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2. Measure the core magnetic moment of the $^{19}F-core$ in teflon.
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4. Measure the gyromagnetic ratio of the proton in glycol.
5. Measure the proton resonance frequency of the hydrogen probation with the lock-in method.

# 2 Theory

## 2.1 Spin

The spin is a attribute of elementary particle and can take integral or half-integral values. Neutrons, electrons and protons have spin $\frac{1}{2}$. Also atomic nuclei, which build up of divers particles, can be attributed to spin. This spin is the sum of the single spins of the divers particles. For the norm of this spin $\mathbf{I}$ is given by:

$$|\mathbf{I}| = \hbar\sqrt{I(I+1)}, \tag{1}$$

where I is the core quantum number. According to the rules of the quantum mechanics is the core spin is quantized to the direction. This means he can take only discrete values to a fixed axis:

$$I_z = \hbar m_I, \tag{2}$$

here is $m_I$ the magnetic quantum number, which also can be integral or half-integral. The possible values for $m_I$ are $-I, -I+1, ..., I-1, I$. So the only possible values for protons or the $^{19}F-core$, with $I = \frac{1}{2}$, are $-\frac{1}{2}\, or\, \frac{1}{2}$. Which means that the spin is parallel or antiparallel to the magnetic flied.

## 2.2 Magnetic moment

If we interpret the spin as a intrinsic angular momentum, a circle electricity is be generated, which induced a magnetic dipole moment. For this effect we get the equation:

$$\mu_I = \gamma\mathbf{I} = \frac{g_I \mu_K}{\hbar}\mathbf{I} \tag{3}$$

$\gamma$ is the gyromagnetic ratio, $g_I$ is the specific core-g-factor, which we will determine in this experiment, and $\mu_K$ is the nuclear magneton. The magneton is defined by:

$$\mu_K = \frac{e\hbar}{2m_p} \tag{4}$$

If we combine equation 2 and 3 we get:

$$\mu_z = (mu_I)_z = \gamma\mathbf{I}_z = \gamma\hbar m_I = g_I \mu_K m_I \tag{5}$$

## 2.3 Spin and magnetic field

The potential energy of a magnetic moment in a magnetic field is given by:

$$E = -\mu B \tag{6}$$

For a magnetic field $B = (0,0,B)$ follows for the potential energy of the magnetic core moment:

$$E = -\mu_z B = g_I \mu_K m_I B \tag{7}$$

A System with a magnetic quantum number $\pm\frac{1}{2}$ has the energetic differential:

$$\Delta E = g_I \mu_K B \tag{8}$$

So this is the energy, which must put into the system to switch the core spin.

## 2.4 Resonance

The core spin can be switched by electromagnetic radiation. The frequency of the radiation must be equal to energetic differential between the two settings for this effect. In this case we speak from resonance. If we use the energy of the radiation $E = h\nu$ and the equation 8, we get:

$$\nu = \frac{\Delta E}{h} = \frac{g_I \mu_K B}{h} \tag{9}$$

Here we can see that the frequency for resonance is proportional to the magnetic field. The radiation can generate two different effects. The spin flip to the energetic lower or higher state. In the first case the energy will released and in the second one it will be absorbed. If there is no difference between the number of cores in this two states, we can't measured this effect. In our case of atomic nuclei the number of cores in the stats will follow the Boltzmann distribution:

$$\frac{N_{higher}}{N_{lower}} = g_I e^{\frac{-\Delta E}{k_B T}} \tag{10}$$

So there will be absorbed more energy than released and we can measured the effect.

## 2.5 Disexcitation

Without any disexcitation the states of our system will be balanced soon and our measurement will be end. Disexcitations are processes, which flip our spin into the lower state without any radiation.

### Spin-lattice disexcitation

Spin-lattice disexcitation means that the energy, which release during the spin flip, will be converted to lattice vibration. The lattice vibration are equal to heat. So the energy is set out to environment without any radiation.

### Spin-spin disexcitaion

Spin-spin disexcitation follow from the interaction between the single spins. This induce basically a enlargement of the adsorption line. The explanation is simple, the core spin induce a magnetic field, so that the rest of the cores are no longer only in the outer magnetic field. The outer magnetic field will be increased and decreased by this, which mean that the resonance frequency of the cores will be split in higher or lower frequency. So the different cores will have different frequency for the spin flip, which induce the enlargement of lines.

### chemical shift

If we look a little bit closer to the resonance frequency of core, we will find that the frequency for a fixed core isn't totally constant. The frequency is dependent on the chemical bond of the atoms. But this effect isn't very strong, so that we can disregard it in our experiment.

## 2.6 hall-effect-sensor

Magnetic field strength can be measured with the hall-effect-sensor. The hall-effect describes the influence form a outer magnetic field to a live conductor. The charge carrier experience the Lorentz force by the magnetic field, and form an unbalance in the conductor. The result is a voltage, which are formed in the conductor. This voltage is named hall-voltage $U_{hall}$. The electric force, which result from this voltage, act against the Lorentz force. After some time there will be a balance between this two forces:

$$F_L = F_E \Leftrightarrow evB = eE \tag{11}$$

After some forming we get for the hall voltage:

$$U_{hall} = \frac{BI}{neb} \tag{12}$$

From this equation follow, that our hall-effect-sensor is more exact for a lower number of charge carrier. That's the reason why we take a semiconductor in our experiment.

## 2.7 Lock-in

The Lock-in method is a technique to get a better signal-noise-relation. We trigger the signal on a fixed frequency, which we will put into the synchronous detector as an reference signal. Then the amplifier will only increase signals with the same frequency. In our experiment we will modulate our magnetic field with a sinus function, so we go through our resonance frequency in periodic intervals. In the amplifier the signal from our measurement and the reference signal will be multiply, so we get a signal with double frequency and an amplitude, which proportional to our signal from the measurement. For this method it is important that our two signals has the same phase. In the end we will integrate our signal over a fixed time to became a signal with a lower disruption.

# 3 Set up

The central point of our experiment is an electric magnet, which is made of with two inductors and a iron core. For the field modulation we use two more inductors with lower energy. For the first set up(Fig. 1) we switching off this inductors. In the second set up we electric driven this inductors with a impulse generator, to get a sinus modulation (Fig. 2). The set up for the last exercise, where we use the lock-in method, is show in Fig. 3.

images/aufbau1.png

Figure 1: Set up for exercise 1
Source: [1]

images/aufbau2.png

Figure 2: Set up for exercise 2-4
Source: [1]

images/aufbau3.png

Figure 3: Set up for exercise 5
Source: [1]

# 4 Analysis

## 4.1 Measurement of the magnetic field with the Hall-probe

The magnetic field which was recorded by the Hall-probe, is plotted at image 4. When the Hall-probe was completely between the magnet, we get a constant value of the magnetic field with $488 \pm 2mT$. The position of the probe could be read fairly exact, so we chose our reading error of $s_x = 0.2mm$. Based on this measurement we chose a depth of $x = 17mm$ for the samples in our other measurements.

images/hall_probe.png

Figure 4: The position of the Hall-probe plotted against the measured magnetic field

## 4.2 Determining the nuclear magnetic moment of $^{19}$F-core of Teflon

Now we compute the nuclear magnetic moment of the $^{19}$F-core. We use the magnetic field which was determine with the Hall-probe with $(0.444 \pm 0.0002T)$. The frequency $\nu$ is our measured value, $\mu_k$ and $h$ are constants.

$$\nu = (17.9 \pm 0.4) MHz \tag{13}$$

$$\mu_k = 5.05 \cdot 10^{-27} J/T \tag{14}$$

$$h = 6.626 \cdot 10^{-34} Js \tag{15}$$

With the equations (9) we get:

$$g_K = \frac{\nu h}{\mu_K B} = 5.27 \pm 0.12 \tag{16}$$

$$\Rightarrow \gamma = \frac{g_K \mu_K}{\hbar} = (2.53 \pm 0.06) \cdot 10^8 T^{-1} s^{-1} \tag{17}$$

We get the error of error propagation with:

$$s_{g_K} = \sqrt{\left(\frac{h}{\mu_K B}\right)^2 s_\nu^2 + \left(-\frac{h\nu}{\mu_K B^2}\right)^2 s_B^2} \quad (18)$$

With equation (5) we get the nuclear magnetic moment:

$$|\vec{\mu}| = \mu_z = \frac{1}{2}\gamma\hbar = (1.33 \pm 0.03) \cdot 10^-26\frac{J}{T} \quad (19)$$

## 4.3 Determining the gyromagnetic ratio of the hydrogen proton

For the hydrogen sample we calculate the gyromagnetic ratio $\gamma$, of the proton. Our magnetic field was $B = (0.444 \pm 0.0002)T$. First we compute $g_K$ similar to the Teflon sample. With the measured frequency $\nu$ we get $g_K$:

$$\nu = (19.0 \pm 0.4)MHz \quad (20)$$

$$g_K = 5.61 \pm 0.12 \quad (21)$$

Now we can calculate $\gamma$ with:

$$\gamma = \frac{g_K \mu_K}{\hbar} = (2.69 \pm 0.06) \cdot 10^8 T^{-1} s^{-1} \quad (22)$$

With the error computed by:

$$s_\gamma = \frac{\mu_K \cdot s_{g_K}}{\hbar} \quad (23)$$

The literature value is $\gamma = 2.675 \cdot 10^8 T^{-1} s^{-1}$. So the difference of our value is just in one standard deviation from the literature value.

## 4.4 Determining the gyromagnetic ratio of the glycol proton

Now we compute the gyromagnetic ratio $\gamma$ also for the glycol proton. We had a magnetic field of $B = (0.439 \pm 0.002)T$ and with the measured frequency $\nu$ we get our $g_K$ again:

$$\nu = (18.7 \pm 0.4)MHz \quad (24)$$

$$g_K = 5.59 \pm 0.12 \quad (25)$$

With the same equations as in 4.3 we get:

$$\gamma = (2.68 \pm 0.06) \cdot 10^8 T^{-1} s^{-1} \quad (26)$$

When we compare this value with our $\gamma$ for the hydrogen proton we can see that is very similar to them. Again, $\gamma$ is just one standard deviation from the literature value.

## 4.5 Determining the gyromagnetic ratio of hydrogen with lock-in method

Now we read out the time difference between the zero crossing point of the sawtooth and the one of the differentiated signals. To get the point exactly of the sawtooth we use the middle of the thick line and use the zero crossing of that fitted line. You can see an example of the whole picture in image 5.

images/bild.png

Figure 5: Red - Differentiated Signal ; Black - Sawtooth Signal

Then we plotted the frequencies of the zero crossing against the time differences, see figure(6) and fitted this with a straight line, which has been expected. With this fit we can find the resonance frequency, this is the point which the time difference is zero. So we get the frequency:

$$\nu = (18.120 \pm 0.013) \tag{27}$$

Now, with the used magnetic field $B = (0.440 \pm 0.002)T$ and compute the gyromagnetic ratio:

$$\gamma = (2.59 \pm 0.06) \cdot 10^8 T^{-1} s^{-1} \tag{28}$$

images/lock_in_new.png

Figure 6: $p0$ = y-intercept; $p1$ = slope of the line

We can compare this result with the value in (4.3) and see, that our $\gamma$ now is smaller then in (4.3), but still close to the literature value (just a gap of two standard deviations).

# 5 Results

In our first measurement we get a value for the magnetic field

$$B = (488 \pm 2)mT$$

but we measured the field for every exercise again, because it changes the whole time. We use this measurement to determine the depth for a constant magnetic field. For the several samples we get following values:

| Sample | $\gamma$ |
|---|---|
| $^{19}$F Teflon | $(2.53 \pm 0.06) \cdot 10^8 T^{-1} s^{-1}$ |
| Hydrogen | $(2.68 \pm 0.06) \cdot 10^8 T^{-1} s^{-1}$ |
| Glycol | $(2.68 \pm 0.06) \cdot 10^8 T^{-1} s^{-1}$ |
| Hydrogen (lock-in) | $(2.59 \pm 0.06) \cdot 10^8 T^{-1} s^{-1}$ |

The literature values for hydrogen and glycol is $\gamma = 2.675 \cdot 10^8 T^{-1} s^{-1}$ and $\gamma = 2.518 \cdot 10^8 T^{-1} s^{-1}$ for the $^{19}$F in Teflon. So our results have a difference of one standard deviation from the literature value, except hydrogen with the lock-in method, there are a difference of two standard deviations.

# References


[1] A.Ortner;M.Kohli;K.Köneke. Versuchsanleitung. http://wwwhep.physik.uni-freiburg.de, 2013.