Contents

1 Physical Background

1.1 Decay Law and Activity

The radioactive decay of a given sample is a statistical process and cannot be predicted deterministically. But since it is a statistical process it is possible to formulate a probability model to describe the phenomenology of decays. This model is the decay law

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda \cdot N, \tag{1}$$

which is a first order differential equation. The solution to this differential equation is

$$N(t) = N_0 \exp(-\lambda t) \tag{2}$$

where N_0 is the number of nuclei at the time $t = 0$ and λ the decay constant. In relation to the decay constant is the mean life time $\tau = 1/\lambda$. Considering eq. (2) it can be seen that the time after which half of the nuclei should be decayed can be calculated by

$$T_{1/2} = \frac{\ln(2)}{\lambda}. \tag{3}$$

To describe the decay properties of a given sample the activity A is defined by

$$A = \frac{N}{\tau} = \frac{\ln(2) \cdot N}{T_{1/2}}. \tag{4}$$

The activity is therefore the expected number of decays per time unit. Another quantity which is going to be referred a lot is the counting rate

$$n = \frac{N}{\Delta t} \tag{5}$$

1.2 Different Types of Radiation

α-Decay

The first and most energetic type of radiation being discussed is the α-Radiation. When a nucleus of an atom X decays the process can be described by

$$^{A}_{Z}\mathrm{X} \longrightarrow \, ^{A-4}_{Z-2}\mathrm{Y} +^{4}_{2}\mathrm{He}. \tag{6}$$

So the α-particle is a helium nucleus which due to quantum tunneling can leave the mother nucleus. Since only 2 particles take part in this decay the energy spectrum of α decay is discrete.

β-Decay

The β-decay is less energetic than the α-decay. There are two different types of β-decays. The first and for the experiment relevant type is the β^--decay which can be described by

$$^{A}_{Z}\mathrm{X} \longrightarrow \, ^{A}_{Z+1}\mathrm{Y} + e^- + \bar{\nu}_e. \tag{7}$$

In detail one of the nucleus' neutrons decays into a proton an electron and an anti-electron-neutrino. Since considering the β-decay three particles interact with each other the spectrum of the β-decay is continuous. The β^+-decay is the same mechanism with the difference

that this time a proton of the nucleus turns into a neutron by emitting a positron and a electron-neutrino. So it can be described by

$$\ce{^A_Z X} \longrightarrow \ce{^{A}_{Z-1} Y} + e^+ + \nu_e. \tag{8}$$
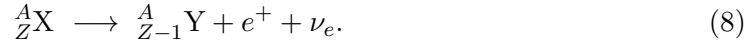
Electron Capture (EC)

The electron capture describes the process when one of the electrons of an inner layer of the atom is captured by the nucleus. So in detail it can be described by

$$\ce{^A_Z X} \longrightarrow \ce{^{A}_{Z-1} Y} + \nu_e. \tag{9}$$

Electron capture is possible because there is a not vanishing probability that the electrons position is inside the nucleus – especially for the inner shell electrons. Inside of the nucleus the electron combines with a proton to a neutron emitting an electron neutrino.

1.3 Correction of the Counting Rates

Samarium – α-decay

Since α-decay is absorbed very easy it has to be considered that some of the radiation is absorbed by the sample itself. To take this into account some further corrections have to be made. α-particles can be detected if the trajectory is shorter than the maximum
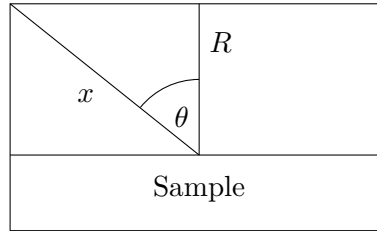

Figure 1: Visualisation of the used parameter for the correction of the counting rate of samarium.

reach R of the radiation in samarium. So looking at fig. 1 one can see that the solid angle Ω in which α-particles can be observed is given as

$$\Omega(x) = \int\limits_0^{2\pi} \mathrm{d}\phi \int\limits_0^{\theta_{\mathrm{max.}}} \sin\theta \mathrm{d}\theta = 2\pi \cdot \left(1 - \frac{x}{R}\right), \tag{10}$$

where $\theta_{\mathrm{max.}} = \arccos(x/R)$ since R is the maximum reach. Considering that solid angle the pulsing rate can be calculated by

$$n = \frac{A}{d} \int\limits_0^R \frac{\Omega(x)}{4\pi} \mathrm{d}x = \frac{A_V F R}{4} \tag{11}$$

where d is the thickness of the sample, A the activity and F the surface. So A_V is the samples activity per volume not considering the correction due to self absorption. Because the maximum reach in samarium is not known it is necessary to use the Bragg-Kleeman relation

$$R \cdot \rho = C\sqrt{m_A} = C\sum_i p_i \sqrt{m_{A,i}} \tag{12}$$

where m_A is the effective atom mass, C is a proportional constant and p_i is the elements relative mass fraction of the used substance. So using eq. (3) and eq. (4) we get for the half-lifetime of a nucleus of the given sample

$$T_{1/2} = \ln(2)\frac{N}{A_V V} = \ln(2)\frac{NR}{4nd} \tag{13}$$

One will recognize that the initial number of radioactive nuclei is unknown, but can easily calculated by

$$N = 2hN_{\text{mol.}} = 2h\frac{\rho RFN_A}{2nM_{\text{mol.}}} \tag{14}$$

where $M_{\text{mol.}}$ is the molar mass of the sample, h is the relative abundance of the isotope and ρ the density of the sample. We now get for the corrected half-life time

$$T_{1/2} = \ln(2)\frac{R\rho hN_A F}{2nM_{\text{mol.}}} \tag{15}$$

Potassium $- \beta$**-decay**

Even though β-radiation is not blocked as easy as α-radiation self absorption has to be considered. So the pulsing rate per surface depending on the mass is defined. So at first the change $\mathrm{d}x$ of a layer x of the sample as shown in fig. 2 is taken into account. Using
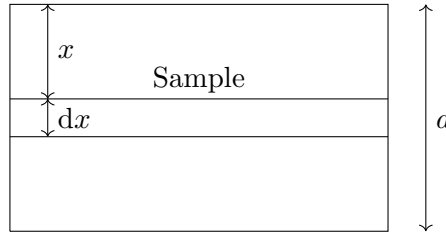

Figure 2: Visualisation of the used parameters to derive the correction needed caused by self absorption for β-radiation

the absorption law we can formulate

$$n \propto A_S F\rho \cdot (1 - \exp(-\mu x))\mathrm{d}x \tag{16}$$

where $A_S = A/m$ is the mass specific activity. Neglecting the energy dependency of μ so assuming that μ is not dependent of x and using $m = \rho Fd$ one can formulate the pulsing rate in dependency of the mass by integrating eq. (16) to the thickness d of the sample. The so received equation for the pulsing rate n is

$$n(m) = \frac{A_S F\rho}{\mu}\left(1 - \exp\left(-\frac{\mu m}{\rho F}\right)\right). \tag{17}$$

To also take backscattering into account we need to add a factor dependent on a reflection factor f_B and the solid angle Ω. Potassium is not only decaying via β-decay it also has to be considered that electron capturing occurs so the total decay constant has to be

$$\lambda_{\text{tot.}} = \lambda_\beta + \lambda_{\text{EC}} = 1.12 \cdot \lambda_\beta \tag{18}$$

even though only β-decay is observable in this experiment. So to calculate the half-life time properly the equation

$$T_{1/2} = \frac{\ln(2)\ N}{1.12\ A(\beta^-)} \tag{19}$$
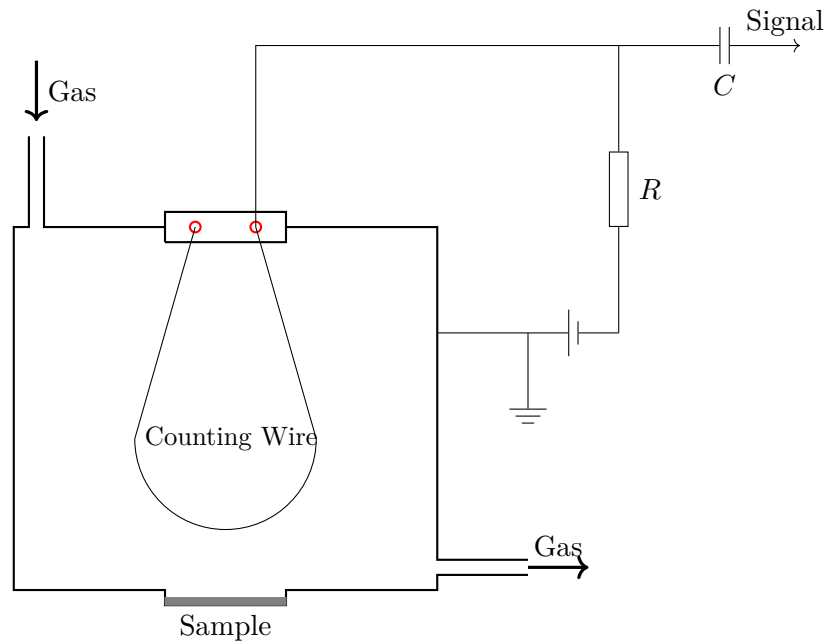
should be used.

Figure 3: In die figure a schematic picture of a counting tube is shown.

2   Setup and Implementation

2.1   Setup

The setup used to perform the experiment consists of a proportional counting tube using floating methane gas for the ionisation, different electronic devices to process the measured signal and an oscilloscope. The measurement can be controlled via LabView on a computer.

2.2   The Proportional Counting Tube

In this experiment a proportional counting tube is used. Inside of the tube there is methane gas which gets ionised when an energetic particle passes through. The methane gas is floating through the tube because the structure of the molecules changes as the get ionised so to guarantee a constant measure behavior the gas is changed constantly. A picture of a counting tube is displayed in fig. 3.

The sample is placed in a turnable wheel so it can be changed easily. A voltage is applied to the tube and the counting wire so that the charge carriers drift to the counting wire. So a current is detected when a energetic particle passes through. When used in the proportionality mode the outlet amplitude of the tube signal is proportional to the detected particles energy loss.

2.3   Implementation

The experiment consists of three parts. In the first part the tube characteristic was measured using a ^{238}U sample which decays by α-decay. The initial voltage was set to $1000\,\text{V}$ while the end voltage was set to $4000\,\text{V}$. For the step length a voltage step of $100\,\text{V}$ while the measuring time was set to $50\,\text{s}$. For this measurement a noise measurement was

done. This measurement is used to correct the other characteristic measurements. For the noise measurement the initial voltage was set to 2000 V and the end voltage was set too 4000 V, with a step voltage of 100 V and a measurement time per step of 100 s.

To determine the half lifetime of samarium at first a measurement of the α-plateau was made. To do so a small aluminium bowl was filled with Sm_2O_3. To flatten the surface of the samarium oxide powder a small stamp was used, since for the analysis a perfect surface is assumed. It is important to turn of the gas flow before changing the sample because if the sample is changed while the gas is still flowing the powder could be blown into the gas tube which leads to a pollution of the tube which makes later measurements impossible. After that a voltage in the middle of the plateau was chosen to measure the pulsing rate in a time period long enough to guarantee that the relative error on that measurement is below 2%, which can be easily approximated because a counting experiment implies a poisson error.

The last measurement was made to determine the half-life time of potassium. This time the β-plateau was measured. Afterwards a voltage in the middle of the plateau was chosen to measure the pulsing rate. This measurement was repeated 10 times with different masses of the sample. The sample was weighted with an analytic scale and noted down. Also the mass of the small aluminium bowl was measured.

3   Analysis

3.1   Counting Tube Characteristics

To find a sensitive voltage for the α- and β-decay the counting tube characteristics of the proportional counting tube was measured with an uranium sample. In fig. 4 the counting tube characteristic is displayed. In the shown counts N the background has already been subtracted. To correct the data of the background the measured counts per
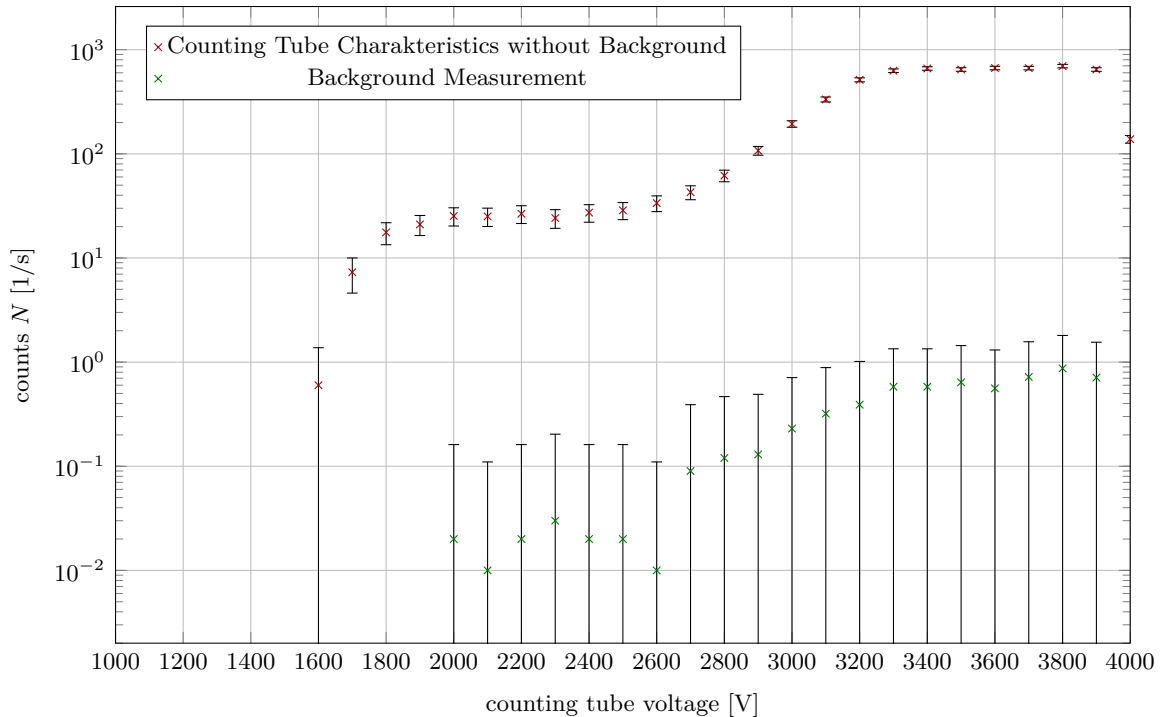

Figure 4: The measured data for the counting tube characteristics has already been subtracted by the measurement of the background. As the raw data of the counting tube measurement covers the displayed data due to the logarithmic plot it is not displayed in the graph.

second are simply subtracted by the background-counts. As the error on the background measurement is extremely small due to the small number of counts it is not taken into account to calculate the total error. Therefore the error can be calculated with

$$s_N = \sqrt{N_{\text{measured}}} \tag{20}$$

The used error on the counting is given by the square root of the total counts as the counts are scattered by a poisson distribution.

With use of fig. 4 and the more accurate measurements of the plateau that are mentioned in section 3.2 and section 3.3 the voltage for measuring the α- and β-decay have been chosen to

$$\begin{aligned} V_\alpha &= 2400\,\text{V} \\ V_\beta &= 3700\,\text{V}. \end{aligned} \tag{21}$$

These values have been chosen during the experiment with the use of a short interim evaluation.

3.2 Determination of the Half-Lifetime of Samarium

To determine the half-lifetime of samarium a long measurement of the α-decay has been made. As the relative error of the measurement was supposed to be under 2% a measurement of the plateau appendix A was made and evaluated during the experiment. Due to the results of the interim evaluation the test time was chosen to $6250\,\mathrm{s}$ as for a roughly determined average rate of $0.4\,\mathrm{counts/s}$ the error should be small enough.

Besides the long measurement of the decay a short background measurement with the same voltage applied to the counter tube has been made. Using the background data the long measurement was corrected by subtracting and as the error on the background is extremely small it was neglected to the error is just determined by the square root of the measured counts.

To calculate the surface area of the sample the diameter of the small bowl was measured. The estimated error on the diameter is $s_d = 0.01\,\mathrm{cm}$. With the mean of the measurements and a gaussian error for the surface area F we find

$$F = (6.375 \pm 0.018)\,\mathrm{cm}^2. \tag{22}$$

To calculate the half-lifetime of samarium eq. (15) was used. The counts per second n are determined as the mean of the counts per second over all measurement periods [1] of the corrected data. As the error using a poisson error would be extremely small it was decided to use the standard deviation over all measurement periods instead. For the overall counting rate we find:

$$n = (0.49 \pm 0.11)\,\frac{1}{\mathrm{s}}. \tag{23}$$

Furthermore the value for the relative frequency has been taken from [Source 1] as $h = 99.75\%$ and the molar mass has been calculated to $M_{\mathrm{mol}}^{\mathrm{Sm}} = 348.8\,\frac{\mathrm{g}}{\mathrm{mol}}$.

Now everything that is needed to determine the half-lifetime has been calculated and we get

$$T_{1/2}^{\mathrm{Sm}} = (1.457 \pm 0.004) \cdot 10^{11}\,\mathrm{years}, \tag{24}$$

where gaussian error propagation was used to calculate the error

$$s_F = \pi d s_d$$
$$s_T = \sqrt{\left(\frac{\ln(2) R \rho h N_A s_F}{2 n M_{\mathrm{mol.}}}\right)^2 + \left(\frac{\ln(2) R \rho h N_A F s_n}{(2 n M_{\mathrm{mol.}})^2}\right)^2}. \tag{25}$$

Also, as already seen the value was transformed from seconds to years.

3.3 Determination of the Half-Lifetime of Potassium Chloride

To determine the half-lifetime of potassium chloride a voltage of $3700\,\mathrm{V}$ was chosen. Same as for samarium first a measurement of the plateau appendix A was done to determine a sensitive measuring time. As the rate is much higher than it was for samarium a measuring period of $700\,\mathrm{s}$ is long enough for a relative error under 2%. Afterwards ten measurements with $700\,\mathrm{s}$ and different weights have been made.

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[1] During the measurement every $50\,\mathrm{s}$ a value was saved, so for $6250\,\mathrm{s}$ measuring time 125 values were saved

As it was only possible to weight the sample in the small aluminium bowl, we also weighted the bowl itself and its weight has been subtracted. The error on the scale has been estimated to $s_{\text{scale}} = 0.0001\,\text{g}$ and the error on the sample can be calculated by simple gaussian error propagation.

To determine the half-lifetime of potassium chloride the counts are subtracted by their background and plotted against the mass. The error on the counts is same as in section 3.2 given by the standard derivation over the measuring periods. Then a fit of the form

$$f(m) = a(1 - \exp(-bm)) \tag{26}$$

has been made with `scipy.optimize.curve_fit` and also the error on the counts were given to the function to optimize the result. The result of the fit is shown in fig. 5. The
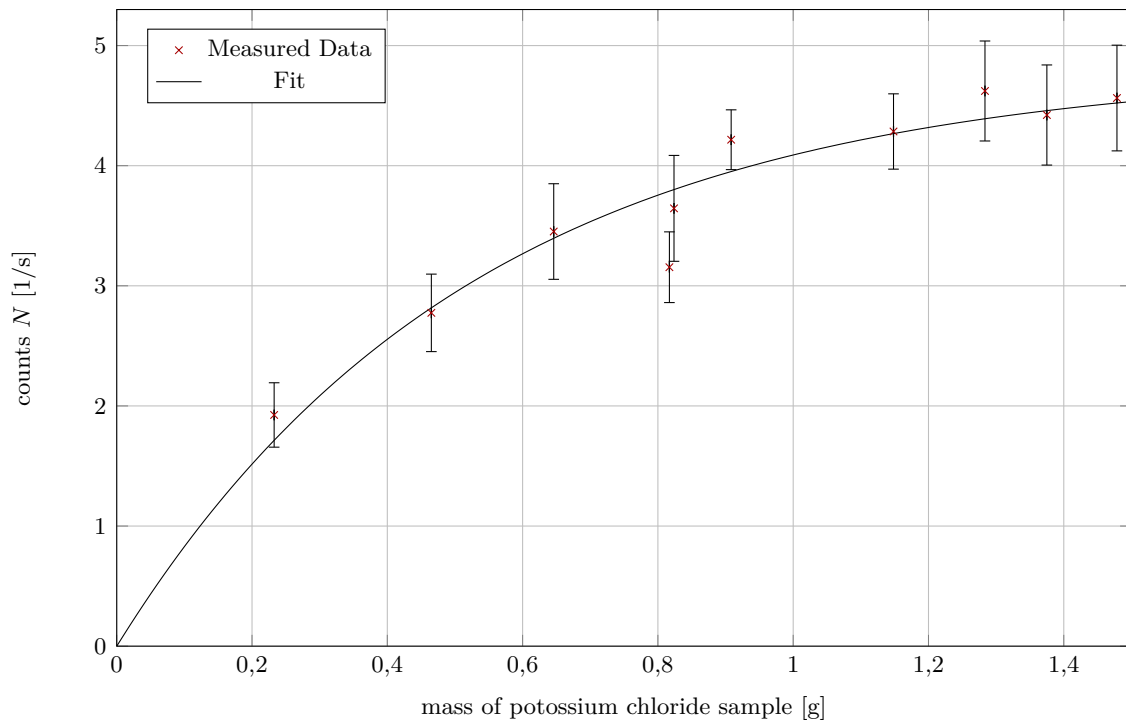

Figure 5: Measured counts per second without the background for the different masses of the sample. The line shows the fit that was made to determine the half-lifetime of potassium chloride.

errors that are shown in the plot were calculated the same way as for samarium. The error on the fit in form of a covariance matrix is also given by the fit program. To evaluate the fit a χ-square was calculated with

$$\chi^2 = \left(\frac{N_{\text{meas.}} - N_{\text{fit}}}{s_N} \right)^2 \tag{27}$$

and the reduced χ-square simply follows by dividing by the free parameters so for the reduced value we get

$$\chi^2_{\text{red.}} = \frac{\chi^2}{8} = 1.13. \tag{28}$$

To find the half-lifetime the slope at $m = 0$ is calculated with

$$\mathfrak{Z} = \frac{\mathrm{d}f}{\mathrm{d}m}\bigg|_{m=0} = a \cdot b. \tag{29}$$

The error on this is calculated with the covariance matrix by

$$s_{\mathfrak{Z}}^2 = \begin{pmatrix} \frac{\partial \mathfrak{Z}}{\partial a} \\ \frac{\partial \mathfrak{Z}}{\partial b} \end{pmatrix} \cdot \begin{pmatrix} 0.13\,\frac{1}{\mathrm{s}^2} & 0.12\,\frac{1}{\mathrm{s\,g}} \\ 0.12\,\frac{1}{\mathrm{s\,g}} & 0.13\,\frac{1}{\mathrm{g}^2} \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial \mathfrak{Z}}{\partial a} & \frac{\partial \mathfrak{Z}}{\partial b} \end{pmatrix}. \tag{30}$$

The specific activity can now be calculated with

$$A_S = \frac{2\mathfrak{Z}}{g} = \frac{2ab}{g}, \tag{31}$$

where $g = 1.12$. The error is given with gaussian error propagation to

$$s_{A_S} = \frac{2s_{\mathfrak{Z}}}{g}. \tag{32}$$

Last the relative frequency $h = 0.0118$ percent and the molar mass of potassium chloride $M_{\mathrm{mol.}} = 74.55\,\frac{\mathrm{g}}{\mathrm{mol}}$ are needed. With use of the specific activity the half-lifetime can be calculated with

$$\begin{aligned} T_{1/2}^{\mathrm{K}} &= \frac{\ln(2)N_A h}{1.12 A_S M_{\mathrm{mol.}}} = (1.33 \pm 0.17) \cdot 10^9 \text{ years} \\ s_T &= \frac{\ln(2)N_A h s_{A_S}}{(1.12 A_S M_{\mathrm{mol.}})^2}. \end{aligned} \tag{33}$$

4 Discussion

In this experiment it was possible to determine the half-life times of potassium and samarium to

$$T_{1/2}^{\mathrm{Sm}} = (1.457 \pm 0.004) \cdot 10^{11} \text{ years and}$$
$$T_{1/2}^{\mathrm{K}} = (1.33 \pm 0.17) \cdot 10^{9} \text{ years.}$$

To determine the quality of the measurement the results can be compared to the literature values [Source 1] which are

$$T_{1/2}^{\mathrm{Sm}} = 1.06 \cdot 10^{11} \text{ years and}$$
$$T_{1/2}^{\mathrm{K}} = 1.28 \cdot 10^{9} \text{ years.}$$

Comparing these values one can see that it was possible to determine the half-lifetime of potassium within a 1σ range. Even tough the relative error on that value is 13% it shows that the error was calculated properly and to a reasonable value. For a more sensitive error propagation one would need to do a lot more measurements for one mass as well as for more different masses. More measurements for one mass would give the opportunity to calculate a mean and a bigger variation in different masses would optimize the fit. So the combination of both would hopefully lead to a more accurate value with a smaller error.

The measurement using samarium was not that accurate since the difference between the measured value and the literature value is 37% of the literature value. Considering that the error was calculated in a reasonable way there might be some systematics which are affecting the measurement:

One possibility is that we did not choose the working voltage properly because this affects the measured pulsing rate to a not quite neglectable extend. Furthermore the backscattering was chosen as constant but it should depend on the hight of the powder in the aluminium bowl. For a thicker layer of samarium the backscattering gets is higher and calculation the half-lifetime with a constant value anyway would lead to an underestimated value. As we did not consider this during the experiment a quite thick layer was chosen as it was easier to flatten the surface for a thick layer of potassium.

But it is still a possibility that it was just statistical "badluck" since the decay of a nucleus is a strictly statistical process.
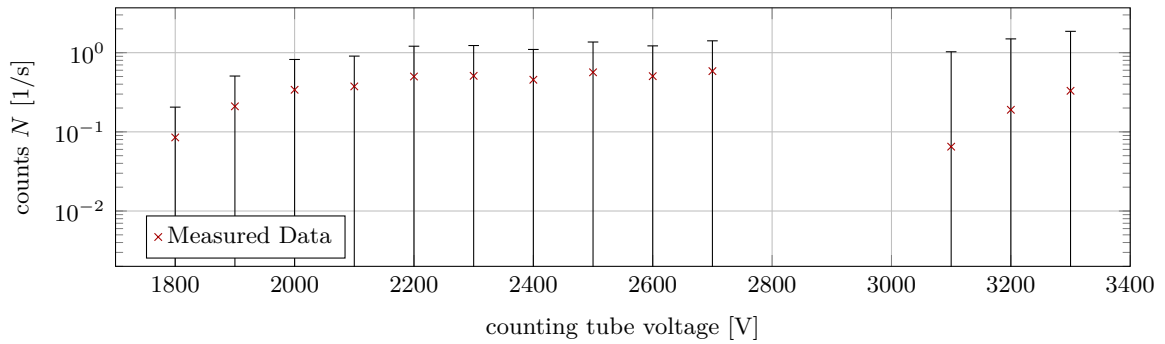
A   Appendix

Figure 6: Measurement of the α-plateau, the background has already been substracted what also lead to some rates dropping under zero. As this does not make sense these rates have been set to zero manually, nevertheless due to the logarythmic axis they are not displayed.
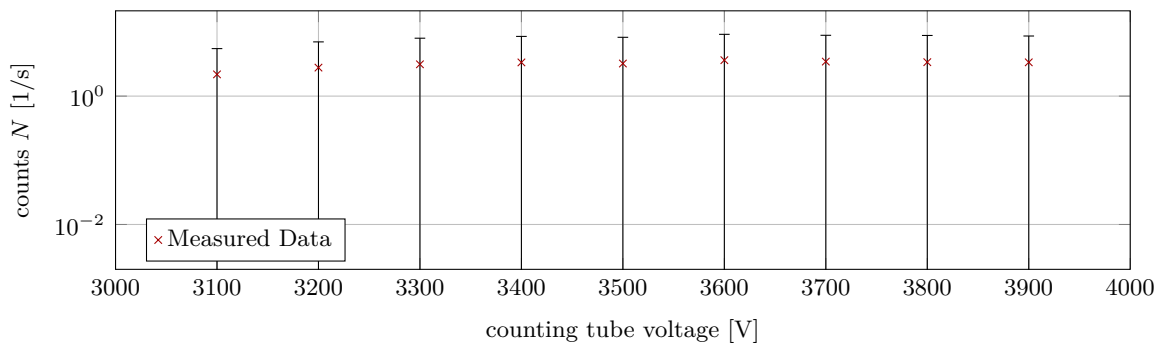

Figure 7: Measurement of the β-plateau, the background has already been substracted in this graph. The shown errorbars come from the poisson-errors for each value.

List of Figures

References

[Source 1] *"Versuchsanleitung Fortgeschrittenen Praktiukum 1, Lange Halbwertszeiten"* Stand 26.06.2012

Lange Halbwertzeiten

aufgenommene Messungen:

zählrohrcharakteristik.txt	2(a)
untergrund _ 1.txt	2(b)
alpha_ 1.txt	3(a)
alpha_ 2.txt	3(a)
alpha_ 3.txt (6250 s)	3(b)
beta_ 1.txt (m₁)	4(a)
beta_ 2.txt (m₁) (7005)	4(b)
beta_ 3.txt m_2	

Durchmesser der Aluminiumschale:

$$= 2,87 \text{ cm}$$
2,885 cm
2,89
2,86
2,83

beta_ 4	m_3	
beta_ 5	m_4	
beta_ 6	m_5	
beta_ 7	m_6	
beta_ 8	m_7	
beta_ 9	m_8	

Masse Aluminiumschale : 1,3250 g

beta_ 10 m_9

m_1 (4 $\overset{a}{b}$) = 2,1420 g
m_2 = 1,5577 g
m_3 = 1,7901 g
m_4 = 1,9710 g
m_5 = 2,1488 g
m_6 = 2,2333 g
m_7 = 2,4733 g
m_8 = 2,6085 g
m_9 = 2,7000 g
m_{10} = 2,7036 g