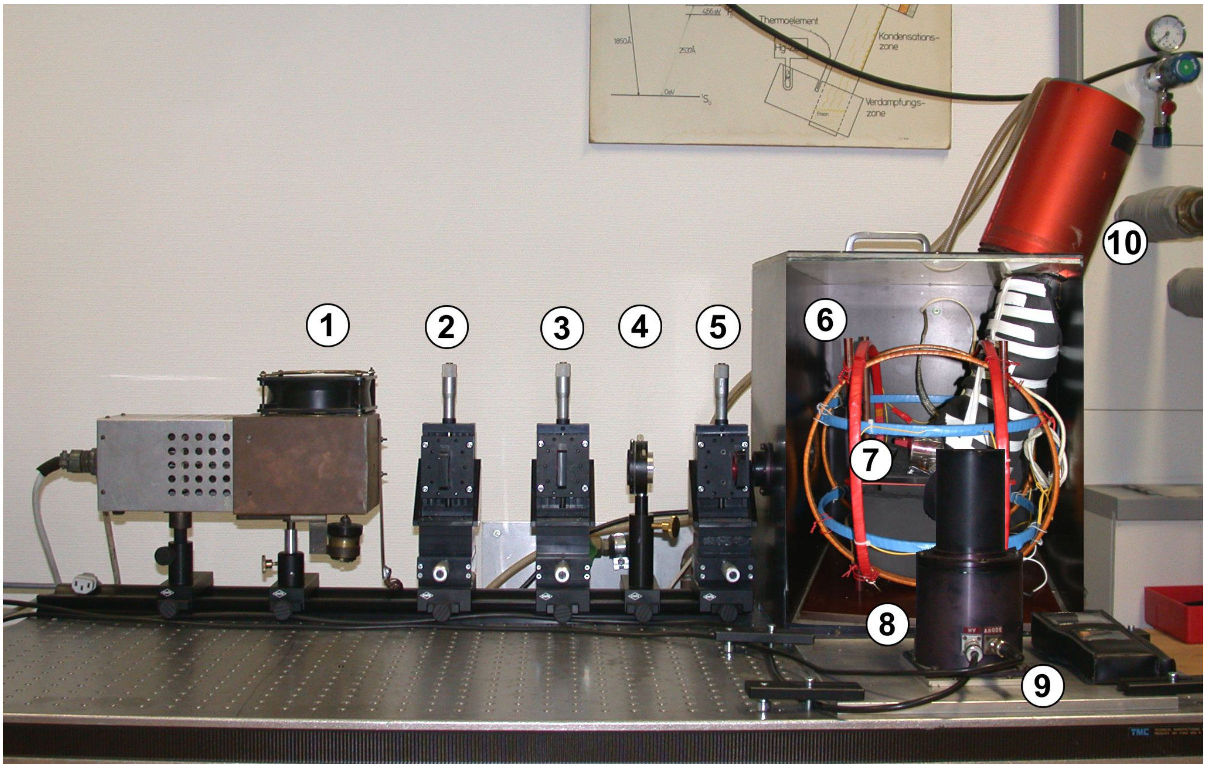

Figure 3: Setup of the experiment from [VeFP, chap. 3.2, p.10].

1) Hg lamp 2) and 5) lenses 3) interference filter 4) polarizer 6) Helmholtz coils 7) Hg 8) photomultiplier 9) thermometer 10) Peltier cooler.

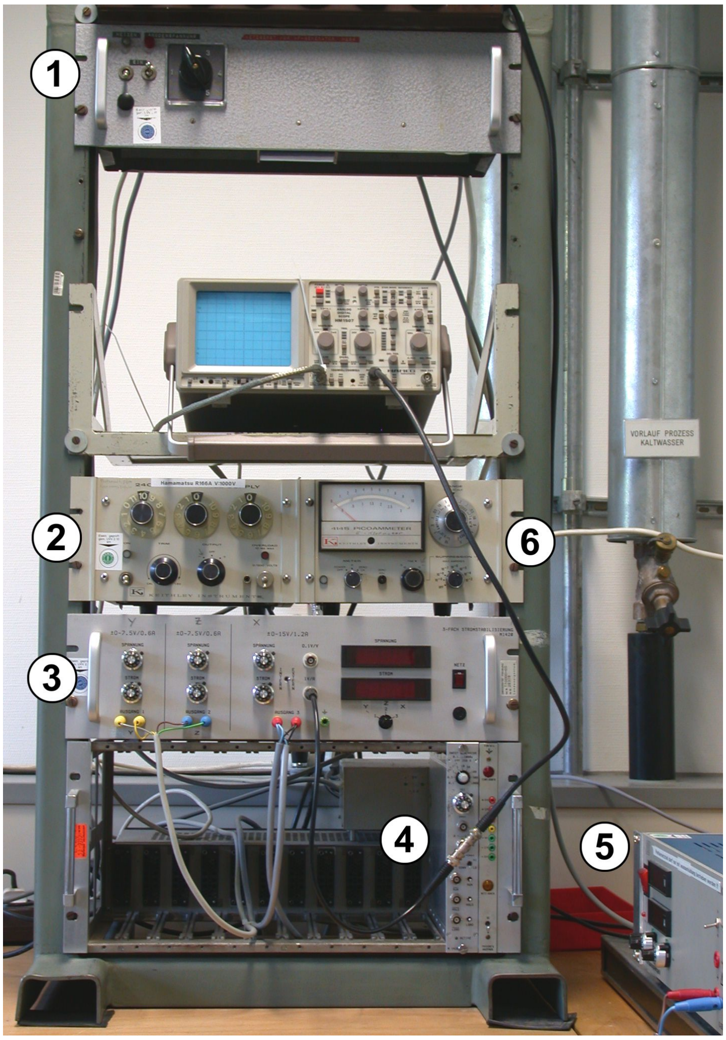

Figure 4: The used electronic components installed in a rack from [VeFP, chap. 3.2, p.11].

1) power supply unit for the Hg lamp 2) photomultiplier-HV 3) power supply unit for the Helmholtz coil pairs 4) ramp generator 5) power supply unit for the Peltier cooler 6) amplifier for the photomultiplier

Procedure

First of all we had to start the cooling process. To do so, we needed to open the pipes for the house water supply. After that we started the passive cooling component. When it reached about 14 °C we turned on all the other components shown in Figure 4. For the Peltier cooler we set the current to 7.39 A. We then had to wait half an hour for the system to cool down to −18 °C. We then proceeded with the adjusting of the magnetic fields. For this we followed the manual in [VeFP, chap. 5.3, p.14]. As a result we had a good looking symmetrical signal. We then started our measurement for 0°, 45° and 90°. Then we turned the Peltier cooling off so the temperature could rise again. Finally we measured for 0° and 90° together. To do so, we turned the polarizer between each measurement. We stopped measuring when the thermometer reached 10 °C.

4. Data analysis

The aim of this experiment is, to determine the lifetime of $^3\mathrm{P}_1$. For this, we measured the Hanle-curves. In this chapter the data we gained will be analysed. The programs we used for this were written in python. The formulas, except for those with which we calculated the errors and made the linear fits, are from [VeFP].

After trying out, how we could get the most symmetric distributions of the photons, we decided, to mark the 5° position as 0° and the 95° as our measurement point for 90°. In the following we will refere to this points as 0° and 90°.

Since we need the full width at half maximum in an intensity-magnetic-flux diagram, we calculated the magnetic flux out of the current of the magnet. For this we used the formula

$$B = I 3.363 \cdot 10^{-4}\,\frac{\mathrm{T}}{\mathrm{A}}.$$

By starting with this calculation, we haven't to take this constant into account for most of the following calculations. After that we plotted the amplitude of the magnetic field against the time. These steps were repeated for each measurement at each temperature. As an example Figure 5 shows the fit for an angle of 0° and a temperature of 1 °C.

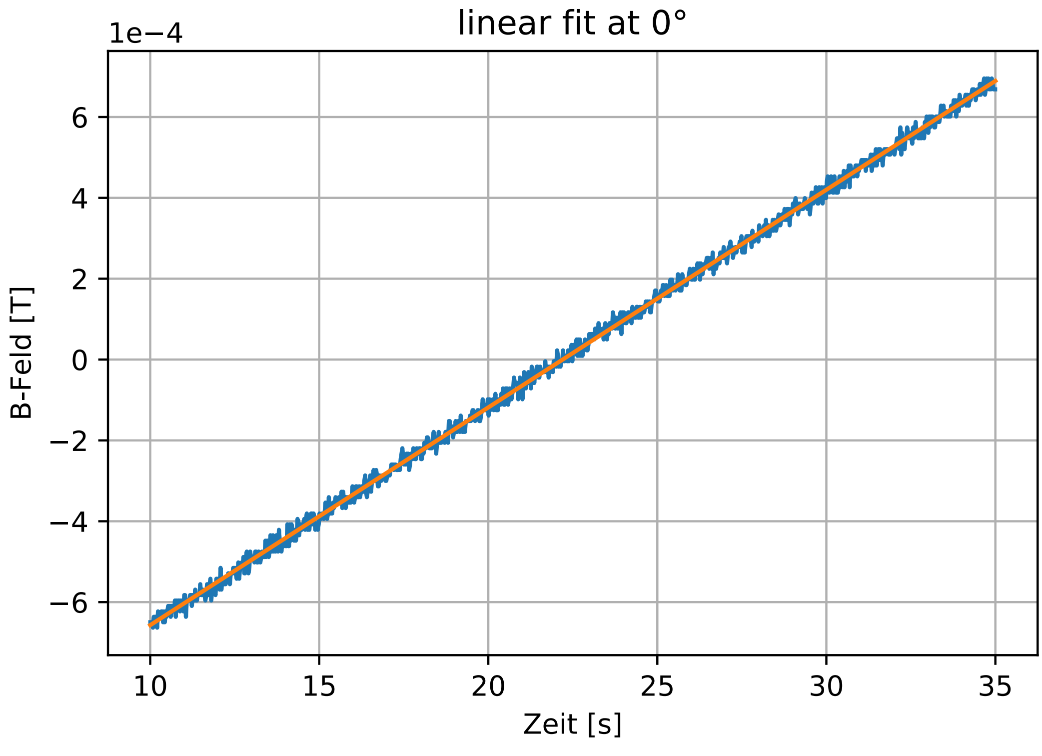

Figure 5: Linear fit to the magnetic field at 0° and −18 °C.

The fit-function in this case is

$$a \cdot x + b,$$

with $a = (5.385 \pm 0.006) \cdot 10^{-5}\,\mathrm{T/s}$ and $b = (-11.95 \pm 0.02) \cdot 10^{-4}\,\mathrm{T}$. With this fit-function we gain a x-axis, over which we plot the measured data. The example, that is shown in Figure 6 has the fit from Figure 5 as x-axis.

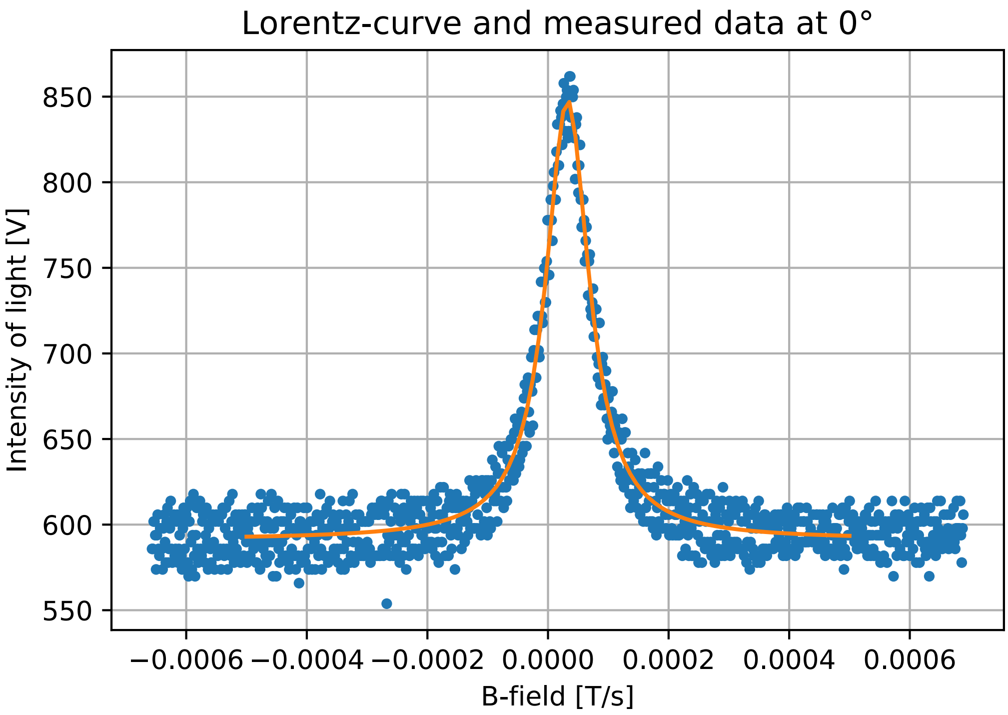

Figure 6: Lorentz fit for 0° and −18 °C.

The function fitted to these Data is

$$I = \frac{C\tau}{2}\left(2 - \frac{(2\omega_L\tau)^2}{1+(2\omega_L\tau)^2}\right), \tag{4.1}$$

with the Larmor frequency

$$\omega_L = g_J \frac{\mu_B}{\hbar} B,$$

with the Landé factor $g_J = 1.4838$ [VeFP] and the magneton of Bohr $\mu_B = 9.274009 \cdot 10^{-24}\,\frac{\mathrm{J}}{\mathrm{T}}$ [VeFP]. We also included a term, that moves the peak of the fit in x-direction and added a term, that takes the offset into account. With the help of the fit parameters one gets the full width at half maximum B_{FW}. With this we can calculate the live time of the state 6s6p

$$\tau = \frac{\hbar}{g_J \mu_B B_{FW}}.$$

The error on τ was calculated, using Gaussian error propagation, as

$$s_\tau = \tau \cdot \frac{s_{B_{FW}}}{B_{FW}}.$$

where we estimated the error for B_{FW} as $s_{B_{FW}} = 0.3 \cdot 10^{-5}\,\mathrm{T}$.

For our second set of data, which was measured at an angle of 90°, all the formulas are the same, except for Equation 4.1. Instead of that, we use the formula

$$I = \frac{C\tau}{2}\left(\frac{(2\omega_L\tau)^2}{1+(2\omega_L\tau)^2}\right).$$

The plots we get here look similar to Figure 7.

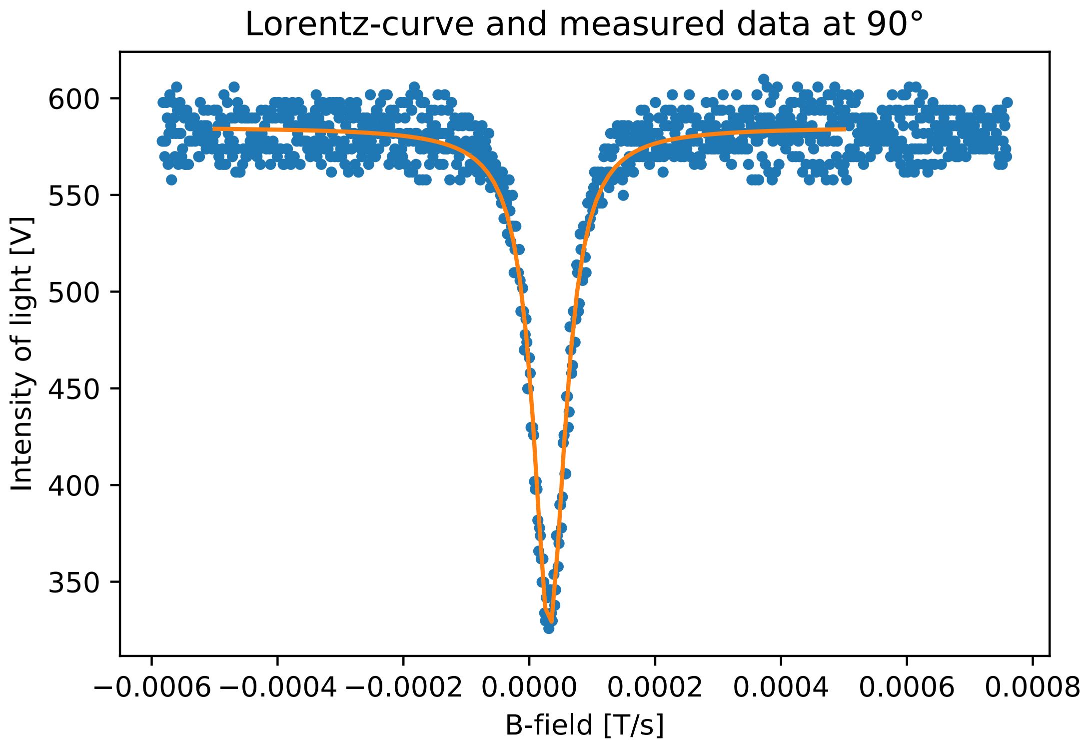

Figure 7: Lorentz fit for 90° and −18 °C.

The life times we got out of these calculations are listed in Table 1 in the appendix.

In order to calculate the live time at zero pressure we plot this data against the pressure. This is necessary in order to reduce the coherence narrowing. For being able to make this plot, we first have to calculate the pressure from the temperature. This we did with the formula

$$\ln\left(\frac{p}{p_c}\right) = \left(\frac{T_c}{T}\right)\left(a_1 T_r + a_2 T_r^{1.89} + a_3 T_r^2 + a_3 T^8 + a_5 T_r^{8.5} + a_6 T_r^9\right)$$

where we used $T_r = 1 - T/T_c$ and the values

$$\begin{array}{lll} a_1 = -4.57618368 & a_2 = -1.40726277 & a_3 = 2.36263541 \\ a_4 = -31.0889985 & a_5 = 58.0183959 & a_6 = -27.6304546. \end{array}$$

from [VeFP]. The error for p can be calculated by Gauss:

$$\begin{aligned} s_p = p \cdot s_t \Bigg(& -\frac{T_c}{T^2}\left(a_1 T_r + a_2 T_r^{1.89} + a_3 Tr^2 + a_4 T_r^8 + a_5 T_r^{8.5} + a_6 Tr^9\right) \\ & + \frac{T_c}{T}\left(\frac{a_1}{T_c} + a_2 \cdot 1.89 \cdot \frac{Tr^{0.89}}{T_c} + 2 \cdot a_3 \frac{T_r}{T_c} + 8 \cdot a_4 \frac{T_r^7}{T_c} + 8.5 \cdot a_5 \frac{T_r^{7.5}}{T_c} + 9 \cdot a_6 \frac{T_r^8}{T_c}\right)\Bigg) \end{aligned}$$

Here we have neglected the errors of the values $a_1, \ldots, a_6$, because, they were, compared to the error on the time neglectable. For the error on the time, we chose $s_T = 1\,°\mathrm{C}$, because the thermometer didn't seem to be very precise. The resulting plot for 0° can be seen in Figure 8 and that for 90° in Figure 9.

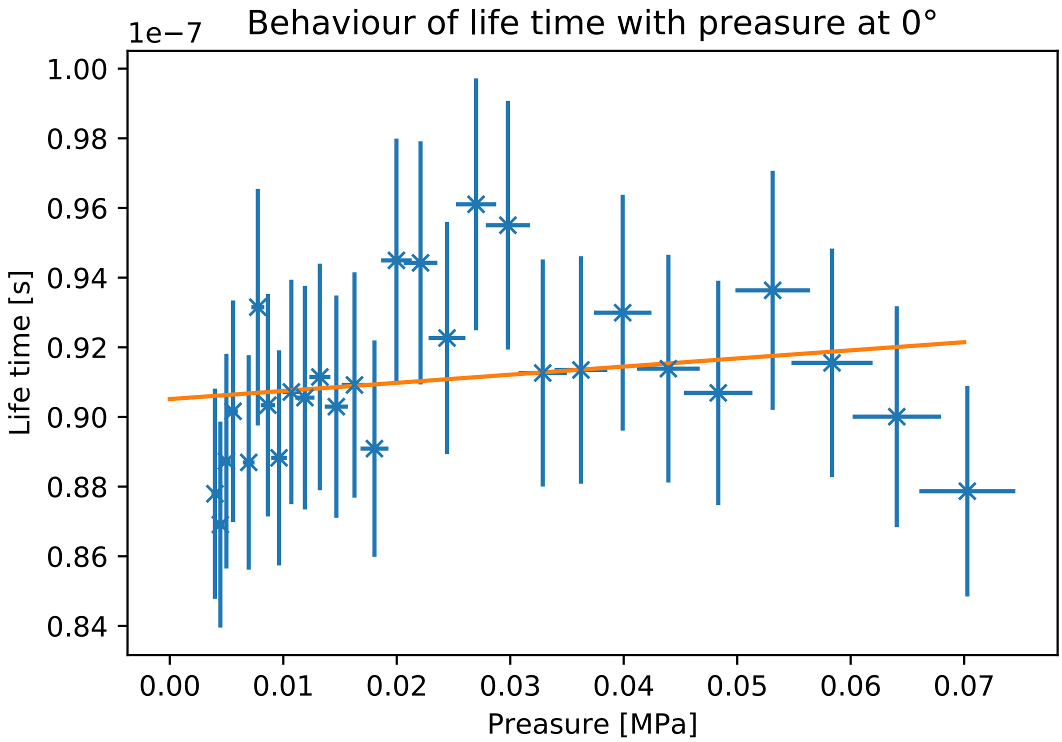

Figure 8: Linear fit for data at 0°.

The fit function has a y-axis-section of $y = (9.05 \pm 0.07) \cdot 10^{-8}$. This corresponds with the life time of the $^3\mathrm{P}_1$ state. So the live time we got here is

$$\boxed{\tau_0 = (0.905 \pm 0.007) \cdot 10^{-7}\,\mathrm{s}.}$$

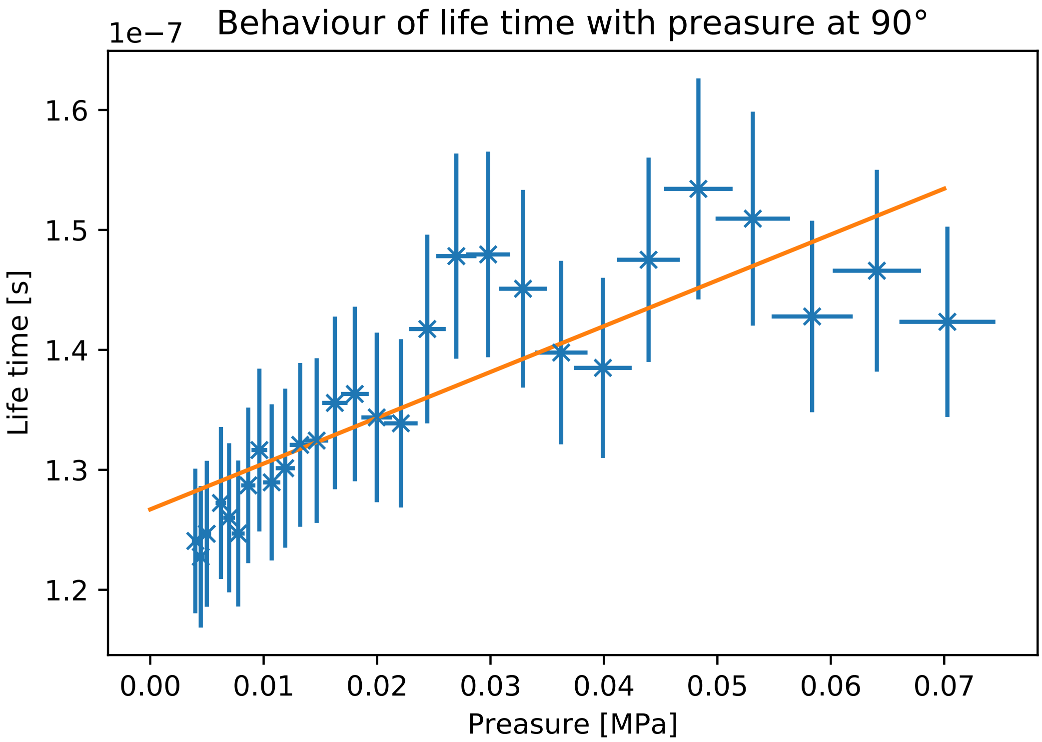

Figure 9: Linear fit for data at 90°.

This time the y-axis-section is $y = (1.27 \pm 0.02) \cdot 10^{-7}$ with a resulting life time of

$$\boxed{\tau_{90} = (1.27 \pm 0.02) \cdot 10^{-7}\,\text{s}.}$$

Finally we plotted one measurement at 45° and $-18\,°\text{C}$. This is shown in Figure 10.

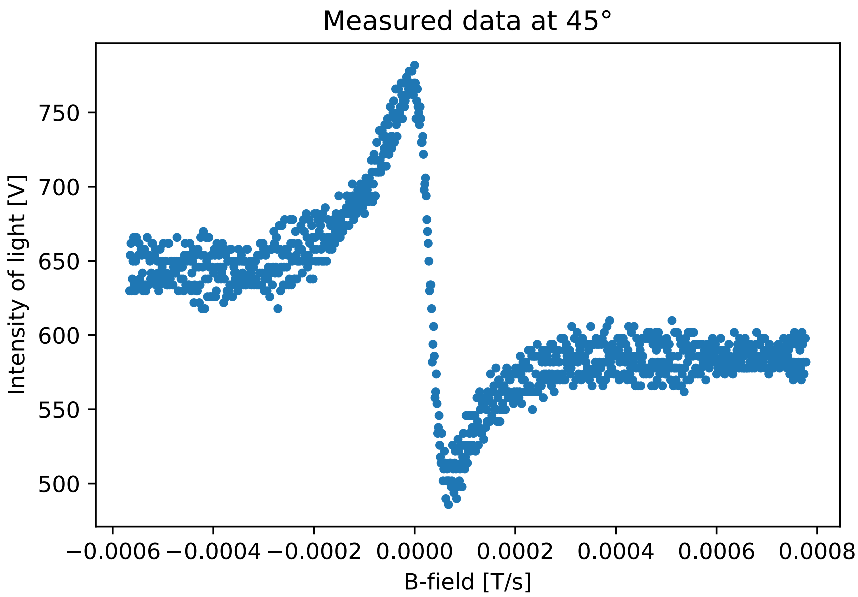

Figure 10: Measurement at $-18\,°\text{C}$ and 45°.

By comparing it with Figure 11, on can see, that this plot corresponds with the black one.

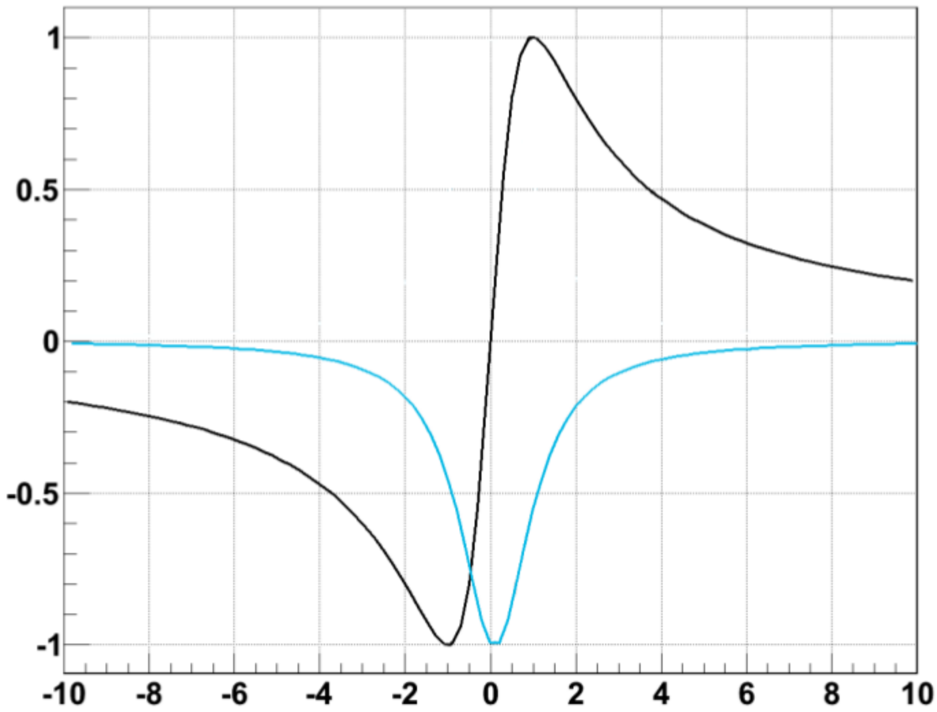

Figure 11: Lorentz plot and dispersion curve [VeFP].

This is the derivation of the Lorentz function, which we fitted to the measurements at 0° and 90°. This function is

$$\frac{d}{d(2\omega_L\tau)}I = C'\left(\frac{(2\omega_L\tau)^2}{(1+(2\omega_L\tau)^2)^2}\right).$$

The fact, that our plot is, compared to Figure 11, mirrored on the y-axis, is simply a matter of defining the sign of C'.

5. Summary and discussion

The aim of this experiment was, to determine the life time of the transition of the state 6s6p to $6s^2$. For this we measured the number of emitted photons at different temperatures. From this we got the life times, which we plotted against the pressure, which we got from the temperature. We fitted a linear function through that data, and extrapolated it to the y-axis intercept, at which point there would be no pressure, which allows to overcome the coherence narrowing effect. The resulting life times are, for 0°

$$\tau_0 = (0.905 \pm 0.007) \cdot 10^{-7}\,\mathrm{s},$$

and for 90°

$$\tau_{90} = (1.27 \pm 0.02) \cdot 10^{-7}\,\mathrm{s}.$$

The literature value from [VeFP] is $\tau_{\mathrm{lit}} = 1.19 \cdot 10^{-7}\,\mathrm{s}$. It can be seen easily, that τ_0 is not very close to this value, to be precise, the difference is $\sigma_0 = 41$. The difference of τ_{90} is, with $\sigma_{90} = 4$, much smaller. In Figure 8 it can be seen, that a much better result for the measurement at 0° could have been achieved, if only the measurements over a pressure of $p = 0.03\,\mathrm{MPa}$ would have been taken into account for the fit. But the linear fit in Figure 9 doesn't indicate, that there is a systematic error for the measurement at low temperatures. One possible reason, why the data of the measurement at 0° is better for higher temperatures is, that the jalousie was lowered, which would reduce the number of possible errors from the light in the room, but Figure 9 gives worse data for the higher temperatures. Therefore this can't be the cause for the deviation. The data in Figure 8 doesn't fit to the linear function very well. A polynomial of the second degree would fit much better, although this contradicts the theoretical prediction completely. The reason, why our measurement at 90° is still 4σ away from the literature value, might be, that the temperature, which was shown by the thermometer wasn't the true temperature of the mercury. The reason for this is, that the thermometer isn't connected directly with the mercury but with a copper block, whose temperature changes more quickly. Therefore our actual y-axis interception would be a bit more to the left in Figure 9, and therefore we would get a smaller mean life time τ closer to the literature value.

We also plotted a measurement at 45°. Its behave was very similar to the theoretical function, which is the derivation of the Lorentz-distribution, which we used in order to fit the data of the measurements at 90°.

A. List of Figures

B. List of Tables

C. References

[VeFP] M.Köhli, *Versuchsanleitung Fortgeschrittenen Praktikum Der Hanle Effekt*, Physikalisches Institut, Fakultät für Mathematik und Physik, Albert-Ludwigs-Universität Freiburg

[Has] W.D. Hasenclever, *Bau einer Aperatur zur Messung von Lebensaduern angeregter Atomzustände mit Hilfe des Hanle-Effekts*

D. Appendix

D.1. Life times

Temperature [°C]	τ [10^{-8} s] for 0°	τ [10^{-7} s] for 90°
-18	8.8 ± 0.3	1.2 ± 0.6
-17	8.7 ± 0.3	1.2 ± 0.6
-16	8.9 ± 0.3	1.3 ± 0.6
-15	9.0 ± 0.3	-
-14	-	1.3 ± 0.6
-13	8.9 ± 0.3	1.3 ± 0.6
-12	9.3 ± 0.3	1.2 ± 0.6
-11	9.0 ± 0.3	1.3 ± 0.6
-10	8.9 ± 0.3	1.3 ± 0.7
-9	9.1 ± 0.3	1.3 ± 0.7
-8	9.1 ± 0.3	1.3 ± 0.7
-7	9.1 ± 0.3	1.3 ± 0.7
-6	9.0 ± 0.3	1.3 ± 0.7
-5	9.1 ± 0.3	1.4 ± 0.7
-4	8.9 ± 0.3	1.4 ± 0.7
-3	9.5 ± 0.4	1.3 ± 0.7
-2	9.4 ± 0.4	1.3 ± 0.7
-1	9.2 ± 0.3	1.4 ± 0.8
0	9.6 ± 0.4	1.5 ± 0.9
1	9.6 ± 0.4	1.5 ± 0.9
2	9.1 ± 0.3	1.5 ± 0.8
3	9.1 ± 0.3	1.4 ± 0.8
4	9.3 ± 0.3	1.4 ± 0.8
5	9.1 ± 0.3	1.5 ± 0.9
6	9.1 ± 0.3	1.5 ± 0.9
7	9.4 ± 0.3	1.5 ± 0.9
8	9.2 ± 0.3	1.4 ± 0.8
9	9.0 ± 0.3	1.5 ± 0.8
10	8.8 ± 0.3	1.4 ± 0.8

Table 1: Lifetimes for 0° and 90° at different temperatures.

D.2. Original data

Versuch: Hanle 13.9.18

Photomultiplier: $U = 1000\,V \pm 1$
$I = 0{,}45\,mA$

Peltierelement: $U = 15{,}4\,V$
$I = 7{,}38\,A$

0° Winkel ist bei 5°

Spulen: $U = -2{,}426\,V$
$I = -0{,}2737\,A$

Messung	Temperatur [°C]	Winkel [°]		
~~1~~	~~-18~~	~~95~~		
~~2~~	~~-18~~	~~5~~		
1	-18	5		
2	-18	95		
3	-18	50		
4	-17	5		
5	-17	95		
6	-16	95		
7	-16	5		
8	-15	5		
9	-14	95		
10	-13	5		
11	-13	95		
12	-12	95		
13	-12	5	← y-pos-div	-6,0
14	-11	5		-6,4
15	-11	95		
16	-10	95		
17	-10	5	← y-pos-div	
18	-9	5		-7,51
19	-9	95		
20	-8	95		
21	-8	5	← y-pos-div	
22	-7	5		-7,95
23	-7	95		
24	-6	95		
25	-6	5	← y-pos-div	
26	-5	5		-8,33
27	-5	95	← y-pos-div	
28	-4	95		-8,67
29	-4	5	←	
30	-3	5		-9,07
31	-3	95	←	
32	-2	95		-9,74
33	-2	5		
34	-1	5		
35	-1	95		
36	0	95		
37	0	5	←	
38	1	5		-10,63
39	1	95		

Messung	Temperatur	Winkel
40	2	95
41	2	5
42	3	5
43	3	95
44	4	95
45	4	5
46	5	5
47	5	95
48	6	95
49	6	5
50	7	5
51	7	95
52	8	95
53	8	5
54	9	5
55	9	95
56	10	95
57	10	5

14.09.2018

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