Albert-Ludwigs-Universität Freiburg
Advanced Physics Lab Course
Summer Semester 2022

Experiment on 12.-13.09.2022

Long Half-Life

Group 13:
14.09.2022

Assistant:

Abstract

In the experiment “long half-life“, two samples with long half-lives on the scale of 10^9 to 10^{11} years are examined and their exact half-lives are calculated. Therefore the used proportional counter is calibrated using Uranium-238 and then measurements with the α-radiator Samarium-147 and the β-radiator Potassium-40 are performed. For the samples, different methods are used to find the half-life: For Samarium-147 it is calculated by considering the surface area of the sample, whereas for Potassium-40 different masses of radioactive material are measured.

For both samples a half-life could be found that matches the scale of the real values but does not correspond with the theoretical expectation perfectly. Hence a detailed discussion part is added to the protocol.

Contents

List of Tables

List of Figures

1 Introduction

In the experiment "long half-life", two samples – Samarium-147 and Potassium-40 – are analysed and their half-lives are determined. The experiment consists of three parts. In a first part the characteristic of the used methane-proportional-counter is studied by using the α- and β-radiator Uranium-238. Then the α-radiator Samarium-147 is examined by using a correction for the self-absorption and finding the half-life by measuring counting rate and surface area of the sample. In a last step the Potassium-40 sample which is a β-radiator is investigated and its half-life is found by measuring the counting rate in dependence of the used mass. For all the measurements the background radiation is measured too, to correct the data afterwards.

2 Theory

The following sections introducing the theory and methodology necessary for the experiment are mainly based on the experiment description, which is provided by the advanced physics lab team [1] and the diploma thesis by Tobijas Kotyk [2].

2.1 Table of the used variables

Tab. 1: Table of the used symbols for the parameters used in the protocol.

Symbol	Parameter
A	activity
A_V	activity per volume
A_s	specific activity
n	counting rate
C	absolute number of counts
$T_{1/2}$	half-life
t	time (of the measurement)
N	number of radioactive atoms
N_0	starting number of radioactive atoms
$N_{Sm_2O_3}$	number of Sm_2O_3 molecules
N_A	Avogadro constant [3]
m	mass of the sample
m_{rel}	relative atomic mass
ρ	density of the used sample
F	surface area of the sample
$\tilde{d}$	diameter of the bowl
d	thickness of the sample
R	range of radiation
Ω	solid angle
λ	decay constant
h_{rel}	relative frequency
f_B	backscatter factor
μ	absorption coefficient
$U_{start/stop/step}$	starting/stopping/step voltage

2.2 Radioactive decay

Radioactivity results from unstable cores that decay with emission of radiation. Especially isotopes (atoms with the same number of protons but another number of neutrons) can decay in a statistical process. This process is characterised by the following differential equation and its solution [2]:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\lambda N, \tag{1}$$

$$N(t) = N_0 e^{-\lambda t}. \tag{2}$$

N is the number of cores that are observed, N_0 the starting amount of cores and λ the decay constant. An important property of an isotope is the half-life. It is defined as the time, after which half of the cores have decayed. One can get the following formula [2]:

$$T_{1/2} = \frac{\ln 2}{\lambda}. \tag{3}$$

By introducing the activity A one can get:

$$A = \lambda N = \frac{\ln 2 N}{T_{1/2}}. \tag{4}$$

To get the half-life of an isotope in this experiment, the activity is measured in two different ways as described in subsection 2.5.

2.2.1 α-decay

The first sort of radioactive decays is the α-decay, where a core emits an α-particle which is a helium-core with two protons and two neutrons. This decay type is the one that has the highest decay energy but also the lowest range in matter, because of its high mass and twice the elementary charge which leads both to high interaction with matter. The α-decay is most likely for cores with high mass numbers of $m \geq 150\,\mathrm{u}$ [2]. For small cores α-decay gets less likely, because the energy gain from α-decay gets smaller too.

2.2.2 β-decay

Another sort of radioactive decay is the β-decay. There are two sorts of β-decay that are distinguished – the β^--decay where a neutron decays into a proton, an electron and an anti-electron-neutrino ($n \to p + e^- + \bar{\nu}_e$) and the β^+-decay where a proton decays into a neutron, a positron and an electron-neutrino ($p \to n + e^+ + \nu_e$) [2]. Because of the resulting particles, β-radiation is less energetic than α-radiation, but instead can travel longer distances in matter.

2.2.3 Electron capture

The last relevant process for this experiment is electron capture. Here an electron in a close shell can be captured by a proton in the core, forming a neutron and an electron-neutrino ($p + e^- \to n + \nu_e$). The gap in the shell than is filled by an electron from a higher shell by sending out roentgen or auger radiation [2].

2.3 Detection of the particles

In the experiment a proportional counter filled with methane is used. In a gas detector, by applying a voltage, an electric field is created. Particles that pass through the detector than can ionise the gas and the charges than drift towards anode or cathode. Thus a charge is also induced in the wires and a pulse can be registered in the voltage. This pulse is proportional to the energy loss of the particle [2].

2.3.1 Gas amplification and proportional area

An important effect in a proportional counter is the gas amplification. The primarily generated charges increase by ionising more gas molecules [2]. Gas amplification is working best for high voltages and a thin wire, which is both used in the experiment.

A proportional counter has multiple areas of work as portrayed in Figure 1. In the first two areas the voltage is too low for gas amplification. In the third area gas amplification leads to a pulse height proportional to the primarily ionised molecules. This is the area where measurements can be taken. The remaining areas leave the proportional sector and provide discharges of anode or cathode.

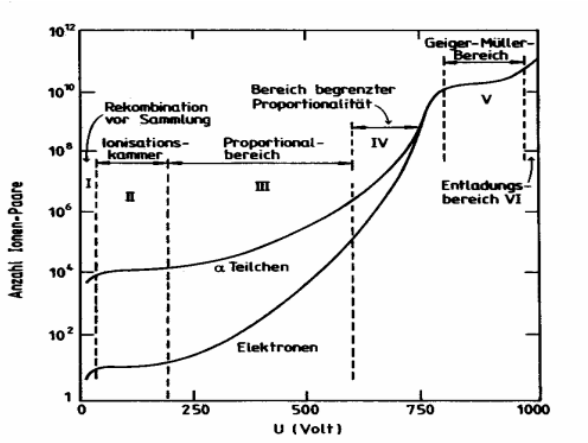


Fig. 1: In the picture one can see the different areas of work of a proportional counter. On the y-axis the number of ion-pairs is plotted against the voltage U on the x-axis. The graphic is taken from [2], p. 35.

As mentioned in subsection 2.2, α-radiation has a higher energy and therefore can be detected with lower voltage compared to β-radiation. This leads to two plateaus being visible in the characteristic of a proportional counter with one of the plateaus beginning when α-radiation is detected and the other one when β-radiation can be measured.

2.4 Characteristics of the used materials

2.4.1 Uranium-238

For calibration purposes a sample of Uranium-238 is used. It has a relative frequency of 99.275% and decays in three main reactions [2]:

1. α-decay from Uranium-238 to Thorium-234
2. β^--decay from Thorium-234 to Protactinium-234
3. β^--decay from Protactinium-234 to Uranium-234

Therefore both sorts of radioactive decay can be measured and a good calibration is possible.

2.4.2 Samarium-147

As an α-radiator Samarium-147 with a relative frequency of 14.87% is used. In the experiment a powder of Samariumoxide (Sm_2O_3) is examined. Samarium-147 α-decays into Neodymium-143, which is a stable isotope [2].

2.4.3 Potassium-40

With a relative frequency of 0.0117% as a β-radiator Potassium-40 is used. In the experiment one uses Potassiumchloride (KCl). Two processes take place in the Potassium-40 sample [2]:

1. β^--decay from Potassium-40 to Calcium-40
2. electron capture (EC) from Potassium-40 to Argon-40

The proportion of the two processes is $EC/\beta^- = 0.12$, so the measurement must be corrected with a correction term as described in the next section.

2.5 Corrections for the α- and β-measurement

2.5.1 Corrections for the α-measurement

For the α-measurement, effects like self-absorption are corrected by using the activity per Volume A_V and then measuring the surface area F of the sample [2]:

$$A_V = \frac{A}{Fd} = \frac{4n}{FR_{\mathrm{Sm_2O_3}}}. \tag{5}$$

d is the thickness of the sample and $R_{\mathrm{Sm_2O_3}}$ the range of the radiation. In addition the number N of atoms can be approximated by the following formula [2]:

$$N = 2N_{\mathrm{Sm_2O_3}}h_{\mathrm{rel}} = 2\frac{mN_A}{m_{\mathrm{rel}}}h_{\mathrm{rel}}, \tag{6}$$

with $N_{\mathrm{Sm_2O_3}}$ being the number of molecules, the relative frequency h_{rel}, the sample-mass m, the relative atomic mass m_{rel} and the Avogadro constant N_A. Plugging all the formulas into the law of radioactive decay and using the density $\rho_{\mathrm{Sm_2O_3}}$ one can get the following formula for the half-life, as used in the analysis part:

$$\begin{aligned}
T_{1/2} &= \ln(2)\frac{N}{A}, && (7)\\
&= \ln(2)\frac{N}{A_V Fd}, && (8)\\
&= \ln(2)\frac{NR_{\mathrm{Sm_2O_3}}}{4nd}, && (9)\\
&= \ln(2)\frac{N_{\mathrm{Sm_2O_3}}R_{\mathrm{Sm_2O_3}}h_{\mathrm{rel}}}{2nd}, && (10)\\
&= \ln(2)\frac{mN_AR_{\mathrm{Sm_2O_3}}h_{\mathrm{rel}}}{2ndm_{\mathrm{rel}}}, && (11)\\
&= \ln(2)\frac{\rho_{\mathrm{Sm_2O_3}}R_{\mathrm{Sm_2O_3}}N_Ah_{\mathrm{rel}}F}{2nm_{\mathrm{rel}}}. && (12)
\end{aligned}$$

2.5.2 Corrections for the β-measurement

As already mentioned, a correction term for the electron capture has to be added to the law of radioactive decay. One can get the correction factor 1.12 for the decay formula by taking the ratio between electron capture and β-radiation [2]:

$$T_{1/2} = \ln(2)\frac{N}{1.12A}. \tag{13}$$

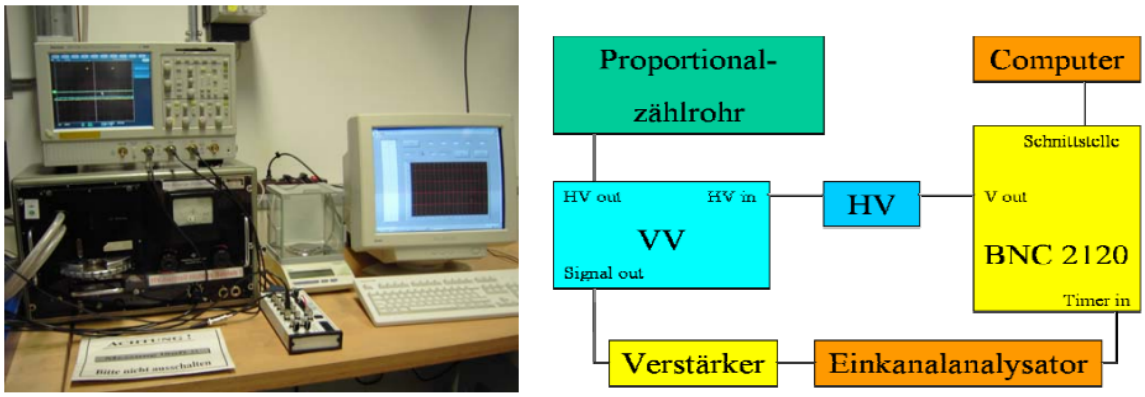
To calculate the activity, a relation between specific activity $A_s = A/m$ and the counting rate n is used [2]:

$$n(m) = f_B \frac{\Omega}{4\pi} A_s \frac{F\rho}{\mu} \left(1 - \exp\left(-\frac{\mu}{F\rho} m\right)\right). \tag{14}$$

f_B is the backscatter factor, $\Omega = 2\pi$ the solid angle in which radiation can be measured and μ the absorption coefficient. As before ρ is the density, m the mass and F the surface area. In the analysis part, the activity can be found by taking measurements for different masses and then fitting a $a(1 - e^{-bm})$-function to the data.

3 Setup and measurements

To be able to calculate the half-life of the given samples, one has to measure the counting rate of different samples of the three materials. Therefore a proportional counter filled with methane is used. The counter is run on a high voltage generator which can vary between 1000 V and 4000 V. After measuring, the signal passes a pre-amplifier and a main-amplifier and enters a single-channel-analyser which outputs a square-pulse for each signal. With an analogue-to-digital converter the signal is than passed to a computer. The signals after pre-amplifier, amplifier and single-channel-analyser are first observed with an oscilloscope to guarantee, that all the signals are counted properly. The setup for the experiment can be seen in Figure 2.



(a) Setup for the measurement

(b) Block diagram of the different electronic parts

Fig. 2: In the left picture the setup for the measurement is presented. In the right picture all the electronic components and their cabling are shown. The proportional counter is connected to the pre-amplifier (VV), the amplifier (Verstärker) and the single-channel-analyser (Einkanalanalysator). In addition the high voltage generator (HV) and the analogue-to-digital converter (BNC 2120) are shown. Both pictures are taken from [2], p. 30.

The setup can be controlled with the `LabVIEW`-Software. A program for the measurement is already provided. In Figure 3 one can find the interface of the program. The following parameters can be configured: starting voltage, end voltage, step voltage, measuring time and time between the measurements. The data then is displayed in a table and a graphic and it can be exported.

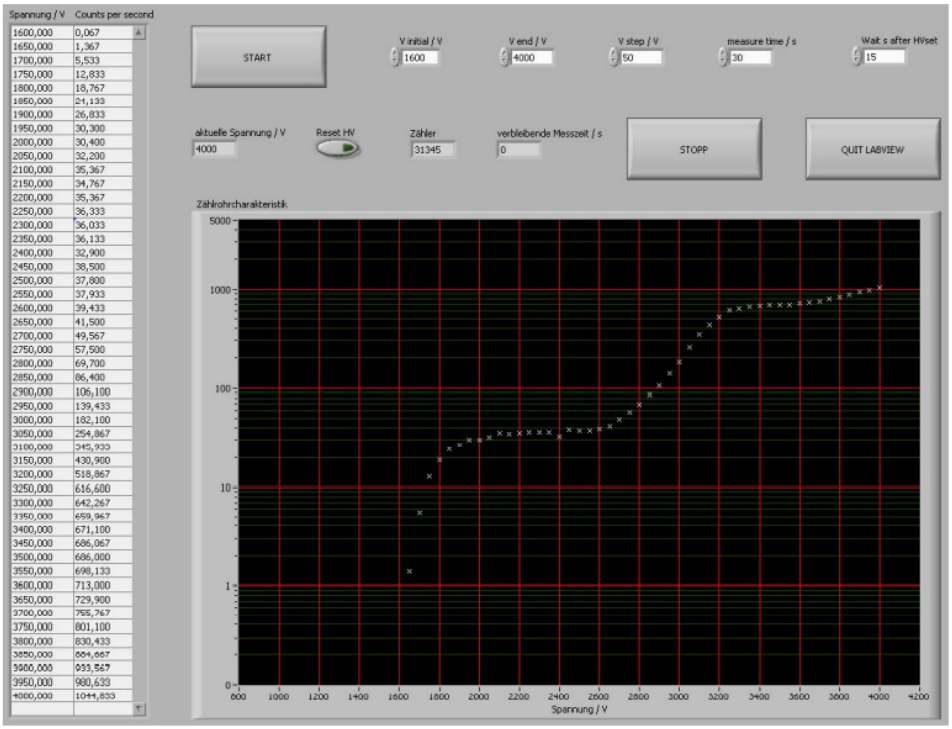


Fig. 3: In the picture one can see the interface of the software `LabVIEW` used for the measurement. On the top the different parameters can be configured, on the bottom a graphic is plotted and on the left hand side the data is summarised in a table. The picture is taken from [2], p. 50.

In a first step two measurements are taken to get the characteristic of the proportional counter. Therefore one measurement is taken with Uranium-238 and one background measurement with an empty bowl. For the Samarium-147, first a measurement of the α-plateau is performed followed by a long measurement of the counting rate in the middle of the plateau. In addition again the background is measured and the surface area of the sample is assessed by measuring the diameter of the bowl. In a last step measurements of Potassium-40 are taken. Again, first the β-plateau is measured, followed by ten measurements at a constant voltage with different masses of Potassium-40, weighed with a high precision weighing balance. To complete the measurements the background is measured for this setup as well.

For all the measurements the parameters are modified to get an improved measurement and guarantee an uncertainty of about 2%. The used parameters are mentioned in the analysis part and can be found in Table 3 in the appendix.

4 Data analysis and discussion of uncertainties

4.1 Counting tube characteristic with ^{238}U

To determine the half-life of ^{147}Sm and ^{40}K their respective plateau has to be found. Therefore the characteristic of the counting tube has to be evaluated. Thus ^{238}U is used, because it emits both α- and β-radiation.

For the measurement of the counting tube characteristic, voltages between 1000 V and 4000 V are applied to the proportional counter with steps of $U_{\text{step}} = 100\,\text{V}$ and a measuring time of $t = 50\,\text{s}$. There is also a waiting time of 15 s between each measurement so that the voltage can stabilise itself. The counts C for each voltage are of statistical nature, so the underlying distribution is the Poisson distribution. As a result the uncertainty for the counts is given as $\sqrt{C}$. The resulting counting rate n of ^{238}U is calculated with Equation 15 and its uncertainty [4] with Equation 16:

$$n = \frac{C}{t}, \tag{15}$$

$$\Delta n = \frac{\sqrt{C}}{t} = \frac{\sqrt{nt}}{t} = \sqrt{\frac{n}{t}}. \tag{16}$$

To find the actual counting rates of each sample the background radiation has to be evaluated, for which a measurement with an empty aluminium bowl is performed. The voltages are again set between 1000 V and 4000 V with steps of $U_{\text{step}} = 100\,\text{V}$. The measuring time is configured to $t = 100\,\text{s}$ and the waiting time to 15 s. The resulting background counting rate is again calculated with Equation 15 and Equation 16. The plotted counting rates n of ^{238}U and the background and their respective uncertainties can be found in Figure 4.

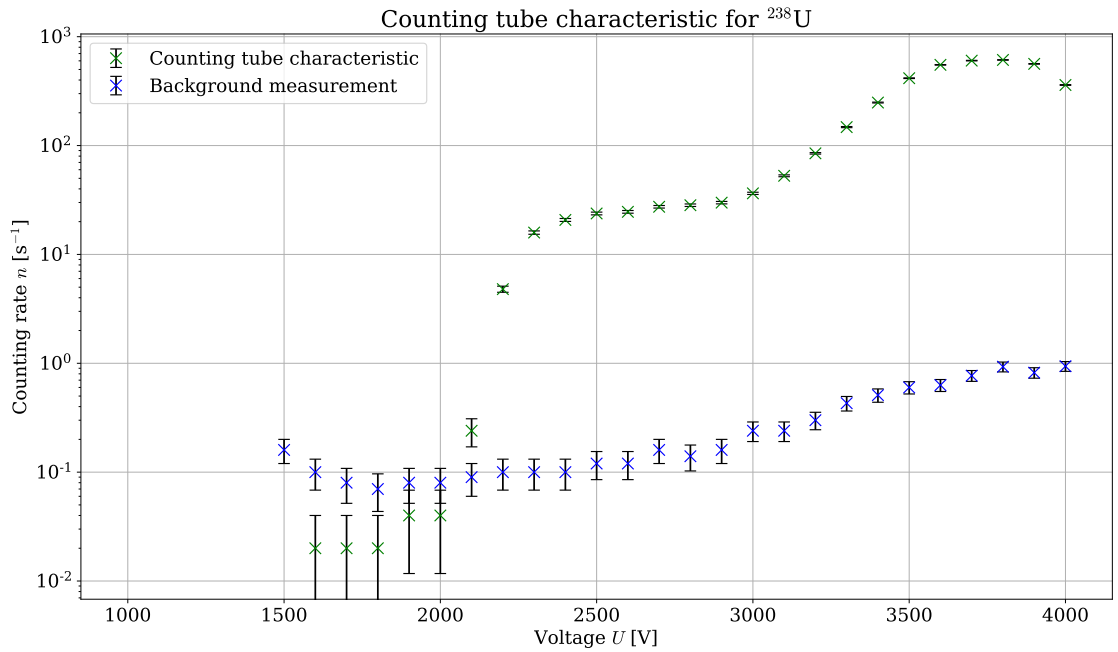


Fig. 4: In this graphic the counting rate n in s^{-1} is plotted against the voltage U in V on a double-logarithmic scale. The green values represent the measurement with ^{238}U and the blue values represent the background counting rate.

In Figure 4 the first detectable counting rate of ^{238}U is at 1600 V, whereas the background counting rate is measurable prior to this voltage and is also higher than the ^{238}U counting rate until 2000 V. Reasons for this behaviour are discussed in subsection 5.3. Nevertheless there are already two plateaus visible for the ^{238}U measurement. To get a more well-found perception of the two plateaus the counting rate n of ^{238}U can be corrected with the background counting rate n_{back} with Equation 17 and Equation 18. The uncertainty can be calculated by Gaussian propagation of uncertainties [4]:

$$n_{\text{corr}} = |n - n_{\text{back}}|, \tag{17}$$

$$\Delta n_{\text{corr}} = \sqrt{(\Delta n)^2 + (\Delta n_{\text{back}})^2}. \tag{18}$$

The corrected counting rate n_{corr} of ^{238}U can be seen in Figure 5.

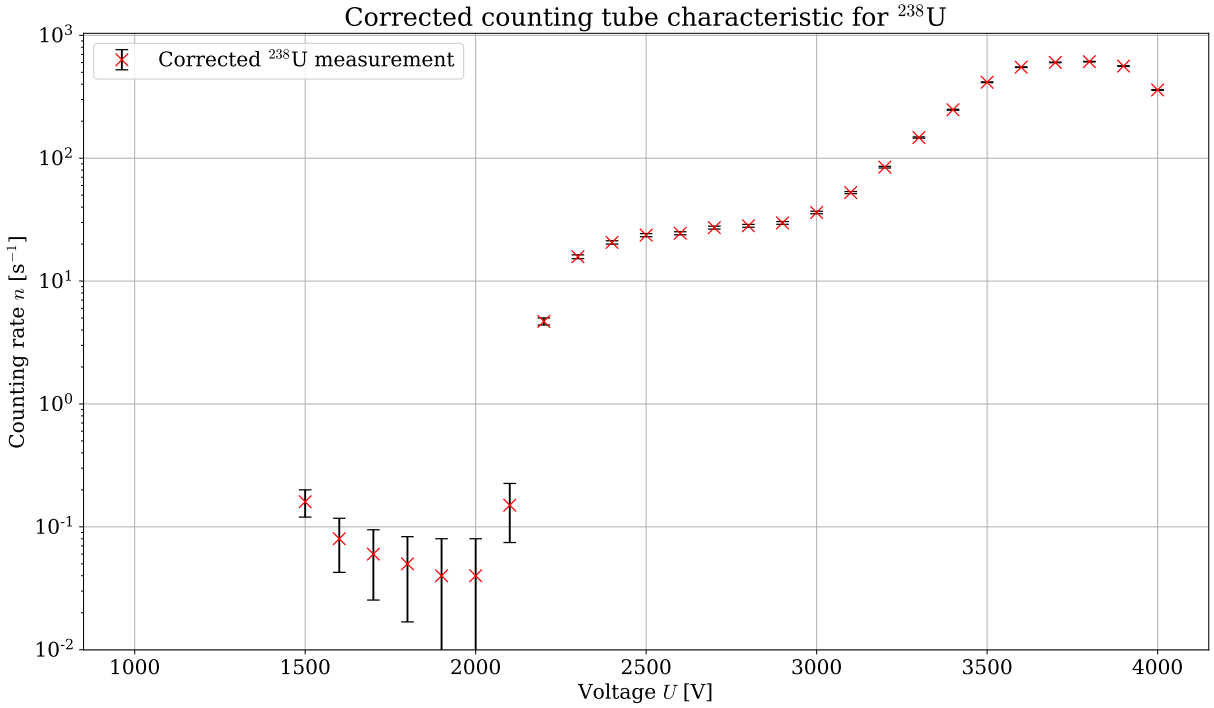


Fig. 5: In this graphic the corrected counting rate of ^{238}U n in s^{-1} is plotted against the voltage U in V on a double-logarithmic scale.

The α- and β-plateau can now be found more easily. The α-plateau reaches approximately from 2300 V to 3000 V and the β-plateau reaches roughly from 3500 V to 3900 V.

4.2 Determination of the half-life of ^{147}Sm

In order to find the half-life of ^{147}Sm a good working voltage in the middle of the α-plateau has to be located, which can later be used to perform a long measurement of the sample and the background. Therefore a measurement from the beginning of the α-plateau at 2300 V to the middle of the β-plateau at 3700 V is taken with a step width of $U_{\text{step}} = 100$ V, a measuring time of $t = 200$ s and a waiting time of 15 s. The assessment of the plateau, with the counting rate n being corrected with the background measurement of ^{238}U as in Equation 17, can be found in Figure 6.

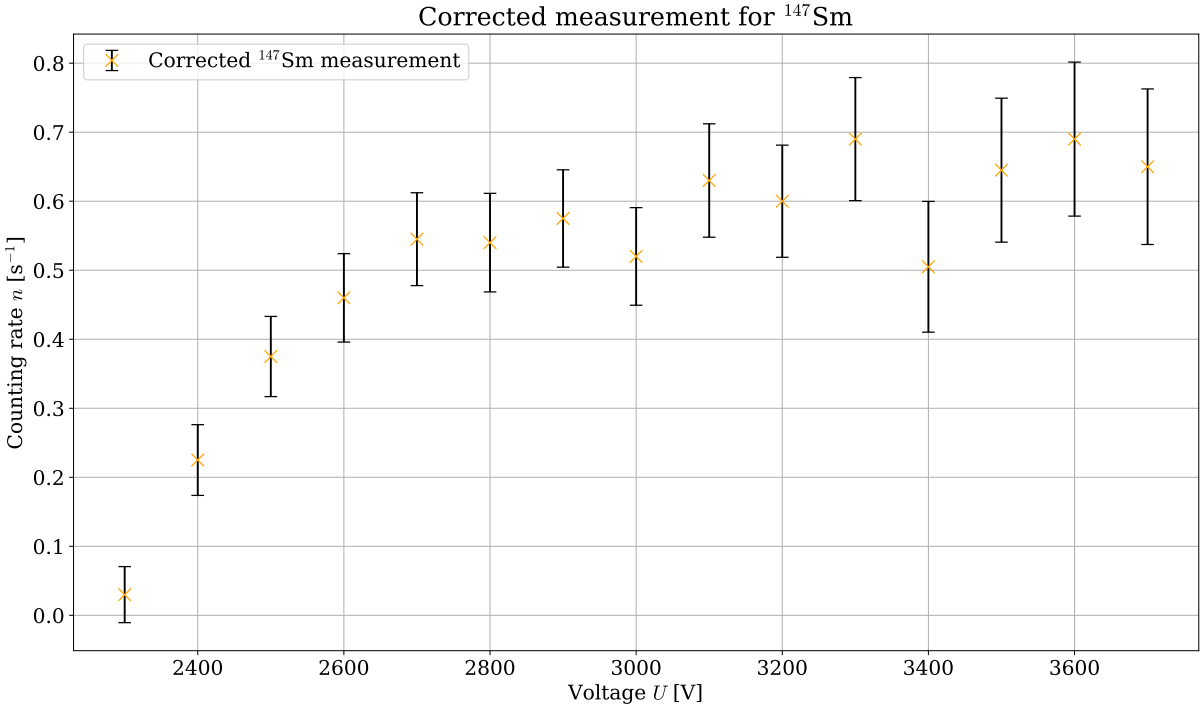


Fig. 6: In this graphic the counting rate n in s^{-1} of ^{147}Sm is plotted against the voltage U in V, which is applied to the proportional counter. The counting rate n is corrected with the background measurement of ^{238}U.

To minimise the uncertainty to about 2% a long enough measuring time has to be selected which can be estimated with Equation 22:

$$\frac{\Delta n}{n} = 0.02, \tag{19}$$

$$\Rightarrow \frac{\sqrt{n}}{n\sqrt{t}} = 0.02, \tag{20}$$

$$\Rightarrow \frac{1}{nt} = 0.0004, \tag{21}$$

$$\Rightarrow \frac{1}{0.0004n} = t. \tag{22}$$

As a working voltage 2800 V is chosen for the long measurement of ^{147}Sm. By plugging in the uncorrected counting rate of ^{147}Sm at 2800 V into Equation 22, which is $n = 0.700\,\mathrm{s}^{-1}$, the resulting measuring time is $t = 3571\,\mathrm{s}$. Therefore a measuring time of 4000 s is chosen. The resulting counting rate, which is calculated with Equation 15 and Equation 16 is:

$$n = (0.699 \pm 0.013)\,\mathrm{s}^{-1}.$$

As a result the relative uncertainty [4] of the counting rate is:

$$\frac{\Delta n}{n} = 1.9\%.$$

Afterwards, a long measurement of the background radiation is conducted, to correct the assessed counting rate of ^{147}Sm. This measurement has to be as long as possible to minimise

the uncertainty of the background radiation. For reasons discussed in subsection 5.3, it had to be taken twice and the shorter measurement is used for analysis. The voltage is again set to 2800 V and the measuring time of the chosen background measurement is 7000 s. The following counting rate of the background could be found analogously as before:

$$n_{\text{back}} = (0.186 \pm 0.005)\,\text{s}^{-1}.$$

With Equation 17 and Equation 18 the corrected counting rate of ^{147}Sm is:

$$n = (0.513 \pm 0.014)\,\text{s}^{-1}.$$

To compute the half-life of ^{147}Sm, the surface area of the sample has to be calculated, for which the used bowl has to be measured. Therefore the mean of six independent measurements is taken and its uncertainty is calculated with the standard deviation of the mean [4]. Thus no uncertainties have to be estimated on each single measurement. The measured diameters and the mean are listed in Table 2:

Tab. 2: Table of the measured diameters of the ^{147}Sm bowl. There are no uncertainties for each measurement of the diameter, because only the mean is important and its uncertainty is calculated with the standard deviation of the mean [4].

Measurement	Diameter
1	2.88 cm
2	2.88 cm
3	2.89 cm
4	2.87 cm
5	2.88 cm
6	2.875 cm
Mean $\tilde{d}$	(2.879 ± 0.003) cm

With the mean diameter $\tilde{d}$ it is now possible to determine the surface of the bowl and calculate its uncertainty with Gaussian propagation of uncertainties [4]:

$$F = \pi \frac{\tilde{d}^2}{4}, \tag{23}$$

$$\Delta F = \pi \frac{\tilde{d}\Delta\tilde{d}}{2}, \tag{24}$$

$$\Rightarrow F = (6.511 \pm 0.012)\,\text{cm}^2.$$

To calculate the half-life of ^{147}Sm the following formula, which is introduced in the theory part in Equation 7, is used with the uncertainty calculated from Gaussian propagation of uncertainties [4]:

$$T_{1/2} = \ln(2)\frac{\rho_{Sm_2O_3} R_{Sm_2O_3} N_A h_{\text{rel}} F}{2 n m_{\text{rel}}}, \tag{25}$$

$$\Delta T_{1/2} = \ln(2)\frac{\rho_{Sm_2O_3} R_{Sm_2O_3} N_A h_{\text{rel}}}{2 m_{\text{rel}}} \cdot \sqrt{\left(-\frac{\Delta F}{n}\right)^2 + \left(-\frac{F\Delta n}{n^2}\right)^2}. \tag{26}$$

Therefore we need to apply the Bragg-Kleeman relation to get $\rho_{Sm_2O_3} R_{Sm_2O_3}$ [2]:

$$\rho_{Sm_2O_3} R_{Sm_2O_3} = \rho_{\text{Air}} R_{\text{Air}} \sqrt{\frac{m_{A,Sm_2O_3}}{m_{A,\text{Air}}}}. \tag{27}$$

The values for m_{rel}, $\sqrt{m_{\text{A,Sm}_2\text{O}_3}}$ and $\sqrt{m_{\text{Air}}}$ can be calculated by using the atom masses from [5] and the relative share p_i of the atoms and the formulas from [1]:

$$\begin{aligned} m_{\text{rel}} &= 2m_{\text{Sm}} + 3m_{\text{O}} = 348.717\,\text{u}, \\ \sqrt{m_{\text{A,Sm}_2\text{O}_3}} &= \frac{2m_{\text{Sm}}}{m_{\text{rel}}}\sqrt{2m_{\text{Sm}}} + \frac{3m_{\text{O}}}{m_{\text{rel}}}\sqrt{3m_{\text{O}}} = 15.908\,\sqrt{\text{u}}, \\ \sqrt{m_{\text{A,Air}}} &= p_{\text{N,Air}}\sqrt{2m_{\text{N}}} + p_{\text{O,Air}}\sqrt{2m_{\text{O}}} + p_{\text{Ar,Air}}\sqrt{m_{\text{Ar}}} = 5.387\,\sqrt{\text{u}}. \end{aligned}$$

The values for the masses of oxygen and nitrogen in the air have to be taken twice, because both of them occur bonded in pairs (O_2 and N_2).

As a result we receive the following half-time for ^{147}Sm with Equation 25:

$$T_{1/2} = (1.46 \pm 0.04) \times 10^{11}\,\text{years}.$$

A comparison with the half-time value in literature can be found in subsection 5.1.

4.3 Determination of the half-life of ^{40}K

To evaluate the half-life of ^{40}K again a good working voltage has to be found. Due to ^{40}K being a β-radiator the β-plateau has to be evaluated. Therefore a measurement with voltages between 3400 V and 4000 V is conducted with a step width of $U_{\text{step}} = 100\,\text{V}$, a measuring time of $t = 100\,\text{s}$ and a waiting time of 30 s. Analogously to ^{147}Sm the counting rate is corrected with the background measurement of ^{238}U with Equation 15 and a plot can be found in Figure 7.

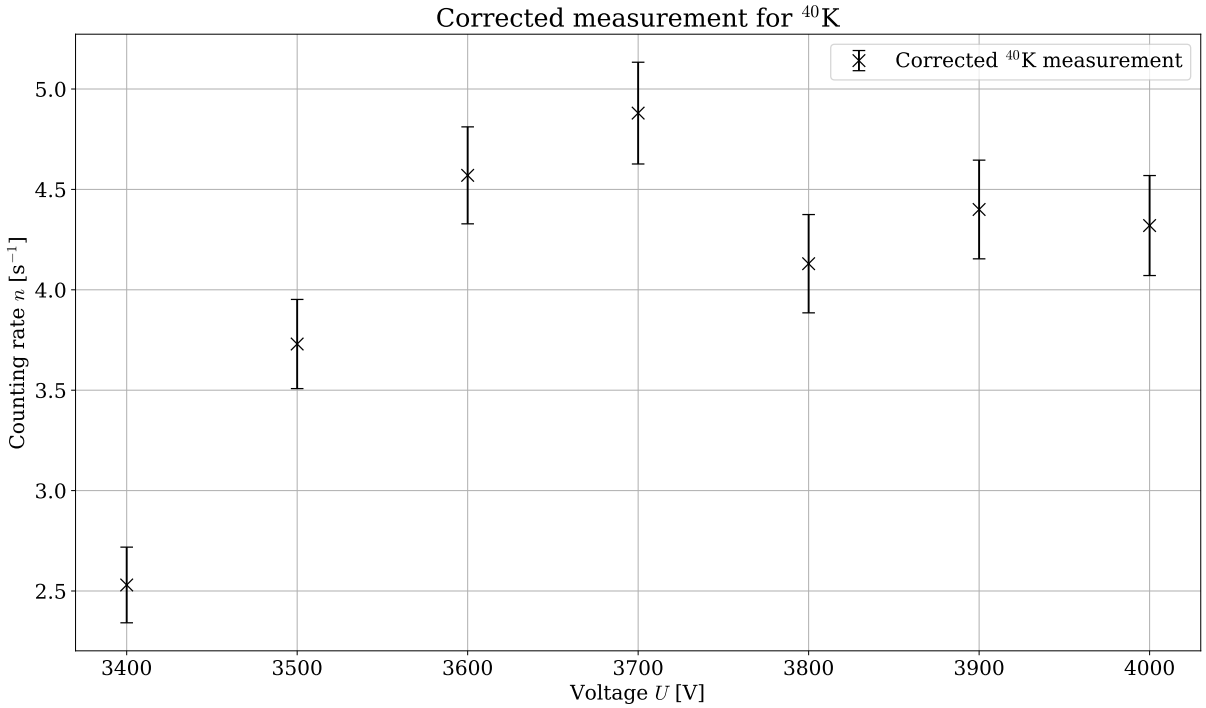


Fig. 7: In this graphic the counting rate n in s^{-1} of ^{40}K is plotted against the voltage U in V, which is applied to the proportional counter. The counting rate n is corrected with the background measurement of ^{238}U.

As a feasible working voltage for the upcoming measurement of the counting rates of different masses of ^{40}K, 3800 V is chosen. To estimate the measuring time of the multiple different

masses, to once again get a relative uncertainty of under 2%, Equation 22 is used. Plugging in the observed counting rate at 3800 V, which is $n = 5.060$, leads to a measuring time of 494 s. The selected masses m, the measuring time t, the counting rate n and the relative uncertainty of each measurement are summarised in Table 4 in the appendix.

To correct these calculated counting rates again a long background measurement is taken with the bowl of ^{40}K. Analogously to the prior calculations, the resulting background counting rate is:

$$n_{\text{back}} = (0.728 \pm 0.016)\,\text{s}^{-1}.$$

In a next step, the counting rates of the different masses are corrected with the background counting rate and then a data fit is performed in order to calculate the half-life of ^{40}K. The expected relation between the counting rate and the mass can be found in Equation 14 in the theory part. The function we use to fit the data is given in [2]:

$$n(m) = a\,(1 - \exp(-bm))\,, \tag{28}$$

$$b = \frac{\mu}{F\rho}, \tag{29}$$

$$a = f_B \frac{1}{2} A_s \frac{F\rho}{\mu} = \frac{f_B A_s}{2b}. \tag{30}$$

The corrected counting rates and the data fit, which was performed with `scipy.optimize.curve_fit`, can be found in Figure 8.

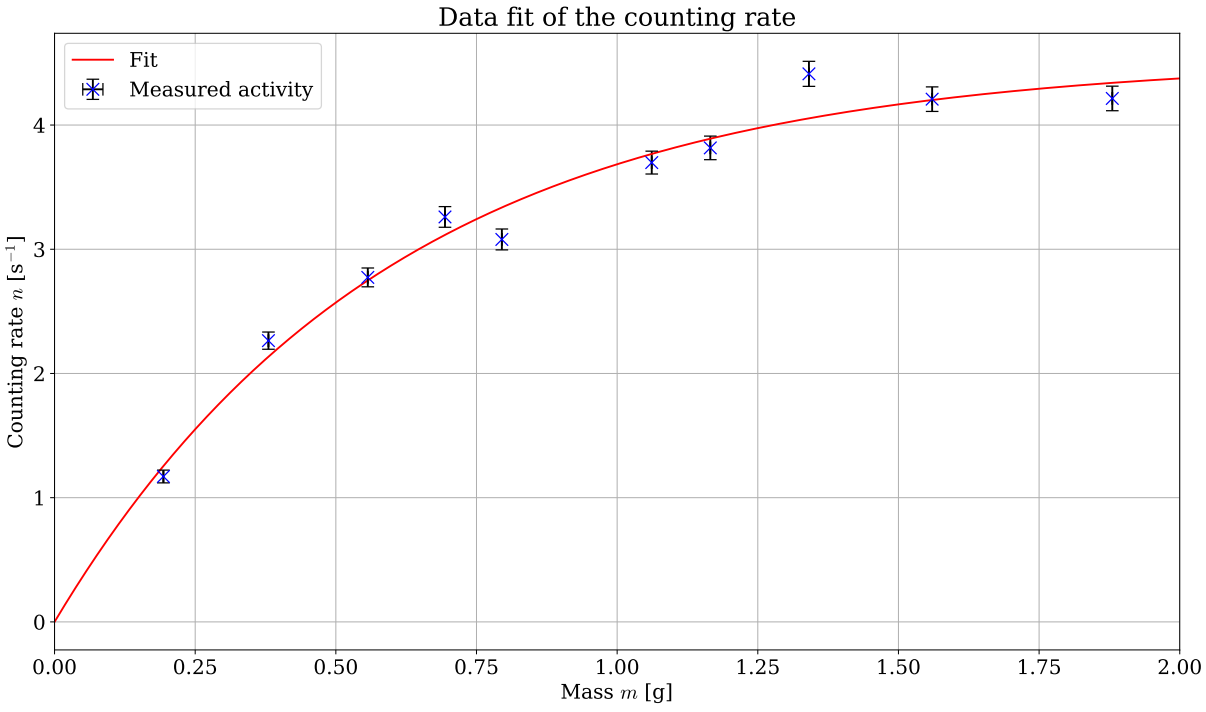


Fig. 8: In this graphic the counting rate n in s^{-1} of ^{40}K is plotted against the mass m of ^{40}K in g. The counting rate n is corrected with the background measurement. Additionally, the data fit is portrayed in the plot.

The values for a and b from the fit and their correlation factor $\mathrm{Cov}(ab)$ [6] are:

$$\begin{aligned} a &= (4.53 \pm 0.09)\,\mathrm{s}^{-1}, \\ b &= (1.67 \pm 0.07)\,\mathrm{g}^{-1}, \\ \mathrm{Cov}(ab) &= -0.006\,\mathrm{s}^{-1}\,\mathrm{g}^{-1}. \end{aligned}$$

A χ^2-value is calculated to give an approximation for the quality of the fit [6]. A χ^2-value of $\chi^2 \approx n - f$ where n is the number of data points – in our case 10 – and f is the number of fit parameters – in our case 2 – characterises a good fit. If $\chi^2 \gg n - f$, the fit is not a good match for the data [6].

$$\begin{aligned} \chi^2 &= 34, \\ n - f &= 8. \end{aligned}$$

The difference in these two values is discussed in subsection 5.3.

To calculate the half-time of ^{40}K Equation 13, that is introduced in the theory part, is used:

$$T_{1/2} = \frac{\ln(2)}{1.12} \cdot \frac{N}{A}. \tag{31}$$

By rearranging the formula for a Equation 30 and with N from [2] we get:

$$A = m\frac{2ab}{f_B}, \tag{32}$$

$$N = \frac{mN_A h_{\mathrm{rel}}}{m_{\mathrm{rel}}}, \tag{33}$$

$$\Rightarrow T_{1/2} = \frac{\ln(2)}{1.12} \cdot \frac{N_A h_{\mathrm{rel}} f_B}{2ab m_{\mathrm{rel}}}, \tag{34}$$

$$\Rightarrow \Delta T_{1/2} = \frac{\ln(2)}{1.12} \cdot \frac{N_A h_{\mathrm{rel}} f_B}{2m_{\mathrm{rel}}} \sqrt{\left(-\frac{\Delta a}{a^2 b}\right)^2 + \left(-\frac{\Delta b}{ab^2}\right)^2 + 2\frac{1}{a^3 b^3}\mathrm{Cov}(ab)} \tag{35}$$

The uncertainty is again calculated by Gaussian propagation of uncertainties, taking into consideration the correlation between a and b [6]. To calculate the half-time we need to compute m_{rel} with masses from [5]:

$$m_{\mathrm{rel}} = m_{\mathrm{K}} + m_{\mathrm{Cl}} = 74.548\,\mathrm{u}$$

Finally the half-life can be determined with the values for h_{rel} and f_B from [1]:

$$T_{1/2} = (1.59 \pm 0.04) \times 10^9\,\mathrm{years}.$$

A comparison with the half-time value in literature can be found in subsection 5.1.

5 Summary and discussion of the results

5.1 Comparison with expectation

A comparison between measured values and the values in literature can be done by using a t-value. The t-value is calculated by dividing the difference between measured and real value with the uncertainty of the measured value. A t-value smaller than two corresponds with a good measurement. For the half-life of ^{147}Sm we get the following t-value with the value in literature – $t_{\text{Lit}} = 1.06 \times 10^{11}$ years – being taken from [1]:

$$t = 10.0.$$

The value in literature for ^{40}K – $t_{\text{Lit}} = 1.28 \times 10^{9}$ years – can also be found in [1] and the t-value is:

$$t = 7.1.$$

Possible reasons for this discrepancy in the t-value are debated in subsection 5.3.

5.2 Summary of the results

In the first part of the experiment the counting tube characteristic was measured with ^{238}U and the α-plateau could be found for voltages from approximately 2300 V to 3000 V and the β-plateau is roughly from 3500 V to 3900 V.

The half-life of ^{147}Sm has been determined by using the method of measuring the diameter of the bowl and thereof calculating the surface area. As a result the half-life of ^{147}Sm is:

$$T_{1/2} = (1.46 \pm 0.04) \times 10^{11} \text{ years}.$$

A comparison with the value in literature lead to a suboptimal compatibility with a t-value of:

$$t = 10.0.$$

In the third part of the experiment the half-life of ^{40}K has been assessed with the method of measuring different masses of the sample and then finding fit parameters in order to plug them in the formula to calculate the half-life. The final value is:

$$T_{1/2} = (1.59 \pm 0.04) \times 10^{9} \text{ years}.$$

The comparison with the value in literature lead to a better compatibility, which is still not that good:

$$t = 7.1.$$

5.3 Discussion of results and uncertainties

Although the results for the half-life for both samples are on the same scale as the real values, there is still a significant discrepancy between the values, which can also be seen, when looking at the t-values calculated in the previous sections. There are two main factors contributing to the unpleasant t-values: On the one hand there are problems in the measurement that lead to wrong results and on the other hand the uncertainties can be underestimated. Reasons for both factors will be discussed in this section.

A first discussion point is the fact that especially for the uranium-measurement the background for low voltages is higher than the measured values, which on first sight should not be possible. Potential reasons are on the one hand statistical factors and on the other hand problems with the

electronics and the setup. Between the measurement of the uranium-sample and the background radiation, the electronics were reconnected to look at the signals on the oscilloscope again. It is possible that some parameters were changed when reconnecting the cables which would lead to other counting rates.

A very important factor is a problem with the counting electronic that occurred during one of the potassium-measurements. For the second measurement the electronics started to count every signal twice, which was noticed, because of only even numbers being counted. This observation was confirmed with an empty bowl. The problem could be easily solved by restarting the `LabVIEW`-Software and the electronics of the setup, but still it can not be ruled out, that double counting occurred during the other measurements, which would lead to falsification of the data. An important example could be the one value in the measuring with different masses that does not fit the theoretical expectation in Figure 8. This data point also leads to the high χ^2-value for the data and could be caused by a measurement with some detectations counted twice.

Another problem, that lead to imprecise measurements was, as already mentioned, that one of the background measurements had to be done twice. The long background measurement for Samarium-147 over night with a measuring time of over 14 hours could not be taken, because the gas supply was shut when arriving in the morning. This means that the measurement had to be redone, but because on the second day only half of the time was available, only a shorter measurement with a greater error could have been taken. This also leads to the fact, that the uncertainties of the background still contribute, although they should not contribute at all.

For the measuring times an approximation was made by finding the time after which the uncertainty is lower than 2% as described in the analysis part. For this approximation the uncorrected data was used. It could have been more sensible to use the corrected data for this approximation, which means that lower counting rates and therefore longer measurements would have been necessary. This could have led to more precise measurements. Because of the correction with the background the uncertainties are all slightly above 2%, which could have been prevented, by using this different approximation.

Another less important factor could be the weighing method used in the experiment. After setting the high precision weighing balance to zero with an empty bowl, the filled bowl was measured. A more precise measurement would have been possible by always weighing the filled bowls and in the end performing several measurements with the empty bowl to subtract its weight.

Some other factors that could have influenced the measurements can be the following. First of all it is possible that remains of radioactive materials are somewhere in the setup that lead to uncertainties in the measurement. Still those remains should not be very influential since they contribute to the background measurements as well. Another more important point is that the radioactive samples could also be polluted. Especially in the potassium-chloride, several clumps were visible that can be the results of pollution. Because every group uses the same material to save costs, it is likely to get contaminations. As a last point one always has to keep in mind that radioactivity is a statistical process which only leads to good results when taking a lot of measurements. Therefore taking longer measurements can always improve the results.

References

[1] Unknown author: *Lange Halbwertszeiten - Versuchsanleitung Fortgeschrittenen Praktikum Teil 1*, (Freiburg im Breisgau: 2012)

[2] Tobijas Kotyk: *Versuche zur Radioaktivität im Physikalischen Fortgeschrittenen Praktikum an der Albert-Ludwigs-Universität Freiburg*, (Freiburg im Breisgau: 2005)

[3] James G. Kushmerick: *The NIST Reference on Constants, Units and Uncertainty* https://physics.nist.gov/cgi-bin/cuu/Value?na (accessed on: 12.09.2022)

[4] Dr. Christof Bartels, Dr. Lukas Bruder, Dr. Thomas Pfohl: *Datenanalyse Teil A - Skript zur Vorlesung am 06.09.2021*, (Freiburg im Breisgau: 2021/22)

[5] W.M.Haynes: *CRC Handbook of Chemestry and Physics - 95th Edition*, (Boca Raton: CRC Press, 2014), p. 33f

[6] Dr. Christof Bartels, Dr. Lukas Bruder, Dr. Thomas Pfohl: *Datenanalyse Teil B - Skript zur Vorlesung am 28.02.2022*, (Freiburg im Breisgau: 2021/22)

7 Attachment

7.1 Tables and graphics

Tab. 3: In the table all the measurements performed are listed and all the relevant parameters for the measurement are shown: The starting voltage U_{start} and the end voltage U_{end}, as well as the step-voltage U_{step}, the measuring time, the waiting-time, the mass m and the counts C. The uncertainty for the mass is always 0.0005 g, other uncertainties are not known. The measurement that were not used in the analysis are also listed but written in italic letters.

Measurement	U_{start} in V	U_{end} in V	U_{step} in V	time in s	waiting-time in s	m in g	C
^{238}U: characteristic	1000	4000	100	50	15	-	-
^{238}U: background	1500	4000	100	100	15	-	-
^{147}Sm: α-Plateau	2300	3700	100	200	15	-	-
^{147}Sm: long measurement	2800	2800	-	4000	30	-	2795
^{147}Sm: wrong background	2800	2800	-	52200	30	-	14309
^{147}Sm: right background	2800	2800	-	7000	30	-	1299
^{40}K: β-Plateau	3400	4000	100	100	30	-	-
^{40}K: mass 1	3800	3800	-	800	30	0.1935	1519
^{40}K: mass 2	3800	3800	-	660	30	0.3802	1975
^{40}K: mass 3	3800	3800	-	640	30	0.5568	2241
^{40}K: mass 4	3800	3800	-	600	30	0.6940	2393
^{40}K: mass 5	3800	3800	-	560	30	0.7951	2132
^{40}K: mass 6	3800	3800	-	540	30	1.0615	2390
^{40}K: mass 7	3800	3800	-	520	30	1.1656	2363
^{40}K: mass 8	3800	3800	-	520	30	1.3409	2673
^{40}K: mass 9	3800	3800	-	520	30	1.5599	2567
^{40}K: mass 10	3800	3800	-	520	30	1.8801	2570
^{40}K: mass counted twice	3800	3800	-	540	30	0.7919	4652
^{40}K: background	3800	3800	-	2700	30	-	1966

Tab. 4: In the table all the measurements with different masses on potassium-40 are presented. In addition to mass m and time t, the counting rate n is given with its absolute and realative uncertainty. The uncertainty for the mass is always 0.0005 g.

Measurement	mass m in g	time t in s	$n \pm \Delta n$ in s^{-1}	Δn_{rel}
mass 1	0.1935	800	1.90 ± 0.05	2.6%
mass 2	0.3802	660	2.99 ± 0.07	2.3%
mass 3	0.5568	640	3.50 ± 0.07	2.1%
mass 4	0.6940	600	3.99 ± 0.08	2.0%
mass 5	0.7951	560	3.81 ± 0.09	2.0%
mass 6	1.0615	540	4.43 ± 0.09	2.0%
mass 7	1.1656	520	4.54 ± 0.09	2.1%
mass 8	1.3409	520	5.14 ± 0.10	1.9%
mass 9	1.5599	520	4.94 ± 0.10	2.0%
mass 10	1.8801	520	4.94 ± 0.10	2.0%

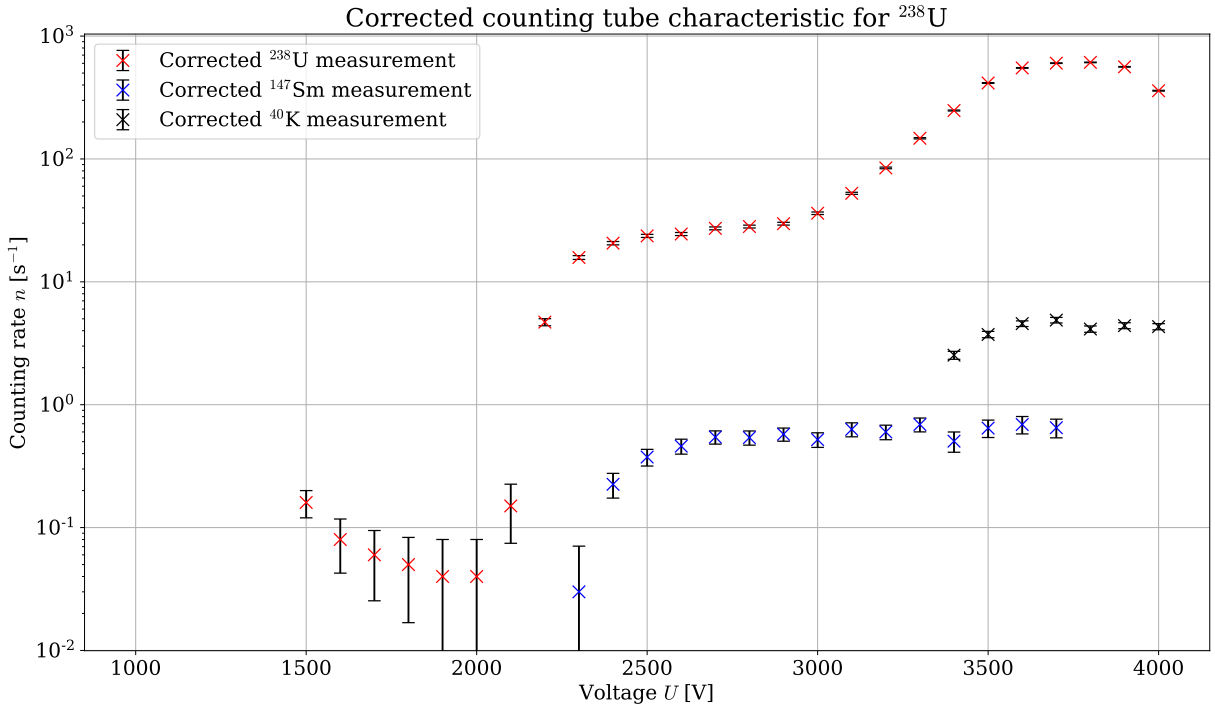


Fig. 9: In this graphic the counting rate n in s^{-1} is plotted against the voltage U in V on a double-logarithmic scale. The counting rates n are corrected with the background measurement of ^{238}U. The red values represent ^{238}U, the blue values represent ^{147}Sm and the values in black are from the ^{40}K measurement.

7.2 Lab notes

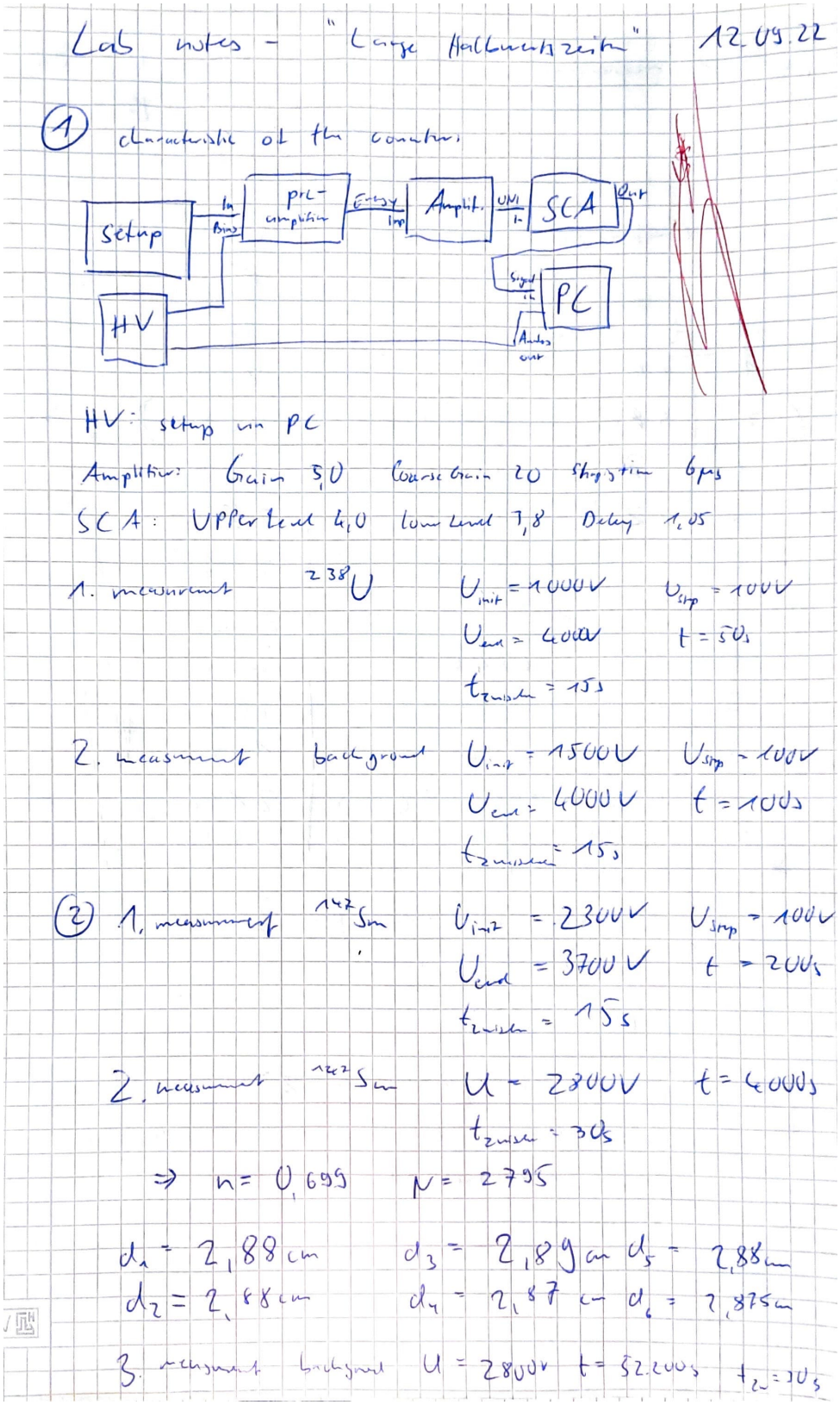
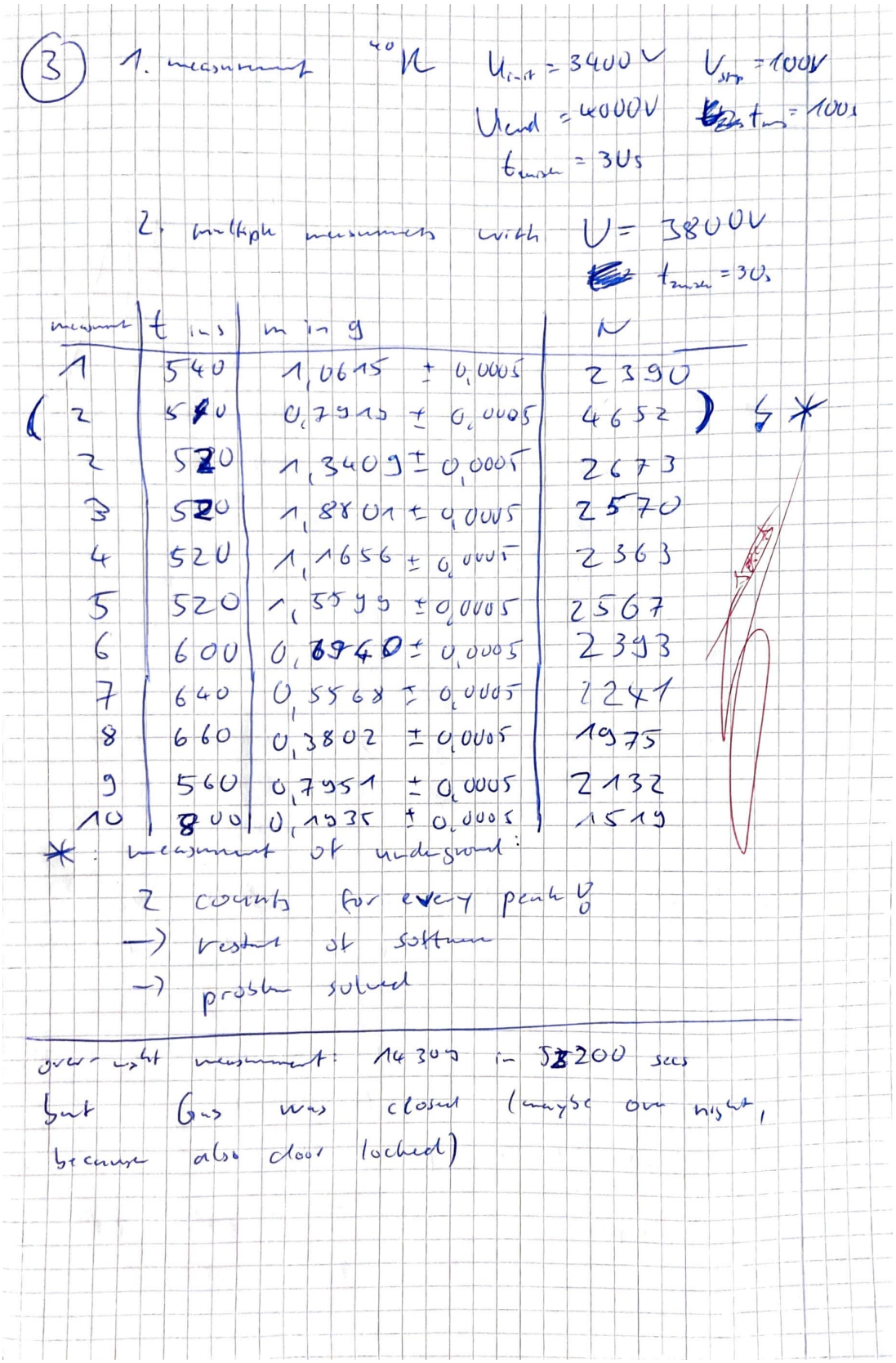


Fig. 10: Lab notes - page 1



(3) 1. measurement ^{40}K $U_{init} = 3400\,V$ $U_{step} = 100\,V$
$U_{end} = 4000\,V$ $t_{meas} = 100\,s$
$t_{meas} = 30\,s$

2. multiple measurements with $U = 3800\,V$
$t_{meas} = 30\,s$

measurement	t in s	m in g	N
1	540	1,0615 ± 0,0005	2390
2	540	0,7913 ± 0,0005	4652) ϟ *
2	520	1,3409 ± 0,0005	2673
3	520	1,8801 ± 0,0005	2570
4	520	1,1656 ± 0,0005	2363
5	520	1,5595 ± 0,0005	2567
6	600	0,8940 ± 0,0005	2393
7	640	0,5568 ± 0,0005	1241
8	660	0,3802 ± 0,0005	1975
9	560	0,7951 ± 0,0005	2132
10	800	0,1935 ± 0,0005	1519

* : measurment of background:
2 counts for every peak U_0
→ restart of software
→ problem solved

over night measurment: 14300 in 58200 secs
but Gas was closed (maybe over night, because also door locked)

Fig. 11: Lab notes - page 2

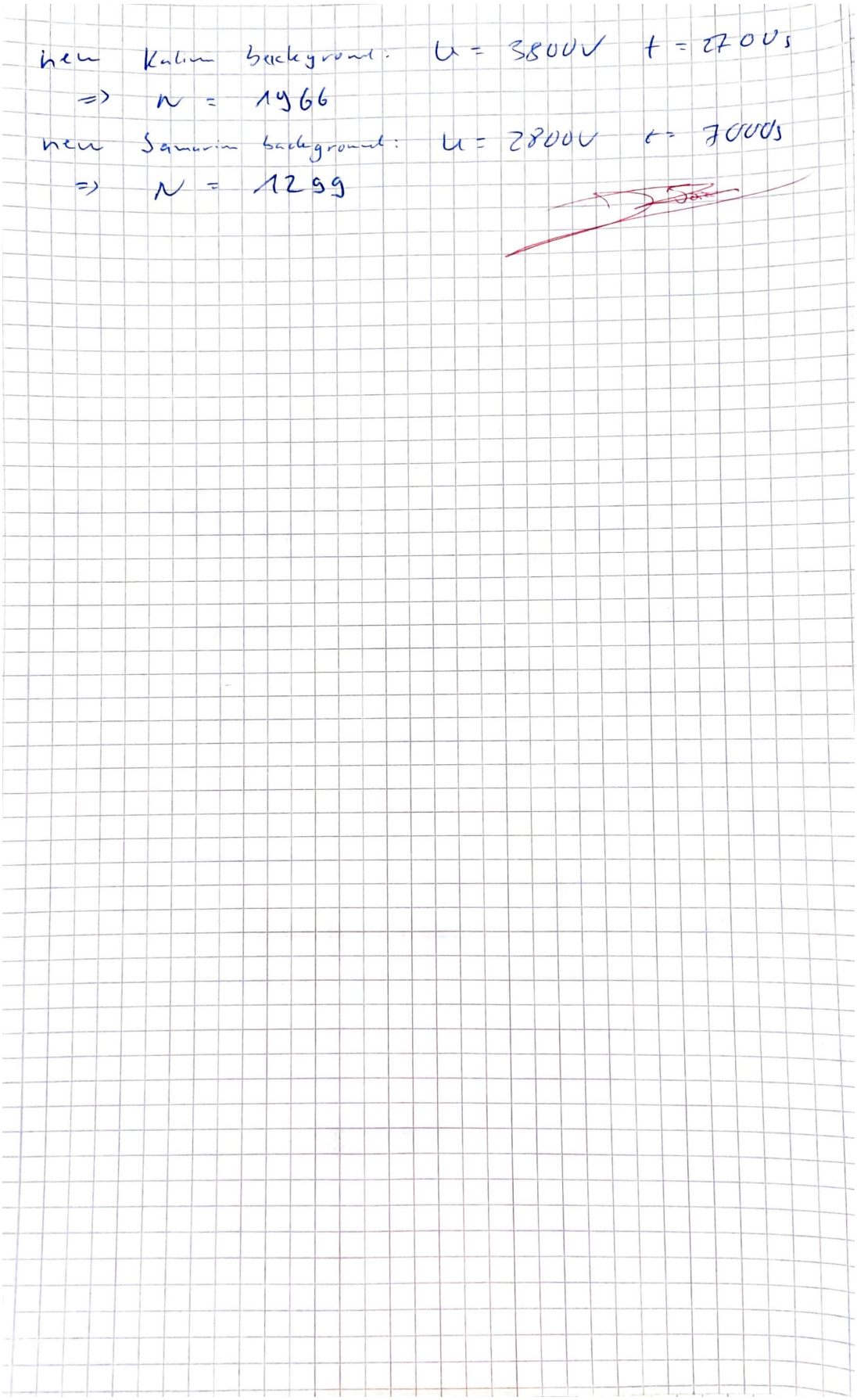


Fig. 12: Lab notes - page 3