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1 Abstract

During this experiment the halflifetimes of the radioactive isotops of ^{147}Sm and ^{40}K is determined. The examined samples are Samariumoxide as a α-radiator and Kaliumchloride as a β-radiator. Therefore the decay law $A = \frac{\ln 2}{T_{1/2}} N$ is used and for Samarium the range of radiation in material, and the dependency of mass for Kalium. The results of $T_{1/2} = (2,53 \pm 0,03) \cdot 10^{11}$years for Samarium and $T_{1/2} = 0,13 \cdot 10^9$ for Kalium are in the expected range of the halflifetimes but make obvious, that not all the experimental setup is good to determine halflifetimes.

2 Theory

2.1 Decay Law

The decay law for radioactive nuclei describes how many of the nuclei in the given material have already decayed after a time t.

$$N(t) = N_0 e^{-\lambda t} \tag{1}$$

$N_0 = N(0)$ is the number of nuclei at the beginning of the decay process and λ is the decay constant which is correlated with the half life time $T_{1/2}$.

$$\lambda = \frac{\ln 2}{T_{1/2}} \tag{2}$$

Another important value of the used sample is its *activity*. It is given as the negative time derivative of the number of nuclei which are available.

$$A = -\dot{N} = \lambda N = \frac{\ln 2}{t_{1/2}} N \tag{3}$$

Because during this experiment the used samples have large halflifetimes $[T_{1/2}(^{40}K) = 1,28 \cdot 10^9$years and $T_{1/2}(^{147}Sm) = 1,06 \cdot 10^{11}$years] it is possible to set the activity and the number of nuclei as a constant. Therefore they are given as

$$A = \lambda N \tag{4}$$

2.2 α-Radiation

Mostly, the decay of α-radiation occurs in heavy nucleids. The characteristic of an α-decay is the release of a Helium-nucleus, which is also called a α-particle. Because of the release the mass number is reduced by 4 and the proton number by two.

$$^A_Z X \rightarrow ^{A-4}_{Z-2} Y + ^4_2 He \tag{5}$$

This effect happens as soon as a Helium-nucleus is able to tunnel through the Coulomb-potential of the mother nucleus. As it is a two-particle decay the α-particle has a discrete energy spectrum. The used samples in this experiment decay as the following:

$$^{147}_{62}Sm \rightarrow^{143}_{60} Nd +^4_2 He \qquad ^{238}_{92}U \rightarrow^{234}_{90} Th^{(*)} +^4_2 He \tag{6}$$
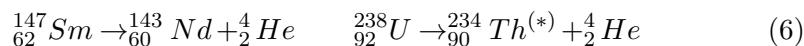
2.3 β-Radiation

There are three different types which are registered as β-decays.

a β^--decay:
   In this process one of the nucleus neutrinos is changed into a proton with an emission of an electron and an anti-electronneutrino.

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{7}$$

b β^+-decay:
   With this process a proton in the nucleus is changed into a neutron with the emission of a positron and a electronneutrino. This effect is only possible inside of a nucleus as the binding energy can provides sufficient amount of energy for the process. While the positron is surrounded by matter it can not exist alone, that's why it's reuniting with an electron and becomes a positriniumatom which then decay into two gammas with an energy of 0,511Mev.

$$p \rightarrow n + e^+ + \nu_e \tag{8}$$

c **Electron Capturing:**
   The nucleus is catching an electron from a close shell (mostly the K-shell) and it is uniting with a proton of the nucleus. A neutron and a electronneutrino are released.

$$p + e^- \rightarrow n + \nu_e \tag{9}$$

   The gap produced by the captured electron is refilled with electrons from higher shells which release the energy difference by gamma radiation or inner conversion.

2.4 Derivation of the Equations for the Half life

2.4.1 Samarium

For the lifetime it has to be taken in account that not all α-particles actually leave the sample and can be detected. Some have a too big distance to the surface and are self-absorbed by the sample before they can reach the surface
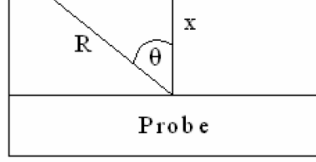
Figure 1: Connection between x, R, θ

and therefore can't be detected. In figure 1 one can see that the connection between the thickness and the distance is limited by the solid angle Ω.

$$\theta_{max} = \arccos \frac{x}{R} \tag{10}$$

Thus, the solid angle is given by

$$\Omega(r) = \int_0^{2\pi} d\Phi \int_0^{\Phi_{max}} \sin \Phi d\Phi = 2\pi \left(1 - \frac{r}{R}\right) \tag{11}$$

For the surface F of a radioactive sample a counting rate of

$$n = A_V \frac{F}{4} R_{Sm_2O_3} \tag{12}$$

is expected. $R_{Sm_2O_3}$ is the range of the released radiation of the sample, $A_V = \frac{A}{V}$ is the activity per volume without self-absorption. The number of ^{147}Sm nucleids is given by $N = 2 \cdot N_{Sm_2O_3} \cdot h_{rel}$ where h_{rel} is the relative number of ^{147}Sm in the Samariumoxide and $N_{Sm_2O_3}$ is the number of Samariumoxide nucleids which is given by

$$N_{Sm_2O_3} = \frac{m \cdot N_A}{m_{rel,Sm_2O_3}} \tag{13}$$

With the relation of Bragg and Kleemann

$$R \cdot \rho = C \sqrt{m_a} \tag{14}$$

where m_a is the effective atommass, ρ is the density and C is a propotional constant (independent of the material) and further transformation where C is eliminated and $\frac{m}{d} = \frac{V\rho}{d} = F \cdot \rho$ the final result becomes:

$$T_{1/2} = \frac{\ln 2 \cdot N_A \cdot h_{rel} \cdot R_{air} \cdot \rho_{air} \cdot F}{2 \cdot n \cdot m_{Sm_2O_3}} \cdot \sqrt{\frac{m_{A,Sm_2O_3}}{m_{A,air}}} \tag{15}$$
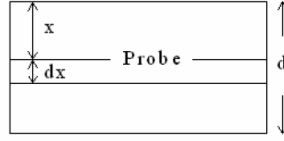
Figure 2: Infinitisimal mass element at position x

2.4.2 Kalium

Looking at an infinitesimal small mass element dm which can be found at a depth of x inside of the material, the counting rate dn is given as

$$dn = A_S \cdot e^{-\mu x} dm = A_S \cdot \rho \cdot F \cdot e^{-\mu x} dx \tag{16}$$

Continuing with the assumption that n(x=0)=0, integration over a thickness of $d = \frac{m}{\rho \cdot F}$ the result becomes

$$n = \frac{f_b \cdot A_S \cdot \rho \cdot F}{2\mu} \left(1 - e^{-\frac{\mu m}{\rho F}}\right) \tag{17}$$

The number $f_B = 1,29$ gives a factor of $\frac{f_B}{2}$ takes the backscattering of the electrons at the aluminium plate into account. Also it is taken into account, that only the radiation which is emitted at the top of the sample can be detected. The unknown values of A_S and μ can be eliminated with the help of a fit where the parameters are a and b.

$$n(m) = a(1 - e^{-bm}) \tag{18}$$

With this fit it is possible to determine A_S and therefore the halflifetime.

$$A_S = \frac{2\mu a}{f_B \rho F} = \frac{2ab}{f_B} \tag{19}$$

2.5 Measurement times

In order to get a relative error of 2% the measurement time to achieve this uncertainty has to be estimated. I can be used, that the counting rate is already known from the characteristics of the counting device. With n being the counting rate at the working point, the time can be approximated using the underlying Poisson-statistics. Therefore the uncertainty on the counting rate can be approximated as $\sqrt{N}$

$$0,02 = \frac{s_n}{n} = \frac{\frac{\sqrt{N}}{t}}{n} = \frac{1}{\sqrt{n}\sqrt{t}} \tag{20}$$

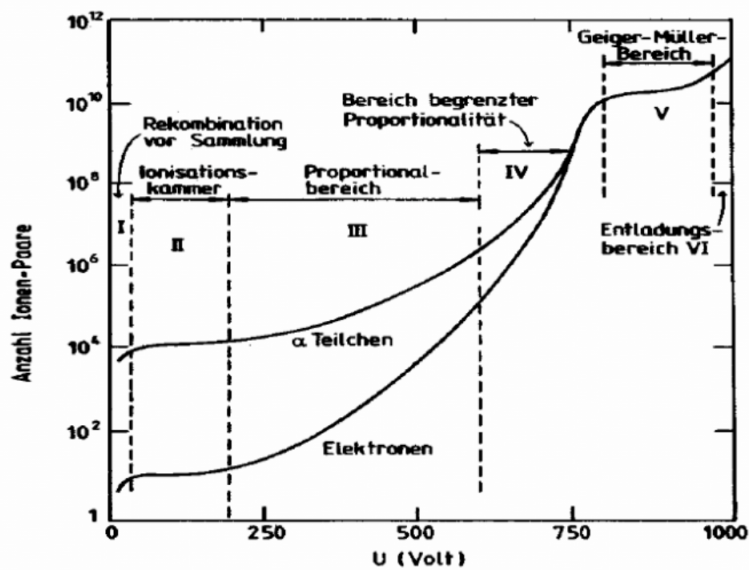$$t = \frac{1}{0,0004 \cdot n} \tag{21}$$

Figure 3: Common charakteristik of a Counting tube

2.6 Counting device

The Counter tube consists of a thin Anode wire and a Kathode mantle. The inside is filled with an ionizable gas. Based on the applied Voltage, the Counter tube works differently. Its characteristic can therefore be separated in different sections, based on how the count rate changes at a given voltage. A typical characteristic is shown in figure 3. The detection of particles with the counting tube is based on the fact, that radiation separates atoms in the gas into electron and ions. Due to the electric field, the ions and electrons drift towards the Kathode and Anode.

For low Voltages, the electrons recombine before they reach the anode. No current or just the current from electrons close to the anode wire can be detected (section 1).

Higher voltages result in currents for all electron ion pairs that were created by primary ionisation. (section 2).

In section 3, the voltage is high enough to create secondary ions. Secondary electron ion pairs are created when the primary ions have enough energy to ionise gas molecules themselves. This increased rate of counts is still proportional to the amount of primary ionisations (gas amplification).

By leaving the proportional section, the gas amplification, the amplification becomes independent from the primary Ionisation (section 4).

In section 5 the electric field is strong enough to ionise the whole gas if a photon ionises an atom.

In section 6 the photon is not needed anymore to ionise the gas. The electric

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field is strong enough to ionise the gas on its own.

3 Experimental Setup

In the experiment a 2π counting tube is used. With voltages between 0 and 1 kV, the operation mode of the tube is the proportionality mode.

The used samples are Uranium (U), Samarium (Sm) and Kalium (K). Emitting mainly α- and β- radiation, the risk of loosing radiation while entering the tube is very high. That's why the sample is put directly inside the Methan-Counting-tube. The exact setup can be seen in figure **??**.

When a α or β particle gets detected, a current that is proportional to the energy can be measured. The signal is transfered to a Preamplifier (VV) where it gets split up. One part goes to a linear gate and the other one to the Single Channel Analyser (SCA). To separate decay events from background noise, the lower level of the SCA can be adjusted. Only Energies above the lower level of the SCA pass the linear gate and are detected with the computer.

To see the signals, an oscilloscope is connected with the amplifier.

In the first part of the experiment, the characteristic of the counting device is measured with uranium. To improve the quality of data, a background measurement is taken for the same voltages.

4 Execution

Looking at the behaviour of the oszilloscope, it is determined, that the lower gate should be set to a level of 0,48 units on the scale of the lower level potentiometer to seperate background noise from relevant data. The shaping amplifier is set to 6 μs.

4.1 Characteristic

To verify the $\alpha-$ and β-plateau of the counting device, a characteristic measurement is taken with uranium as a sample. Voltages from 1 kV to 4 kV are measured in steps of 100 V with a measurement time of 50 s and a delay of 100s before the measurement for a certain voltage starts to ensure that the voltage is stable in the experimental setup. Furthermore a background measurement is taken for the same voltages. This time, the uranium is removed from the counter and the measurement time is set to 100s.

4.2 Half-life period of Sm

To measure the Half-life period of Sm, the α- plateau of the characteric is measured once more with Samarium as sample. At a voltage in the middle of the plateau, the Counts are measured for a 4200 s.

4.3 Half-life period of Kalium

To measure the Half-life period of Ka, the β- plateau of the characteric is measured once more with Kalium as sample. For 10 different masses, the Counts in 560 s are measured at a voltage in the middle of the plateau.
To reduce the impact of background radiation, two background measurements with 7 hours each are taken at the two chosen voltages in the plateaus.

5   Data Analysis

5.1   Characteristic of the counting device

The counts per second n are measured with the counting device. For the characteristic, the rates of voltages in range of 1000 V - 4000 V are measured in steps of 100 V. For each voltage the time of measurement is $t = 50s$. The error on the total amount of counts N can be approximated due to the underlying Poisson distribution as $s_N = \sqrt{N}$. The uncertainty on the count-rate can therefore be calculated as

$$n = \frac{N}{t} \qquad \rightarrow \qquad s_n = \frac{\sqrt{N}}{t} = \sqrt{\frac{n}{t}} \qquad (22)$$

The measurement time for the background measurement is 100s for each Voltage. The improved data can therefore be calculated as

$$N = N_m - \frac{t_m}{t_b} \cdot N_b = N_m - \frac{1}{2}N_b \qquad (23)$$

N_m are the counts of the main measurement, N_b are the counts on the background measurement. Because $N_m >> N_b$, the uncertainty on N_b can be ignored, as it is given as $\sqrt{N}$.
The improved characteristic is shown in figure 5. The original data from the main- and backgroundmeasurement are shown in figure 4.
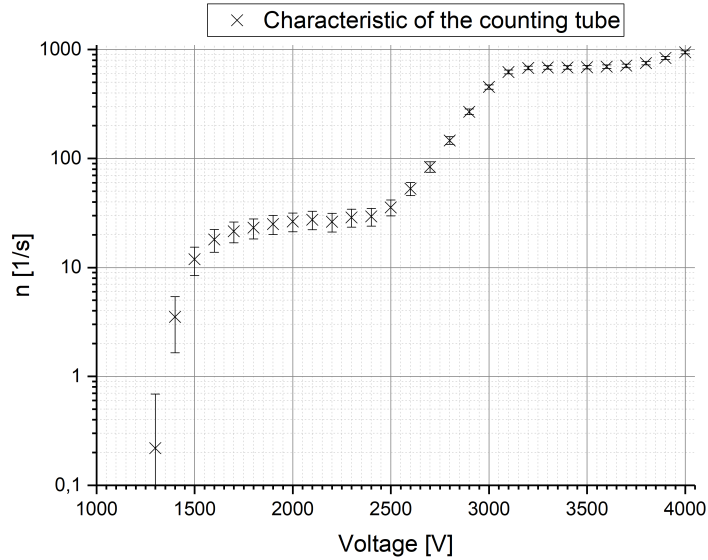

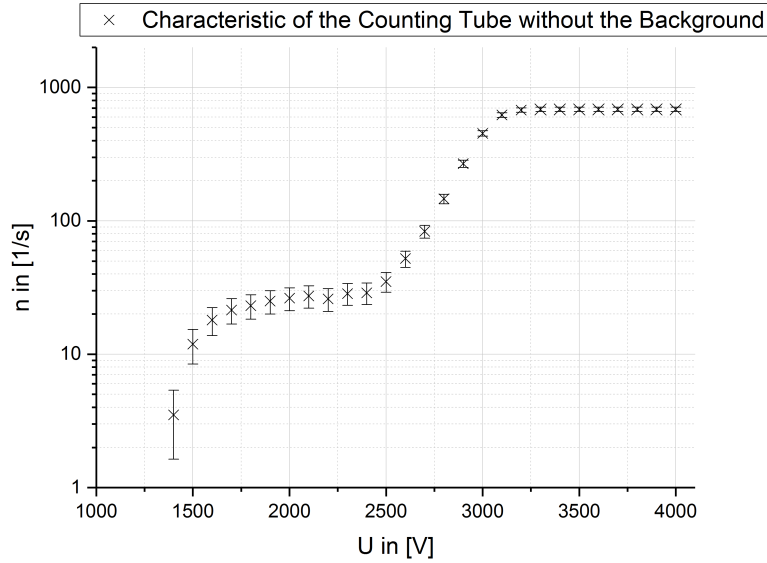Figure 4: Characteristic with background

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Figure 5: Characteristic without Background

Based on this characteristic, the working points are selected for the α-plateau (U=2300V) and the β- plateau (3400 V). Unfortunately, the highest step that can be selected during the background measurement is 1000 V. Therefore, we couldn't measure the background at the exact same points of the plateau. Instead the background at $U = 2350$ V and $U = 3350$ V were measured. Looking at figure 4, one can see that the counting rates are not very different for small surroundings of the correct Voltage. Therefore, this shouldn't influence our results. If the influence was underrated, we would be able to see a systematic offset on the counting rate in the plot of the Kalium analysis.

5.2 Half-life period of Samarium

In order to calculate the half-life period of Samarium, it has to be secured, that the voltage of the counting device is set to the middle of the α- plateau. Therefore a measurement of the count rates n is taken in range of 1500V to 3000V in steps of 100 V.

As a working voltage the middle of the α-plateau was approximated and set at $U = 2300V$. For the following measurement a measuring time of $\Delta t = 2100s$ is set. which results in a counting rate of

$$n' = 0,471\frac{1}{s} \qquad s_{n'} = \sqrt{\frac{1}{n' \cdot \Delta t}} = 0 \qquad (24)$$

The error on the counting rate is not existent because in the background
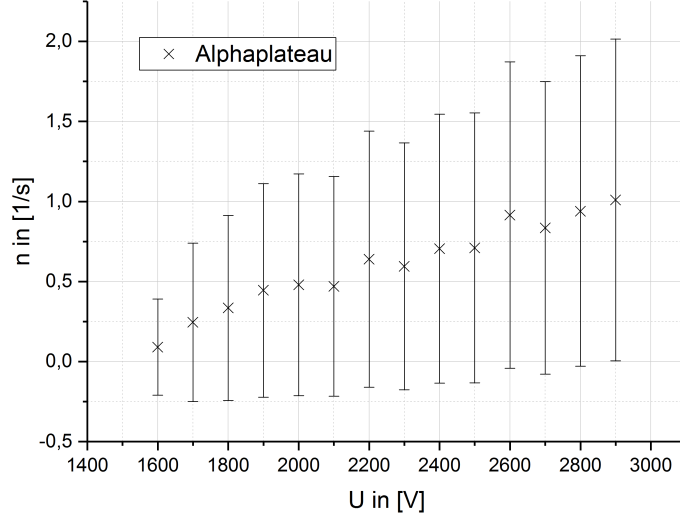
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Figure 6: α-plateau

measurement there is no count at a voltage of $U = 2300$V. For more details the background datas can be found in the attachment 4.

The background measurement is taken overnight and results in a counting rate of

$$n_B = 0{,}026 s^{-1} \qquad \Delta t = 7\text{h} \qquad s_{n_B} = n_B \cdot \sqrt{\frac{1}{n_B \cdot \Delta t_B}} = 1{,}016 \cdot 10^{-3} \quad (25)$$

As the error on the background counting is neglectable the counting rate is given by

$$n = n' - n_B = 0{,}391 s^{-1} \tag{26}$$

The diameter of the used aluminium plate is measured five times with the following results:

measurement	diameter in [cm]	s_d
1	2,9	0,01
2	2,9	0,01
3	2,9	0,01
4	2,9	0,01
5	2,89	0,01
average	2,898	$4{,}47 \cdot 10^{-3}$

Table 1: Diameter of the Aluminium Plate

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The error on the average is calculated by

$$s_d = \frac{s_d(seperate)}{\sqrt{5}} = 4,47 \cdot 10^{-3} \tag{27}$$

With this result it is now possible to determine the surface of the plate.

$$F = \pi \left(\frac{d}{2}\right)^2 = (6,6 \pm 0,09)cm^2 \qquad s_F = d \cdot \pi \cdot s_d \tag{28}$$

With the equation already given in the theory and the error on it, it is now possible to determine the halflifetime.

$$T_{1/2} = \frac{\ln 2 \cdot N_A \cdot h_{rel} \cdot R_{air} \cdot \rho_{air} \cdot F}{2 \cdot n \cdot m_{Sm_2O_3}} \cdot \sqrt{\frac{m_{A,Sm_2O_3}}{m_{A,air}}} \qquad s_{T_{1/2}} = T_{1/2} \cdot \sqrt{\left(\frac{s_F}{F}\right)^2 + \left(\frac{s_n}{n}\right)^2}$$
$$\tag{29}$$

For this calculation the following constants are needed:

- $N_A = 6,022 \cdot 10^{23} mol^{-1}$

- $h_{rel} = 0,1487$

- $R_{Air} = 1,13 cm$

- $\rho_{Air} = 0,001226 \frac{g}{cm^3}$

- $\sqrt{m_{A.Air}} = 3,833\sqrt{u}$ (due to the relation between Bragg and Kleemann)

- $\sqrt{m_{A,Sm_2O_3}} = 11,125\sqrt{u}$

The molare mass of the Samariumoxide can be determined by

$$M_{Sm_2O_3} = 2M_{A,Sm} + 3M_{A,O} = 348,717\frac{g}{mol} \tag{30}$$

Finally, the halflifetime results as

$$\textbf{Samarium:} T_{1/2} = (2,53 \pm 0,03) \cdot 10^{11} \text{years} \tag{31}$$

5.3   Half-life period of Kalium

Being a β-radiator, a working point in the approximated middle of the β-plateau is selected.

The measured counting rates have to be validated with the background measurement and the mass of the sample holder.
For each measurement with the weigh scale, an error of 0,0003 g is taken

due to the fluctuation of the value.
The sample holder has a mass of

$$m_0 = (2{,}3922 \pm 0{,}0003)\ \text{g} \tag{32}$$

Therefore the mass is

$$m = m_{mess} - m_0 \qquad\qquad s_m = 0{,}0003 G \tag{33}$$

The statistical uncertainty s_m is not increased, because the uncertainty on m_0 is a systematic uncertainty as the mass of the object holder doesn't change during the experiment.

The counting rate n has to be improved with the knowledge of the background measurement

$$n = n_{mess} - n_0 \tag{34}$$

The uncertainty of the background measurement can be ignored, because it is more than one magnitude smaller than the one of the main measurement. This is because of the longer measurement time and smaller counting rate

$$s_n = \sqrt{\frac{n}{t}} \tag{35}$$

The improved values can be seen together with their uncertainties in figure 7. A Fit is made for the values. As explained in the theory, it is useful to fit the function

$$n(m) = a \cdot (1 - e^{-b \cdot m}) \tag{36}$$

For the analysis, the value at $m = 1{,}2$ g is ignored because it doesn't fit in the trend of the other values.
The fitting results are shown in table 2.

a	s_a	b in [1/g]	s_b	Kor. R square
4,81	0,12	1,87	0,11	0,98637

Table 2: Caption

A systematic offset can not be seen. Also the Kor. R square value shows that the chosen fit function is very likely to fit to the results. This shows that the slightly different voltage of the underground measurement doesn't influence the result significantly. The halflifetime can be calculated

$$\textbf{Kalium:}\ T_{1/2} = \frac{f_B \cdot \ln(2) \cdot N_A \cdot h_{rel}}{1{,}12 \cdot 2 \cdot a \cdot b \cdot M_{KCl}} = 0{,}13 \cdot 10^9 \text{years} \tag{37}$$
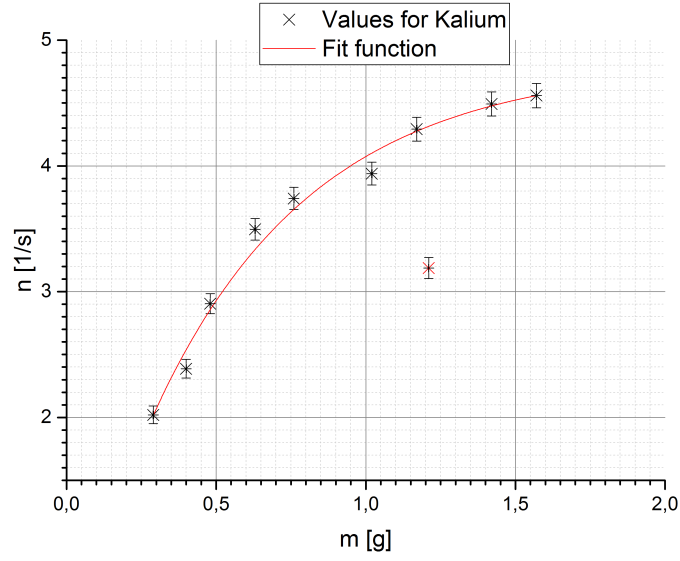
Figure 7: Dependence of counting rate and mass for Kalium

The error is calculated by

$$s_{T_{1/2}} = T_{1/2} \cdot \sqrt{\left(\frac{s_a}{a}\right)^2 + \left(\frac{s_b}{b}\right)^2 + 2 \cdot \frac{s_a}{a} \cdot \frac{s_b}{b} \cdot \rho} \qquad (38)$$

and the final solution becomes

$$\textbf{Kalium:} T_{1/2} = (0,13 \pm 0,01) \cdot 10^9 \text{years} \qquad (39)$$

with $h_{rel} = 0,0000118$, $f_B = 1,29$ and $M_{KCl} = 74,55 \frac{g}{mol}$.

6 Discussion

Isotop	$T_{1/2}$ in years	Literature Value
^{147}Sm	$(2,53 \pm 0,03) \cdot 10^{11}$	$1,06 \cdot 10^{11}$
^{40}K	$(0,13 \pm 0,01) \cdot 10^{9}$	$1,28 \cdot 10^{9}$

Table 3: Caption

Looking at the received halflifetimes for Samarium and Kalium it becomes obvious, that both values could not be determined correctly. It can be seen that the value for the Samarium is bigger than the literature value and that the value for Kalium is smaller than the expected value. This seems strange as the same electronical setup was used for both measurements.
A possible reason for the too big value of the Samarium sample could be a wrong measurement of the surface of the Aluminium plate. As the measurement should be quite accurate as it was taken with a calliper, another reason could be that the Samarium was not spread very well inside of the plate. Therefore the counting rate could be decreased as not all the radiation could leave the sample because it was reabsorbed before.
Other possible problems could have been caused be wrong setups of the electronic. If the Lower Level for the discriminator is too big, actual signals that should be detected will be filtered and don't add into our counting rate. As the result for the Kalium is smaller and because the same setup was used, this also seems unlikely (as already mentioned).
The most likely reason for the shifted value is the age of the sample itself. As the powder is already used since quite a while and is put back into the box after each group, it is possible that the measurement was not taken with pure Samariumoxide and pure Kaliumchloride. Also, for the measurement of the Kalium it is an act of balance to weigh the sample and transport it from the scale to the measuring device. It is possible that the sample dropped several times with other groups. When collecting the powder from the table, dirt and dust can be collected as well and therefore ruin the sample step by step. Thus, it is very likely that the activity is decreased.

References

[1] [instructions]
Instruction paper published at `http://omnibus.uni-freiburg.de/`
`~phypra/fp/Versuche/FP1/FP1-7-LangeHalbwertzeiten/` 09.10.2017
13:51

[2] [exam paper]
Tobijas Kotyk: "Versuche zur Radioaktivität im Physikalischen Fort-
geschrittenen Praktikum an der Albert-Ludwigs-Universität Freiburg"
`http://omnibus.uni-freiburg.de/~phypra/fp/Versuche/FP1/`
`FP1-7-LangeHalbwertzeiten/` 09.10.2017 13:51

7 Attachment

1800,000	0,000
1900,000	0,020
2000,000	0,010
2100,000	0,020
2200,000	0,010
2300,000	0,000
2400,000	0,000
2500,000	0,030
2600,000	0,010
2700,000	0,040
2800,000	0,130
2900,000	0,120
3000,000	0,260
3100,000	0,320
3200,000	0,530
3300,000	0,640
3400,000	0,770
3500,000	0,640
3600,000	0,620
3700,000	0,610
3800,000	0,770
3900,000	0,770
4000,000	0,870

Table 4: Background measurement