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1 Experimental overview

This experiment gives a rough overview of the principle of nuclear magnetic resonance. It is split up into five single parts.

1. The magnetic field is measured with a hall probe and its homogeneity is to be confirmed.
2. A teflon (^{19}F) sample is inserted in the magnetic field and by finding the resonance frequency the nuclear magnetic moment will be determined.
3. This part of the experiment is about the gyromagnetic ratio of the proton in a ^1H sample. Which will be compute by the given magnetic field and the resonance frequency which has to be found experimentally in this task.
4. With the same two quantities like in task 3 the gyromagnetic ratio of the protons in glycol will be examined.
5. In this last task the proton resonance frequency of hydrogen will be obtained from the ^1H sample by the lock-in method.

2 Theoretical background

In the following chapter, all the equations and pictures used, can be found in [1]

2.1 Spin

In this experiment we will encounter protons and neutrons which form the nuclei inside atoms. Both are fermions and as we know from the lecture they have an intrinsic spin \vec{I} which is given by the combination of quarks inside of those fermions. Protons and neutrons both have a spin with the value of $\frac{1}{2}$. The quantum numbers for the nuclear spin are I and m_I . I determines the absolute value of the spin vector, whereas m_I represents the projection on the z-axis.

$$|\vec{I}| = \hbar\sqrt{I(I+1)} \quad (1)$$

This means, that if we observe for example the nucleus spin in z-direction, this becomes

$$I_z = \hbar m_I \quad (2)$$

Where the possible values for m_I are given by $-I, -I+1, \dots, 0, \dots, I-1, I$, there are $2I+1$ possibilities for the spin projections.

2.2 Magnetic moment

Due to the quark composition of the nucleons (protons, neutrons) which form a nucleus, a magnetic dipole moment is induced, which leads us to the following equation:

$$|\vec{\mu}_I| = \gamma \vec{I} = \frac{g_I \mu_k}{\hbar} \vec{I} \quad (3)$$

where γ is the gyromagnetic ratio given by

$$\gamma = \frac{g_I \mu_K}{\hbar} = \frac{2\pi \nu_{res}}{B_z} \quad (4)$$

and $\mu_k = \frac{e\hbar}{2m_p c} = 5.05078 \cdot 10^{-27} \frac{J}{T}$ [2] and g_I is a proportionality factor, called g-factor. With the equations 2 and 3 we obtain the following equation for the magnetic dipole moment in z-direction:

$$\mu_z = \gamma I_z = \gamma \hbar m_I = g_I \mu_k m_I \quad (5)$$

2.3 Spins and their behaviour in a magnetic field

When a magnetic moment is exposed to a magnetic field the potential energy is given by:

$$E_{pot} = -\vec{\mu} \cdot \vec{B} \quad (6)$$

so for the z- direction we get with equation 5:

$$E_{pot} = -g_I \mu_k m_I B_z \quad (7)$$

Given the selection rule for quantum mechanical states and the information that all the probes in the experiment have a spin of $I = \frac{1}{2}$, $\Delta m_I = \pm 1$ the energy change has to be:

$$|\Delta E| = g_I \mu_k B_z \quad (8)$$

Therefore photon of this energy will be needed to bring the nucleus into the higher energy state. But it is important to point out that nuclei with anti-parallel spin towards the B-field have a higher potential energy than the ones with parallel spin.

2.4 Resonance

Transitions between different energy levels can be induced by electromagnetic radiation when an incoming photon has got the same frequency as the resonance frequency of the sample. This frequency results from the known equation:

$$E = h\nu \quad (9)$$

So, if we now take equations 5,8 and 9 we get the frequency which we must apply in order for a resonance to occur.

$$\nu_{res} = \frac{\Delta E}{h} = \frac{g_I \mu_k B_z}{h} \quad (10)$$

If the radiation has got the frequency above we can use the Zeeman effect. This means that the nucleus absorbs a quantised unit of energy (a photon) and "flips" to a state with more potential energy. On the other hand, it is possible that spins flip back from the higher energy state to a lower one, by emitting a energy back as radiation. This process is called induced absorption/emission. An excited nucleus can also emit a photon by spontaneous emission while dropping back into the lower energy state. In thermal equilibrium the occupation number of a given energy in a state is Boltzmann distributed, which means that the ratio between the occupation numbers of two different states is given by:

$$\frac{n_{high}}{n_{low}} = e^{-\frac{\Delta E}{k_B T}} \quad (11)$$

Where k_B is the Boltzmann constant and T is the temperature in kelvin. Having this in mind it is obvious that more particles are in the lower state than in the excited one. From which we can conclude that there is more radiation absorbed by the sample than the probe emits back to the radiation field.

2.5 Relaxation processes

There are more ways for an excited particle to relax. The most important ones in this experiment are the following:

1. Spin-lattice relaxation

This process is occurring, when a lattice is exposed to a magnetic field and anti-parallel spin pairs flip back to their more relaxed state, they give their energy to the so called lattice field (a field induced by the rotation and vibration of the many nuclei) in order to restore the state of equilibrium. This recovers in an exponential behaviour. During this process there is no photon emitted.

2. Spin-spin relaxation

The nuclear magnetic moment of a nucleus induces a magnetic field at the location of another nucleus. Which then experiences a slightly higher or lower magnetic field than the one applied from the outside. This leads to the effect that absorption lines appear a bit wider on the screen than expected, because the resonance frequency is changed by the slightly different magnetic field. Which results together with the spin-lattice relaxation in a damped harmonic oscillator.

3. Chemical shifting

The resonance frequency of a nucleus is shifted by the molecule structure around it. Because the molecule binding disturbs the magnetic field and so the resonance frequency is changed as well.

2.6 Hall effect

This effect is used in the experiment, by the Tesla meter, which we use to measure the magnetic field with a so called Hall-probe.

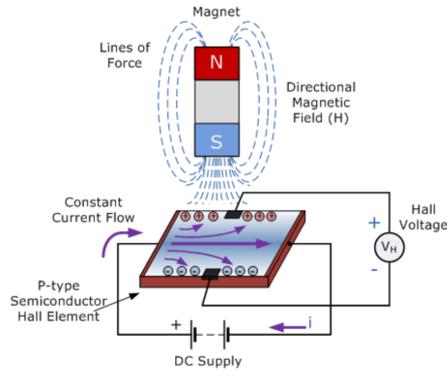


Figure 1: This picture shows the effect of the Hall-probe. [3]

Basically the Hall-probe is a current-carrying conductor which induces a electric field as soon as it is exposed to a magnetic field. The electrons in the Hall-probe are drifting with a velocity of \vec{v} when exposed to a magnetic field, they are pushed aside due to the Lorentz force and cause an electric field inside the conductor, similar to a capacitor:

$$F_L = e(\vec{v} \times \vec{B}) = q \cdot \vec{E} = \vec{F}_E$$

The corresponding Hall Voltage is the following:

$$U_{hall} = vBd \quad (12)$$

Where d is the thickness of the Hall-probe. If we substitute the current intensity equation $I = nevBd$, into equation 12:

$$U_H = \frac{I}{ne}$$

With the equations above it is easy to calculate the magnetic field strength by the induced Voltage.

2.7 Measuring methods

Three different measuring methods will be used in this experiment.

2.7.1 Measuring the magnetic field

For the determination of the magnetic field strength the Hall-probe has to be put inside of a non-modulating magnetic field. Bit by bit while pulling it back out of the sample hole we can verify, by measuring the field strength with the coupled tesla-meter, the homogeneity of the field.

2.7.2 Absorption method and sine modulation

As we know from the previous text and equation 10 the resonance frequency ν_{res} strongly depends on the magnetic field strength. So to find ν_{res} we modulate the magnetic field by a sin wave which increases and decreases the magnetic field periodically by a relatively small amount. That is very helpful to find the right frequency because even if the magnetic field is static, the sine wave scans a small range around it. That means, that per period of the sin-wave the resonance frequency is hit twice. This is not true only in three cases: Firstly when the B-field modulation is not sufficient enough to scan-pass the searched frequency. Secondly the resonance frequency is exactly at the maximum of the sine and the of course when it is at the minimum. If now those absorption-minima are equidistant, we can examine the correlation between the resonance frequency and the B-field, because then the resonance is at the zeroes of the sin modulation where the B-field is known from the Hall-probe measurement.

2.7.3 Lock-in method

The lock-in method is a measuring technique, which amplifies very small signals underneath a large amount of noise. In the so called synchronous detector, the input signal from the radiation field (NMR-signal) is multiplied by a phase-shifted reference signal of the B-field modulation. Instead of a simple sin-wave modulation the lock-in method makes use of an additional sawtooth function

$$U = U_{st} + U_0 \sin(\omega_{input}t)$$

The two signals (NMR-signal and reference-signal ($U_{ref} = U_0 \sin(\omega_{ref}t)$)) are then integrated by a low-pass-filter and amplified before it becomes visible on the oscilloscope. The sine-wave and the added sawtooth will give the magnetic field a modulation in the form of a slowly rising sin-wave function. The absorption curve $A(U)$ can then be written as a taylor series, if the sine is small in comparison to the sawtooth.

$$A(U(t)) \approx A(U_{st}) + \frac{dA}{dU} U_0 \sin(\omega_{input}t)$$

With the mathematical principle of orthogonality also every signal without the right phase shift and frequency will be filtered. Due to the following integral it becomes clear that the sines where $\omega_{input} \neq \omega_{ref}$ are getting cancelled out:

$$\int_{-\infty}^{\infty} \sin(\omega_{input}t) \cdot \sin(\omega_{ref}t) dt = \delta(\omega_{input} - \omega_{ref})$$

With the resulting differentiated absorption curve, we can now simply measure the

time interval between the points where the NMR-signal and the sawtooth-sine-signal pass zero and plot those as a function of the adjusted frequencies. To get the resonance frequency we use the linear dependence of the B-field and the resonance frequency (equation 10), to calculate the y-axis-intersection.

3 Experimental setup

The main part of the experiment is an electromagnet with a small hole for samples in between. This electromagnet is connected to a power supply unit, which makes it possible to change the voltage and thus the magnetic field strength.

For the measurement of the magnetic field the setup is then completed by a Teslameter with an attached Hall-probe cf. Figure 2

For the measurement with the absorption method a NMR-Oscillator, generates high frequencies of roughly 19 MHz. Right next to it the sine modulation is sent to small modulation-coils between the magnetic poles. With an Oscilloscope the NMR-signal as well as the sine-modulation wave is made visible. cf. Figure 2

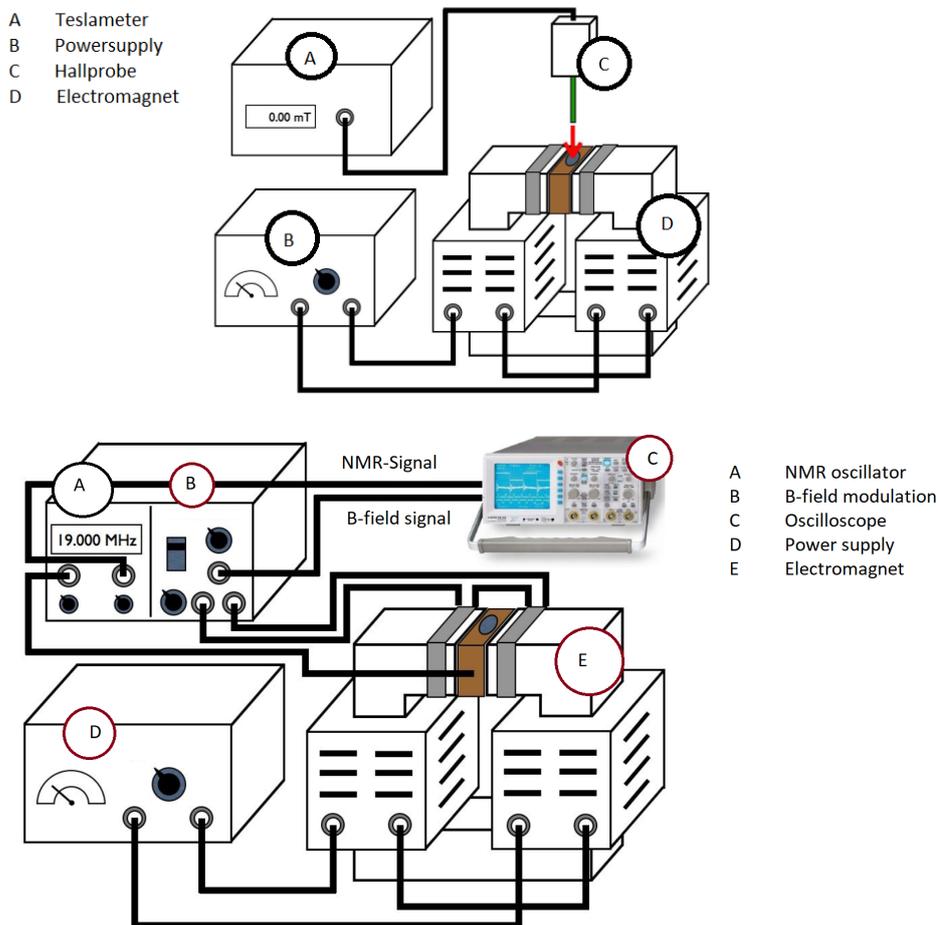


Figure 2: On the top, the setup for the measurement (2.7.1) on the bottom the setup for the second part (2.7.2)

Lock-in Method In the lock-in method setup the setup of the absorption method is complemented by the already introduced synchronous detector (see 2.7.3) and a

machine which is able to generate the sawtooth-, sin-signal, and also to add them up.
 The combined signal then goes to the oscilloscope, see the picture below for details:

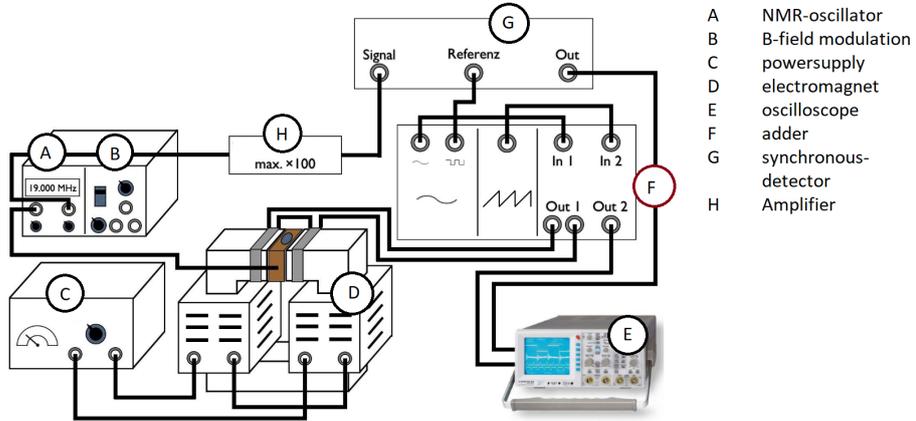


Figure 3: Experimental setup for the lock-in method

4 Data analysis

The uncertainty in the measurements used in section 4.2 strongly depends on the uncertainty in determining that the peaks are equidistant. The display of the frequency generator showed a systematical error by "jumping" at various frequencies. We considered this systematic error by measuring the minimum and maximum frequencies ν_{min} , ν_{max} of subjectively still equidistant peaks (considering the jumps in frequencies) and by following the gaussian error propagation (adding the squared uncertainties) of the systematical and the statistical uncertainties.

In these cases we will use the newer data for our analysis.

4.1 Measurement of the magnetic field

The measurement of the magnetic field depending on the vertical position of the Hall-probe shows a good homogeneity in the range between $x = 10\text{mm}$ and $x = 30\text{mm}$. The data is shown in fig. 4. We estimated the B-field with the Hall probe as $B = (472 \pm 0.3)\text{T}$. Where the error of the B-field is given by: $\frac{0.5\text{T}}{\sqrt{3}}$.

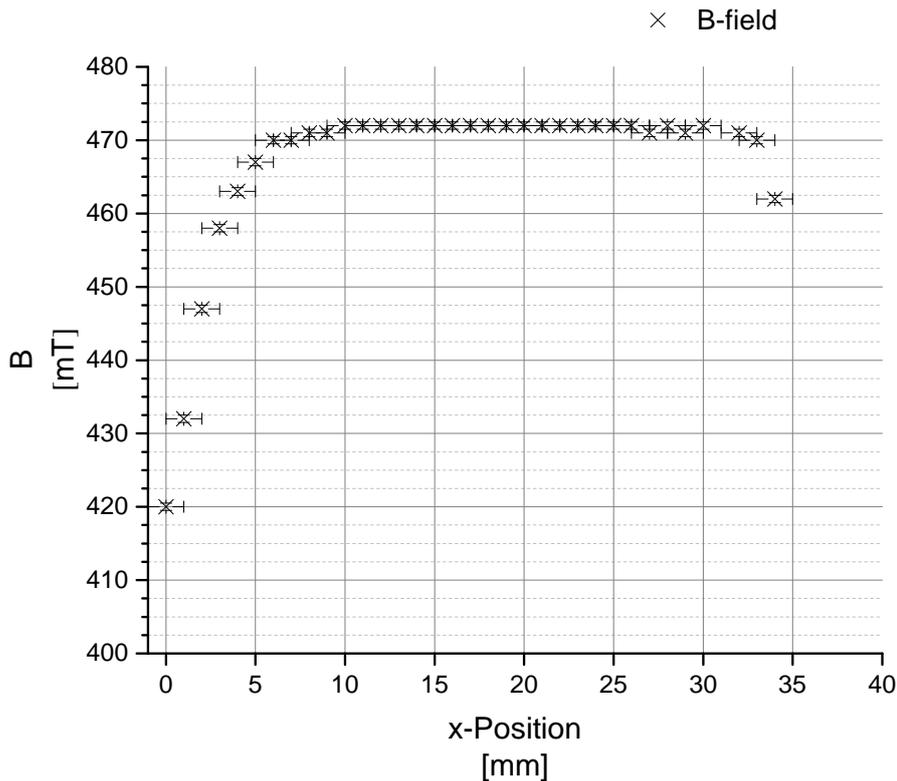


Figure 4: The B-field as a function of the x-position from the hall-probe inside the magnet

4.2 Measurements using a sine modulation

In this section we calculate the nuclear g-factor g_I and the gyromagnetic ratio γ of ^{19}F (sections 4.2.1 and 4.2.2) and ^1H (sections 4.2.3 and 4.2.4) respectively, according to equations 10 and 4:

$$g_I = \frac{h\nu}{\mu_K B}$$

$$\gamma = \frac{2\pi\nu}{B}$$

We also calculate the z-component of the nuclear magnetic moment μ_z of the respective nucleus, according to equation 5, in units of the nuclear magneton:

$$\mu_z = g_I \mu_K m_I$$

Besides, we calculate the respective $\frac{\nu}{B}$ ratio to be able to compare the accuracy of the ^1H /Glycol measurements to the Lock-in-method. The error propagation leads to:

$$\Delta_{g_I} = g_I \sqrt{\left(\frac{\Delta_\nu}{\nu}\right)^2 + \left(\frac{\Delta_B}{B}\right)^2} \quad (13)$$

$$\Delta_\gamma = \gamma \sqrt{\left(\frac{\Delta_\nu}{\nu}\right)^2 + \left(\frac{\Delta_B}{B}\right)^2} \quad (14)$$

$$\Delta_{\mu_z/\mu_K} = m_I \Delta_{g_I} \quad (15)$$

$$\Delta_{\nu/B} = \frac{\nu}{B} \sqrt{\left(\frac{\Delta_\nu}{\nu}\right)^2 + \left(\frac{\Delta_B}{B}\right)^2} \quad (16)$$

$$\Delta_\nu = \frac{\nu_{max} - \nu_{min}}{2} \quad (17)$$

We combined the systematical and statistical errors by adding the squares and taking the square root of the sum.

4.2.1 ^{19}F sample

For the ^{19}F -sample we found that the distance between two peaks was equidistant for a frequency of $\nu = (19.37 \pm 0.03)\text{MHz}$ with a magnetic of $B = (428 \pm 0.3)\text{mT}$. We also estimated an error for the equidistance with a maximum and minimum frequency which we determined with equation 17 and the following values:

$$\nu_{min} = 19.3484\text{MHz}$$

$$\nu_{max} = 19.4001\text{MHz}$$



Figure 5: The captured picture from the oscilloscope for the ^{19}F -sample

The results are the following:

$$\frac{\nu}{B} = (40.18 \pm 0.12)\text{MHz} \cdot \text{T}^{-1}$$

$$g_I = 5.272 \pm 0.015$$

$$\gamma = (25.25 \pm 0.07) \cdot 10^7 \text{rad} \cdot \text{s}^{-1} \text{T}^{-1}$$

$$\frac{\mu_z}{\mu_K} = 2.636 \pm 0.008$$

4.2.2 Teflon sample

The teflon sample was also measured with a magnetic field of $B = (428 \pm 0.3) \text{mT}$ and with a frequency of $\nu = (19.34 \pm 0.02)\text{MHz}$. The extrema for the error estimation were found also with equation 17 and the following values:

$$\nu_{min} = 19.3181\text{MHz}$$

$$\nu_{max} = 19.3592\text{MHz}$$

With this we found the following results

$$\frac{\nu}{B} = (40.12 \pm 0.09)\text{MHz} \cdot \text{T}^{-1}$$

$$g_I = (5.264 \pm 0.012)$$

$$\gamma = (25.21 \pm 0.06) \cdot 10^7 \text{rad} \cdot \text{s}^{-1} \text{T}^{-1}$$

$$\frac{\mu_z}{\mu_K} = 2.632 \pm 0.006$$

4.2.3 Glycol sample

For the glycol sample we had to adjust the magnetic field to a strength of $B=(452 \pm 0.3)$ mT. We found equidistancy at a frequency of $\nu = (19.31 \pm 0.02)$ MHz the maximum and minimum for the frequency could not be measured due to an interference inside of the HF-generator. The frequencies were likely to jump around and although the peaks always moved in one direction, the resulting minimum frequency was always higher than the one previously set. So we just estimated in comparison to the ones before an error of $\Delta\nu = 0.02$ MHz

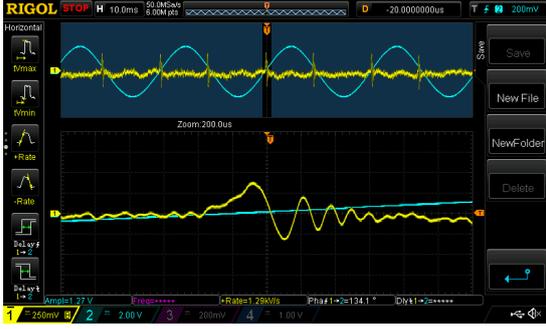


Figure 6: The captured picture from the oscilloscope for the Glycol sample

The results are the following:

$$\frac{\nu}{B} = (42.71 \pm 0.06)\text{MHz} \cdot \text{T}^{-1}$$

$$g_I = (5.603 \pm 0.008)$$

$$\gamma = (26.84 \pm 0.04) \cdot 10^7 \text{rad} \cdot \text{s}^{-1} \text{T}^{-1}$$

$$\frac{\mu_z}{\mu_K} = 2.802 \pm 0.004$$

4.2.4 ^1H sample

For the ^1H sample we found with a magnetic field of $B=(452 \pm 0.3)$ mT a frequency of $\nu = (19.34 \pm 0.02)$ MHz. In Figure 7 the combined effect of spin-spin relaxation (ssr) and spin-lattice relaxation (slr) is good visible. We have already talked about it in section 2.5 but here is a more precise explanation:

If we would only observe one of the two effect ssr and slr the effect of the damped harmonic oscillation would not be visible. Lets play this through let us assume only slr would apply, then there would only be a peak and an exponential decay visible on the oscilloscope, because all energy would be given to the lattice on the way back to the equilibrium state. But of course we can not neglect the ssr. Which is shifting the resonance frequency of a few atoms. This leads to an oscillating effect, because the steady rising magnetic field will hit those ones delayed. They will then again start to relax, and due to slr lose energy exponentially. Which results in the damped harmonic oscillating effect.

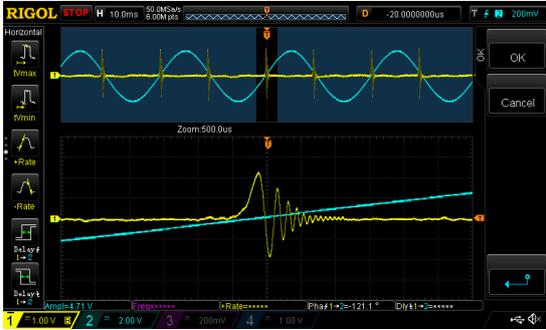


Figure 7: The captured picture from the oscilloscope for the Hydrogen sample

those are the calculated results:

$$\frac{\nu}{B} = (42.78 \pm 0.06)\text{MHz} \cdot \text{T}^{-1}$$

$$g_I = (5.612 \pm 0.008)$$

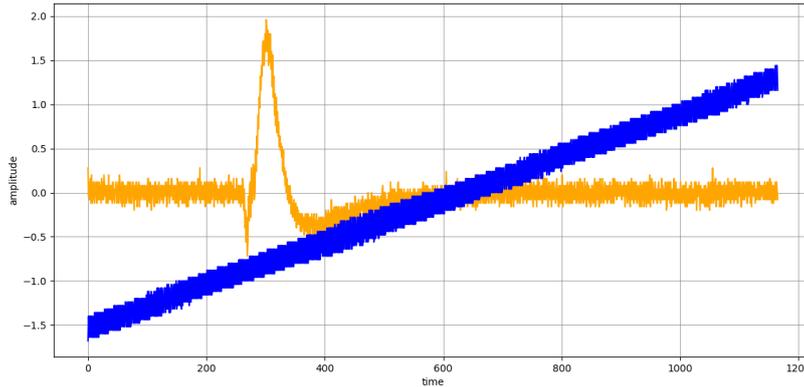
$$\gamma = (26.88 \pm 0.04) \cdot 10^7 \text{rad} \cdot \text{s}^{-1} \text{T}^{-1}$$

$$\frac{\mu_z}{\mu_K} = 2.806 \pm 0.004$$

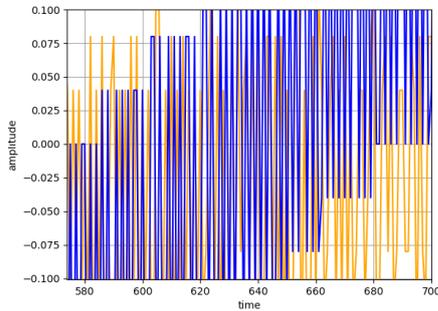
4.3 Measurements using the lock-in-method

To calculate the resonance frequency using the data obtained with the lock-in-method, we use the matplotlib module in python 2.7 to plot the data and the built-in zoom function to determine the zeroes of the two curves.

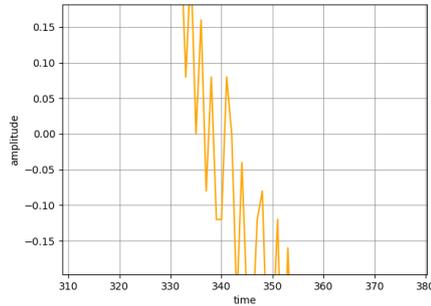
As an example of this graphical method, we take our first measurement, plot it and zoom in to the zeroes (cf fig 8).



a) quick plot of a lock-in-measurement with matplotlib. Time values are given in divs k of the oscilloscope, amplitudes in V.
Blue: Sawtooth function / Orange: NMR signal



b) magnified view the zero of the sawtooth function



c) magnified view of the zero of the NMR signal

Figure 8: Graphical determination of the zeroes

We refrain from fitting either one of the curves since with this method we obtain the zeroes to a sufficient accuracy compared to the relatively large uncertainty in measuring the frequencies. The time axis in the data obtained from the oscilloscope has units of incremental integers k which are multiplied by the value marked "increment" in the .csv file to obtain actual time values. We then determine the time difference Δt between the zeroes of the two curves. Since the HF curve still has some statistical fluctuations and the zero of the sawtooth is also somewhat blurry from the sine modulation, we obtain uncertainties Δ_k on the zeroes which propagate to the Δt values. The matplotlib environment allows to determine the x-y-position of the cursor. Using this method we obtain a minimum and maximum value for the

zero of the NMR signal (fig. 8.c):

$$t_{min} = 335\text{div}$$

$$t_{max} = 342\text{div}$$

We obtain the zero by averaging over these, and take half the distance between t_{min} and t_{max} as the uncertainty of the measurement.

All the data obtained is shown in appendix 6.1. Since ν_{res} and $B = B_0 + B_{mod}(t)$ show a linear dependence according to eq. 10 and the $B_{mod}(t)$ sawtooth function increases linearly with t , we expect the resonance frequency at $\Delta t = 0$ where the B-field reduces to the known $B = B_0$. Therefore we can interpolate the resonance frequency using a linear fit.

The linear fit made by Origin [5] with the built in weighted linear regression, is shown in Figure 9. The following errors are taken straight from the regression of Origin and propagate as shown in equation 16

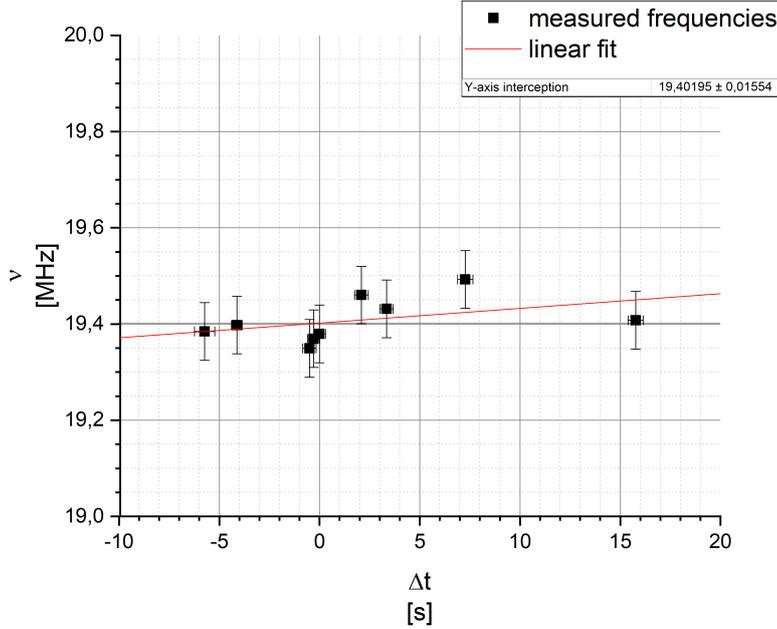


Figure 9: $\nu - \Delta t$ - diagram with linear fit

We obtain the y-intercept,

$$\nu_{res} = (19.402 \pm 0.016)\text{MHz}$$

or rather the $\frac{\nu_{res}}{B}$ ratio:

$$\frac{\nu_{res}}{B} = (42.55 \pm 0.07) \frac{\text{MHz}}{\text{T}}$$

5 Summary and discussion of the results

In this section we summarise our results and compare the values requested in the tasks (section 1) to reference values.

5.1 Measuring the magnetic field

In section 4.1 we observed a clear homogeneity of the magnetic field at the location of the samples at a value of $B = (472 \pm 0.3)$ mT which allowed us to perform the other measurements.

5.2 Sine modulation method

In section 4.2 we obtained the following results:

5.2.1 ^{19}F Sample

$$\begin{aligned}\frac{\nu}{B} &= (40.18 \pm 0.12)\text{MHz} \cdot \text{T}^{-1} \\ g_I &= 5.272 \pm 0.015 \\ \gamma &= (25.25 \pm 0.07) \cdot 10^7 \text{rad} \cdot \text{s}^{-1} \text{T}^{-1} \\ \frac{\mu_z}{\mu_K} &= 2.636 \pm 0.008\end{aligned}$$

5.2.2 Teflon sample

$$\begin{aligned}\frac{\nu}{B} &= (40.12 \pm 0.09)\text{MHz} \cdot \text{T}^{-1} \\ g_I &= (5.264 \pm 0.012) \\ \gamma &= (25.21 \pm 0.06) \cdot 10^7 \text{rad} \cdot \text{s}^{-1} \text{T}^{-1} \\ \frac{\mu_z}{\mu_K} &= 2.632 \pm 0.006\end{aligned}$$

The result for $|\vec{\mu}|$ of the ^{19}F nucleus, as indicated by Teflon sample coincides within 1σ with the following reference value[4]:

$$\frac{\mu_{z^{19}\text{F}}}{\mu_K} = 2.6288688 \quad (18)$$

5.2.3 Glycol sample

$$\begin{aligned}\frac{\nu}{B} &= (42.71 \pm 0.06)\text{MHz} \cdot \text{T}^{-1} \\ g_I &= (5.603 \pm 0.008) \\ \gamma &= (26.84 \pm 0.04) \cdot 10^7 \text{rad} \cdot \text{s}^{-1} \text{T}^{-1} \\ \frac{\mu_z}{\mu_K} &= 2.802 \pm 0.004\end{aligned}$$

The result for the gyromagnetic ratio γ of the proton as indicated by the Glycol sample coincides within 2σ with the referenced value [6]

$$\gamma_{\text{H}} = 26.75221900 \cdot 10^7 \text{rad} \cdot \text{s}^{-1} \text{T}^{-1} \quad (19)$$

The larger discrepancy could be explained by the great difficulty at adjusting the resonance frequency due to the interference inside the HF Generator during the Glycol measurement.

5.2.4 ^1H sample

$$\begin{aligned}\frac{\nu}{B} &= (42.78 \pm 0.06)\text{MHz} \cdot \text{T}^{-1} \\ g_I &= (5.612 \pm 0.008) \\ \gamma &= (26.88 \pm 0.04) \cdot 10^7 \text{rad} \cdot \text{s}^{-1} \text{T}^{-1}\end{aligned}$$

$$\frac{\mu_z}{\mu_K} = 2.806 \pm 0.004$$

The result for the gyromagnetic ratio γ of the proton as indicated by the ^1H sample coincides within 3σ with the above referenced value.

5.3 Lock-in method

In section 4.3 we obtained the following result for the $\frac{\nu_{res}}{B}$ ratio of the proton by using the data from the ^1H sample:

$$\frac{\nu_{res}}{B} = (42.55 \pm 0.06)\text{MHz} \cdot \text{T}^{-1}$$

Which coincides within 1σ with the reference value [7]

$$\frac{\nu_{res,^1H}}{B} = 42.5781\text{MHz} \cdot \text{T}^{-1} \quad (20)$$

Although we have to admit that the data points scatter by relatively large distances around the linear fit, the y-interception still delivers a good result.

This result is in good accordance with the reference value and shows that the Lock-in method allowed us better measurement than the sine modulation, where the result has a 3σ discrepancy from the reference value. One could improve the measurements by avoiding the detour via the NMR-oscillator and send the NMR signal after amplifying it straight into the oscilloscope and reduce the interference inside the NMR-oscillator by doing so.

6 Appendix

6.1 lock-in method

file name	NMR-Signal		average	error
	min	max		
1	335	342	6,77	0,07
2	382	388	7,70	0,06
3	882	888	17,70	0,06
4	523	528	10,51	0,05
5	367	372	7,39	0,05
6	727	732	14,59	0,05
7	428	435	8,63	0,07
8	615	620	12,35	0,05
9	880	885	17,65	0,05
	Sawtooth			
	min	max	average	error
1	600	650	12,50	0,50
2	578	604	11,82	0,26
3	78	115	1,93	0,37
4	525	555	10,80	0,30
5	355	384	7,39	0,29
6	738	771	15,09	0,33
7	248	280	5,28	0,32
8	496	530	10,26	0,34
9	500	538	10,38	0,38
	Δt [s]	error on Δt [s]	Frequency [MHz]	
1	-5,73	0,50	19,3844	
2	-4,12	0,27	19,3977	
3	15,77	0,37	19,4076	
4	-0,29	0,30	19,3694	
5	0,00	0,29	19,3795	
6	-0,50	0,33	19,3493	
7	3,35	0,33	19,4314	
8	2,09	0,34	19,4602	
9	7,27	0,38	19,4926	

6.2 Lab-notes

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27.8.2018

Nuclear Magnetic Resonance

1.) Measurement of magnetic field (Hall-Probe)
 $\Delta x = 1 \text{ mm}$ $\Delta B = 0,5 \text{ mT}$

x [mm]	B_z [mT]	x [mm]	B_z [mT]
34	462	9	471
33	470	8	471
32	471	7	470
30	472	6	470
29	471	5	469 467
28	472	4	463
27	471	3	458
26	472	2	447
25	472	1	432
24	472	0	420
23	472		
22	472		
21	472		
20	472		
19	472		
18	472		
17	472		
16	472		
15	472		
14	472		
13	472		
12	472		
11	472		
10	472		

2.) Resonance frequency of Teflon

$$\nu = 18,3512$$

$$\Delta\nu = 0,0001$$

3.) new Magnetic B-field: 453 mT

Res. freq. Hydrogen

$$\nu = 19,3496$$

$$\Delta\nu = 0,0001$$

4.) Res. freq. Glykol

$$B = 453 \text{ mT}$$

$$\nu = 19,3882$$

$$\Delta\nu = 0,0001$$

~~Left~~ Right side of frequency jump

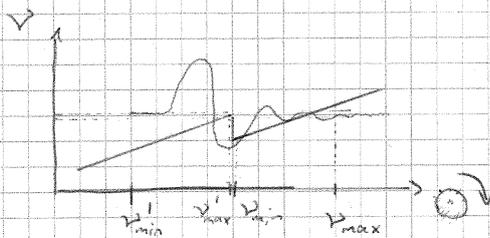
$$\nu_{\text{max}} = 19,3495$$

$$\nu_{\text{min}} = \cancel{19,3782} \rightarrow 19,3032$$

~~Right~~ Left side

$$\nu_{\text{max}} = 19,3980$$

$$\nu_{\text{min}} = 19,3186$$



5.1)

$B_0 = 454 \text{ mT}$ (Hydrogen Probe)

$\Delta B_0 = 0,5 \text{ mT}$

Name of Data	Sawtooth Period	Freq. \sin	H F (MHz)	Time const.
1	10s	54 Hz	19,4300	0,3
2	100s	54 Hz	19,4100	1
3	100s	54 Hz	19,3876	1
4	100s	25 Hz	19,3970	1
5	100	25 Hz	19,3676	1
6	100	25 Hz	19,3468	1
7	100s	25 Hz	19,3148	1
8	100s	54 Hz	19,3149	1
9	100s	54 Hz	19,4800	1
10	100s	54 Hz	19,3290	1
11	100s	54 Hz	19,2023	1
12	100s	54 Hz	19,2792	1
13	100s	54 Hz	19,3918	1
14	100s	54 Hz	19,3217	1
15	100s	54 Hz	19,3388	1
16	100s	54 Hz	19,4238	1

$B_0 \text{ (end)} = 450 \text{ mT}$

28.8.2018

New Measurement with Hydrogen / Calc-in-Med
Sweep Rate: 30s, B freq: 54 Hz, Time const: 1, B₀ = 456 mT

File Name	HF freq [MHz]
di_1	19,3846
di_2	19,3977
di_3	19,4076
di_4	19,3694
di_5	19,3795
di_6	19,3493
di_7	19,4314
di_8	19,4602
di_9	19,4926

New Measurement with ~~Teflon~~ Sinus Modula
(and ^{19F} sample)

B = 456 mT → Resonance out of range

B = 482 mT

• ~~19F~~ ^{19F} Sample: B = 482 mT

$\nu = 19,3688$ MHz

Δν from ~~min~~ equidistance of Absorption pec

$\nu_{\min} = 19,3484$ MHz

$\nu_{\max} = 19,4001$ MHz

• Teflon Sample: B = 482 mT

$\nu = 19,3387$ MHz

$\nu_{\min} = 19,3181$ MHz

$\nu_{\max} = 19,3582$ MHz

• Glykol Sample: B = 452 mT

$\nu = 19,3058$

~~Δν~~ Δν = 0,002 MHz

• ^1H sample:

$$\nu = 19,3361$$

$$\Delta\nu = 0,02$$

After the experiment: $B = 453 \text{ mT}$

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S. Lippert

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