Universität Freiburg
Advanced Physics Lab Course
FP1 2024-2

Experiment 7
Michelson Interferometer

Short Report

[redacted]

August 20, 2024

Lab Work conducted: August 19

Assistant: [redacted]

Contents

1 Objectives

In this experiment, a Michelson Interferometer was set up and subsequently used to perform two different measurements. The objective of the first measurement was to determine the wavelength λ of a laser, and during the second part the thermal expansion coefficient α of an aluminium rod was determined.

2 Equipment used

The experiment was conducted using components from the *THORLABS EDUMINT2 Michelson Interferometer Kit* [THO]. More specifically, both parts of the experiment used the same green laser, a beamsplitter, one plain adjustable mirror, a convex lens and a screen. All the components in their respective holders were positioned on a breadboard. For the first part, a translatable mirror was used in addition. During the second part of the experiment, this mirror was replaced by an aluminium rod with a heating foil attached to it. Moreover, a digital rod thermometer as well as a laboratory power supply unit were used.

3 Procedure

Before any measurements could be taken, the Michelson Interferometer had to be set up and adjusted. This was done by following the recommended steps explained in the Kit Manual [THO]. The interference pattern was observed on a screen. As the convex lens, with the task of diverging the laser beam, was placed between the laser and the beamsplitter, the resulting interference pattern consisted of concentric rings.

During the first part of the experiment, a mirror which could be translated in the direction of the interferometer arm by turning a micrometer screw was positioned opposite to the laser on the breadboard at approximately the same distance to the beamsplitter than the adjustable, but stationary, second mirror. A picture of the setup for this part can be found in fig. 1.

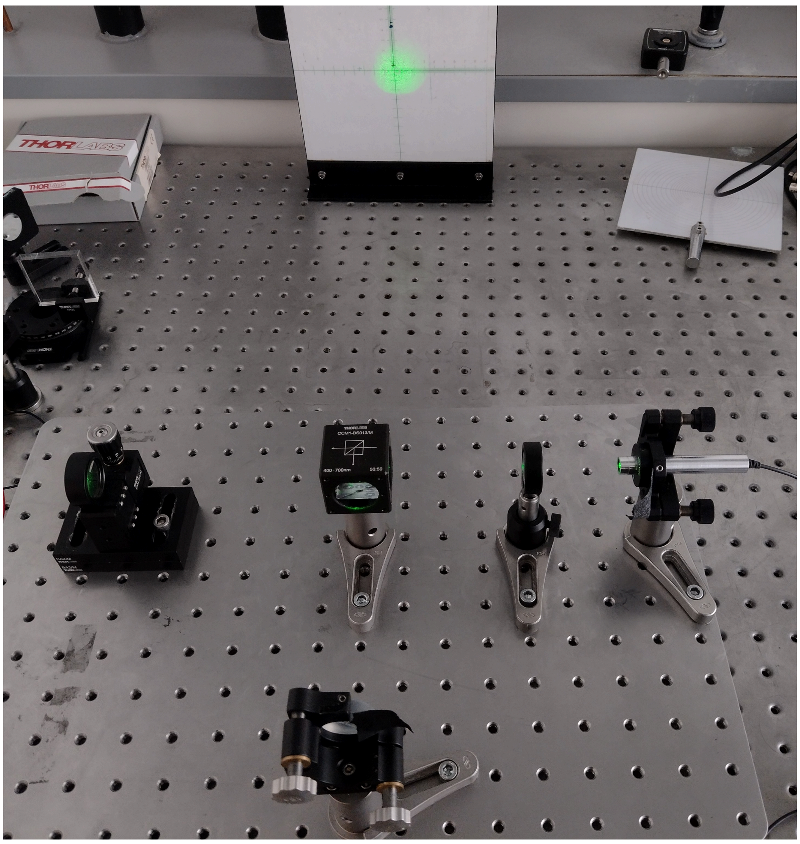


Figure 1: Setup for part 1 of the experiment

Once the interference pattern on the screen appeared clearly and steady, a series of measurements was taken during which the micrometer screw was turned by a bit, thus changing the length of the corresponding interferometer arm, and the number N of light-dark-light transitions was noted. This was performed for 9 different N and after reaching the desired number of transitions, the distance Δx of translation in µm was noted. Before the next measurement, the screw was turned back to 0 to make it easier to read off Δx of the next measurement. Throughout the entire series of measurements it was made sure that the table and the interferometer components were touched as little as possible by, for example, turning the micrometer screw from the top, without resting the hand anywhere on the setup. The full set of data taken can be found in Table 1 of the lab notes in the appendix (section 8).

Before the second part of the experiment could be started, the interferometer setup had to be modified by exchanging the translatable mirror for the Al rod component. This consisted of an aluminium rod around which a foil heater was already wrapped and taped. On one of the sides of the rod, a plain mirror was attached and on the other side, there was an indentation in the rod, into which the touch probe of a digital thermometer could be inserted. The two outer cables of the heating foil were connected to a power supply unit via banana plugs. In fig. 2, a picture of the setup for this part can be found.

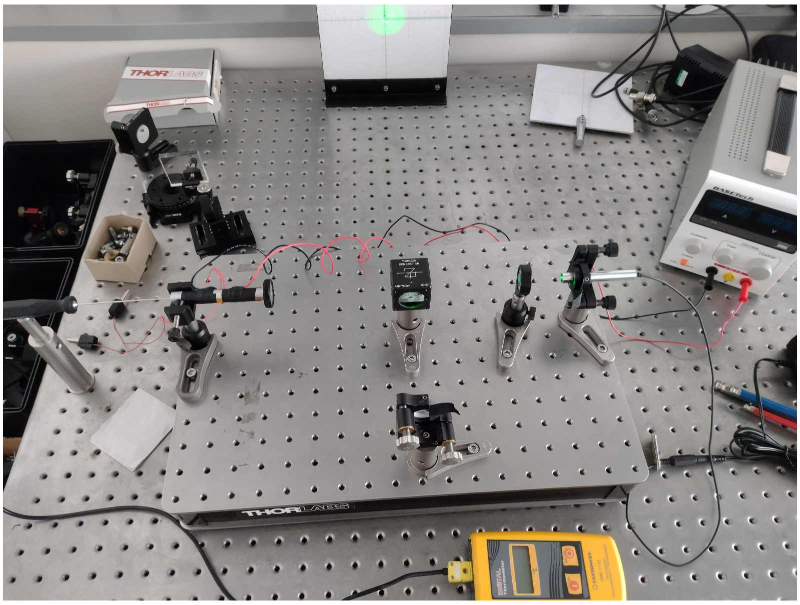


Figure 2: Setup for part 2 of the experiment

After the interferometer was readjusted to obtain a clear interference pattern on the screen, the power supply was turned on and the current was set to 2 A.
The voltage was originally set to around 1 V. First, a start temperature T_{start} was taken. This was done exactly at the point in time, when the display of the digital thermometer showed a first temperature increase and at the same time the counting of the light-dark-light transitions on the screen was started. After each temperature increase by one °C, the temperature T_{end} as well as the number of so far occurred transitions N was noted. The voltage was increased a bit whenever the motion of the interference pattern had slowed down exceedingly. In total, the voltage was varied between 1 V and 7.8 V, corresponding to a temperature change from 26 °C to 46°C. The total set of data taken during this part can be found in Table 2 in the lab notes in section 8

4 Observations, Data and Analysis

4.1 Analysis Part 1: Determining the Wavelength λ of the Laser

As the screw on the translatable mirror is turned, the length of one of the interferometer arms changes, resulting in a varying phase shift between the two beams recombining after the beam-splitter and thus a change in the observed interference pattern. Each light-dark-light transition corresponds to a change in the optical path length of one wavelength λ, but since the distance between the translatable mirror and and the beamsplitter is crossed twice, the expected linear relationship ([Dem17]) between the observed number N of transitions and the distance Δx is given by:

$$\Delta x = \frac{\lambda}{2} \cdot N. \tag{1}$$

From the performed measurements it is thus possible to determine the laser wavelength λ.

While performing the measurement series, the interference rings were, overall, moving inwards, which means that we were increasing the difference between the partial beams' path lengths by moving the mirror. However, the interference pattern was often fluctuating and very sensitive to external influence such as movement of the table or the optical components whenever we were touching them. This made it more difficult to count the light-dark-transitions, so we assume a relative uncertainty of 5% on N. We also observed that the pattern was often going in the opposite direction (rings moving outwards) for a short time ($\approx$ 1-2 transitions) when we were beginning to turn the screw before it started moving in the expected direction. This might probably be due to the same reasons (sensitivity to touch), and we just ignored these transitions. It is however possible that there remains a systematical offset in N due to this anomaly.

Finally, at the end of the measurement series, we conducted two additional measurements after a short break, but those deviate so clearly from the data taken before that we suppose some external factors have changed and we decided not to include them in the analysis.

For the distance Δx, we assume an uncertainty $\sigma_{\Delta x} = 1\mu\text{m}/\sqrt{6} \approx 0.4\,\mu\text{m}$, based on a triangular distribution of width 2µm. We justify this by the fact that even if the scale on the screw theoretically had a 1µm division, it was not quite realistic to stay within that scale uncertainty, due to factors like the human reaction time when stopping to turn after a certain number N was reached or involuntary tiny movements with the hand.

The resulting data is listed in table 1 in section 8 and plotted in fig. 3. As implied by eq. (1), we could fit the data with a model $\Delta x = aN$. In order to exclude a possible systematical error like the one noted above, we perform instead a two-parameter linear fit using the model $\Delta x = aN + b$ with an additional offset b. This results in optimal parameter values

$$a = 0.347(8)$$
$$b = 0.7(3)\ .$$

The wavelength can then be calculated as

$$\lambda = 2a = 0.694(16)\ \mu\text{m} \tag{2}$$

4.2 Analysis Part 2: Determining the Thermal Expansion Coefficient α of Al

The linear thermal expansion coefficient α of a solid is defined by the following differential equation [THO]:

$$\alpha = \frac{1}{L}\frac{dL}{dT},$$

where dL is the relative linear expansion, dT the change in temperature and L the total length of the expanding object. Solving this differential equation gives the expected relationship between the length L after a change in temperature by ΔT:

$$L = L_0 \exp(\alpha\, \Delta T), \tag{3}$$

where L_0 denotes the original length.

In this experiment, the change in length ΔL was determined using the knowledge from part 1 and the relationship between the number N of observed light-dark-light transitions as well as using the reference wavelength $\lambda_{\text{ref}} = 532\,\text{nm}$, as given in the manual [THO]:

$$\Delta L = \frac{N \cdot \lambda_{\text{ref}}}{2}. \tag{4}$$

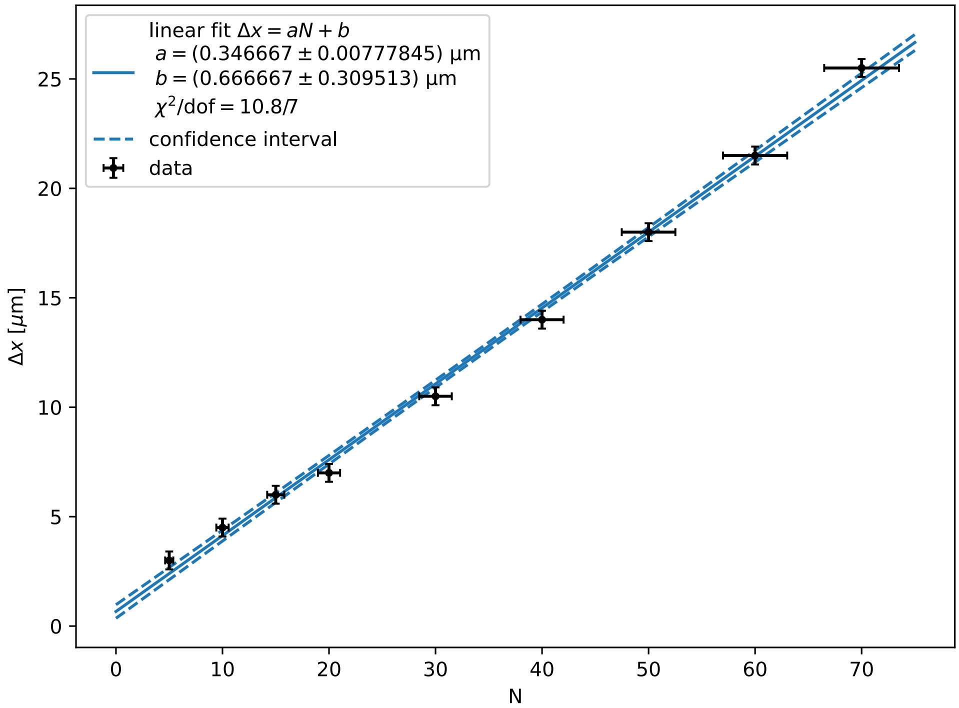


Figure 3: Data and fit: distance Δx versus number N of light-dark-light transitions

Assuming the uncertainty on the reference wavelength to be negligible, the error on ΔL was obtained by the following error propagation:

$$\sigma_{\Delta L} = \frac{\sigma_N \cdot \lambda_{\mathrm{ref}}}{2}$$

The counting error on N was again estimated to be around 5%.

For the uncertainty on the start and end temperatures, the accuracy of the digital thermometer display was used. This was estimated to be distributed with a rectangular distribution with a width of 1 °C, resulting in a standard deviation of

$$\sigma_{T_{\mathrm{start}}} = \sigma_{T_{\mathrm{end}}} = \frac{1\,^\circ\mathrm{C}}{2 \cdot \sqrt{3}}.$$

According to simple Gaussian Error Propagation, this resulted in an uncertainty on the temperature difference $\Delta T = T_{\mathrm{end}} - T_{\mathrm{start}}$ of around 0.4 °C.

The length L_0 of the aluminium post prior to heating above room temperature was given in the kit manual as

$$L_0 = 9\,\mathrm{cm}, \tag{5}$$

and was assumed to have a negligible uncertainty. It was noticed, however, that the heating foil did not cover the full length of the rod, and that it was not specified in the manual at which exact temperature this length was measured, which gives rise to a potential systematic error.

The diagram in fig. 4 shows the change in rod length ΔL as a function of temperature change ΔT. The data points are shown with their errorbars in both directions. From the naked eye, it does not seem as though a linear first order approximation for a fit would hold in this case. Therefore, an exponential fit of the form

$$\Delta L = L_0 \cdot \exp(\alpha\, \Delta T) - L_0 \tag{6}$$

was performed, in which L_0 was assumed to be the constant given reference value and α being the only fit parameter. The fit was performed in python using *scipy.optimize.curvefit*, which is based on least-square-optimization and resulted in the following best parameter value:

$$\alpha = 2.79(7) \cdot 10^{-5} \frac{1}{\mathrm{K}}. \tag{7}$$

The fit is illustrated as well in fig. 4.

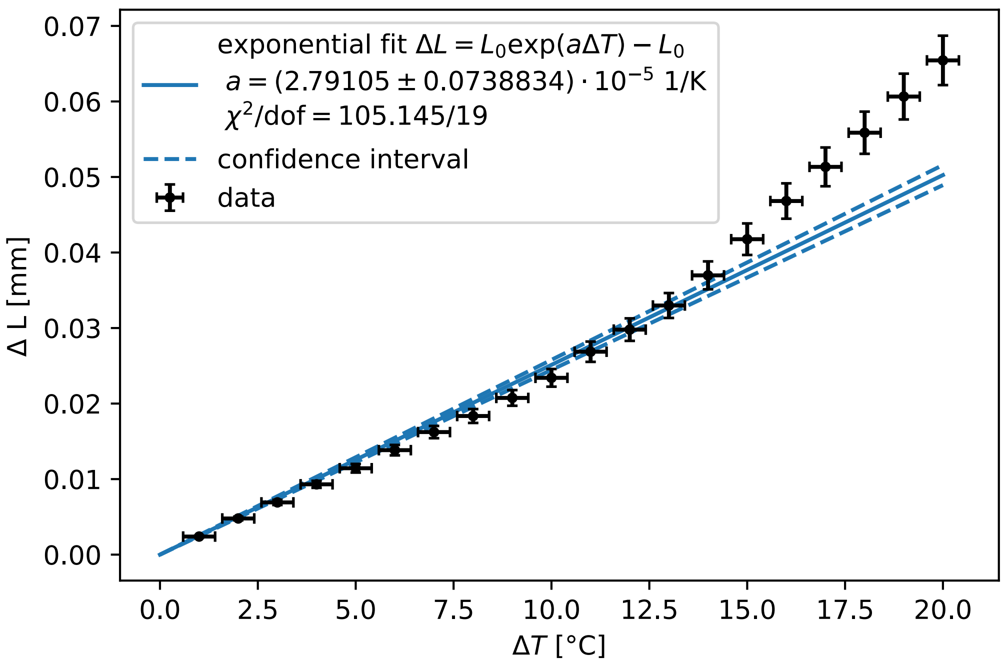


Figure 4: Data and fit: length increase ΔL versus temperature difference ΔT

To assess the goodness of the fit, a χ^2- test was performed, resulting in

$$\frac{\chi^2}{\mathrm{dof}} \approx \frac{105}{19},$$

corresponding to a p-Value of

$$p \approx 6.2 \cdot 10^{-14}.$$

5 Discussion

5.1 Discussion Part 1: Determining the Wavelength λ of the Laser

The quality of the fit we used to determine the laser wavelength is acceptable with a reduced χ^2 value of $\chi^2/\mathrm{dof} \approx 1.5$, corresponding to a p-value of $\approx 15\%$. This matches also the impression one gets when looking at fig. 3: all the data points touch the linear fit or at least the confidence band with their error bars.
The offset $b = 0.7(3)\,\mu\mathrm{m}$ that we found indeed implies a systematical uncertainty as we suspected, though it could also be due to simple statistical fluctuation. The result for the laser wavelength, however, differs from the expectation:

According to the manual [THO], the true laser wavelength is $\lambda_{\mathrm{ref}} = 0.532\,\mu\mathrm{m}$. The value that we calculated in section 4.1, $\lambda = 0.694(16)\,\mu\mathrm{m}$, strongly deviates from this reference value:

$$t = \frac{\lambda - \lambda_{\mathrm{ref}}}{\sigma_\lambda} \approx 4.5.$$

At a confidence level of 0.05, the result of this t-test would indicate that our result is not compatible with the reference value.

The relative difference in the values is

$$\frac{\lambda - \lambda_{\text{ref}}}{\lambda_{\text{ref}}} \approx 30\%.$$

As mentioned before, the setup was extremely sensitive to all kinds of unwanted external influences. The thereby caused fluctuations in the interference pattern are probably the main reason for why the result differs quite significantly from the expectation. A second strongly limiting factor is the fact that the distance was changed by hand with the micrometer screw, which was almost impossible to do very accurately without affecting the setup.

5.2 Discussion Part 2: Determining the Thermal Expansion Coefficient α of Al

The goal of this part of the experiment was to determine the thermal expansion coefficient α of Al, which was obtained here as the best fit value of the exponential fit in fig. 4. In the kit manual [THO], a reference value of

$$\alpha_{\text{ref}} = 2.31 \cdot 10^{-5}\,\frac{1}{\text{K}} \tag{8}$$

was given. This value can be compared with the here obtained value (eq. (7)) via a t-test:

$$t = \frac{\alpha - \alpha_{\text{ref}}}{\sigma_\alpha} = \frac{2.79 - 2.31}{0.07} \approx 6.5.$$

At a significance level of 0.05, this would imply a rejection range of $t > 2$ and one would thus need to conclude that our obtained result is not compatible with the reference value.
The relative difference is

$$\frac{\alpha - \alpha_{\text{ref}}}{\alpha_{\text{ref}}} \approx 21\%.$$

Taking into account the simple means and many ambient influences out of the control of the experimenters, one might say that this is is nevertheless an acceptable result. The accuracy could probably be improved by performing the experiment in a more controlled environment and with more acurate measuring devices (e.g. thermometer).

Besides this result, it is important to discuss the quality of the fit attempted. As one can see from the naked eye, the exponential fit in fig. 4 does not really seem to describe the data very well, especially at higher temperatures. This is also confirmed by the χ^2-test. At a confidence level of 0.05, the obtained p-value is extremely low and suggests that the data cannot be described sufficiently accurate by the proposed exponential shape.

It is also interesting to see, that the exponential best fit result is so shallow, that it almost can be mistaken for a linear fit.

One alternative approach would have been to fit the same exponential function, but without assuming the original rod length L_0 to be fixed and using this instead as a second free parameter. This was attempted, but did however not result in an improvement of the overall quality of the fit. It seems as though an additional unknown component to the model might have been necessary to describe the data more accurately.

As mentioned before, it is noticeable that the deviation from an exponential behaviour seems to increase with increasing temperature. One possible explanation could be that more complex processes are beginning to happen at this regime. One can speculate that this might have to do with the heating foil then actually also heating the surrounding air, thus changing the density along the path of the laser. Similarly, the observed affect might have something to do with the general conditions (temperature, pressure) in the lab changing over the course of the heating process.

6 Improvements and Suggestions

One big source of uncertainty during this experiment was the fact that the setup was extremely sensitive to all kinds of unwanted influences, from changes in density and ambient temperature to potential errors caused by someone touching the optical components of the board. One way of increasing the stability of the interference pattern would be to place the whole setup on a higher quality optical table. Besides, it might have been a good idea to close window shutters: both to be able to see the interference pattern better at lower ambient light and also to prevent temperature in the room to fluctuate as much due to sunlight entering the lab at varying intensities.
If one strives after obtaining clearer more highly resolved interference patterns, it would be advisable to use a better collimated and stronger laser of higher quality. A higher coherence length in the laser would also make it easier to find an interference pattern during the setup, as this would not require the lengths of the arms to be as closely the same.

Another major source of uncertainty was the fact that the micrometer screw of the translatable mirror had to be turned by hand. This implied on the one hand a limited accuracy with which the distance could be changed, as well as a considerable uncertainty in reading off the moved distance on the micrometer scale. One suggested improvement could be to use a piezo crystal instead of a mechanical screw for changing the distance in one of the interferometer arms.

In part 2 of the experiment, a rod thermometer was used, which was only loosely inserted into the hole of the aluminium probe. An alternative and possibly a bit more reliable way of measuring the temperature would have been to use the thermistor sensor of the foil heater by connecting the two respective cables to a multimeter and by reading off the temperature on this. In addition to the aspect of reliability, this might also offer a more finely resolved digital display.

In order to obtain a value for the original length L_0 of the Al rod at the start temperature, other than the one given in the manual, one could have measured the length using a caliper gauge with a precise Vernier scale. In the here presented case this was, however, not really feasible, as the plain mirror was glued permanently to one end of the rod, making it rather challenging to measure the actual length.

7 Conclusion

In this experiment, a Michelson Interferometer was set up and subsequently used to perform two partial experiments. In the first part, the laser wavelength λ was determined by fitting the linear relation between the number N of light-dark-light transitions and the corresponding distance Δx, by which a translatable mirror had been moved (see fig. 4). This resulted in a best value of

$$\lambda = 694(16)\,\mathrm{nm}, \tag{9}$$

which was later compared with the reference value of

$$\lambda_{\mathrm{ref}} = 532\,\mathrm{nm}. \tag{10}$$

The relative difference is around 30%. A t-test indicates a significant deviation.

In the second part, the thermal expansion coefficient α of an Al rod was determined by heating the rod by temperature differences ΔT and observing the number N of light-dark-light transitions, from which a corresponding change in length ΔL was calculated. An exponential model was fitted to the data according to the theoretical expectation (see fig. 4), resulting in the following best fit value:

$$\alpha = 2.79(07) \cdot 10^{-5}\,\frac{1}{K}, \tag{11}$$

which was within 21% of the reference value

$$\alpha_{\mathrm{ref}} = 2.31 \cdot 10^{-5}\,\frac{1}{K}, \tag{12}$$

resulting in a t-value of around 6.5.
In this case the quality of the fit was rather unsatisfactory, especially for higher temperatures.

Different reasons for the deviations of the experimental results from the expectations were discussed and some suggestions for possible improvements were made.

The biggest limiting factor and potential source of errors was identified to be the high sensitivity of the whole setup towards different environmental conditions and fluctuations, like temperature, pressure or unavoidable touching of the micrometer screw.

References

[Dem17] Wolfgang Demtröder. *Experimentalphysik 2: Elektrizität und Optik*. 7th ed. Springer Spektrum Berlin, Heidelberg, 2017. DOI: https://doi.org/10.1007/978-3-662-55790-7.

[THO] THORLABS. *Edumint 2 Michelson Interferometer Kit User Guide*. URL: https://ilias.uni-freiburg.de/goto.php?target=file_3498737_download&client_id=unifreiburg.

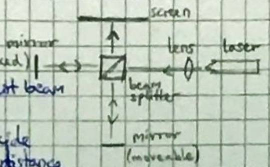
8 Appendix A: Signed Lab Notes

Michelson interferometer

[redacted]

objectives: - assemble interferometer
- determine laser wavelength
- determine thermal expansion coefficient of Al rod

setup:



- position translateable mirror and laser, tilt so that reflected beam aligns with incident beam
- install beam splitter and screen, make primary and secondary reflexions coincide
- install second mirror at approximately same distance, tilt this mirror to make beams coincide on screen
- install lens and move translateable mirror to get clear interference pattern

[redacted]

first part: wavelength

Turn screw on translateable mirror while counting number N of light-dark-light transitions, note distances x.
Scale on translateable mirror: $1\mu m$, estimated uncertainty (triangle distribution): $0{,}2\mu m$

N	10	20	30	40	50	60
x (μm)	3	6	10	13	18	23

Restart the measurement series since we noted there are too many unwanted influences: now we reduce contact to the board as far as possible, and restart the distance measurements at zero for every value of N

Table 1:

N	10	20	30	40	50	60	70	5	15	25	35
x (μm)	4,5	7	10,5	14	18	22	~~26~~ 25,5	3	6	11,5	15,5

$\underbrace{11{,}5 \quad 15{,}5}$ seem to deviate from former measurements, potential change in surroundings?

We note that sometimes, when we begin to turn the screw, the pattern initially changes in the wrong direction (inwards instead of outwards) before it changes directions. We are ignoring these anomalies.

We assume the uncertainty while counting transitions to be $\frac{\Delta N}{N} \approx 5\%$.

second part: thermal expansion coefficient

The aluminium rod is $L_0 = 9cm$ long. (It is fixed to the post $\approx 1cm$ from its end.)
We now use a slightly different setup where the translateable mirror is replaced by a mirror attached to the rod with the heating foil. The setup procedure is analogous to the one before.
We control the temperature by controlling the voltage on the heating foil via a lab mains adapter* and measure the temperature value with a thermometer.
The rod is heated ~~stepwise~~ continuously, we count the number of light-dark-light transitions while waiting for the temperature to reach the next value.
Start temperature: 26°C

*voltage range: 1V - 7,8V

~~T_{start} (°C)~~	~~24 25~~
~~T_{end} (°C)~~	~~27~~
~~N~~	~~18~~

restart measurement with new procedure (one common start temperature)

Estimated uncertainty (rectangle distribution): $\Delta T \approx 0{,}3°C$

Table 2:

T (°C)	27*	28	29	30	31	32	33	34	35	36	37	38	39	40
N	9	18	26	35	43	52	61	69	78	88	101	112	124	139

T (°C)	41	42	43	44	45	46
N	157	176	193	210	228	246

In this part of the experiment, the interference pattern didn't fluctuate as much as before, so we assume an uncertainty $\frac{\Delta N}{N} \approx 1\%$, lower than in part 1.

9 Appendix B: Python code

In [12]:

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats
import scipy.optimize

# part 1: determining the wavelength of the laser

# data
lambda_ref = 532/1000 #µm

N = np.array([10,20,30,40,50,60,70,5,15]) # (25,35)
Nerr = 0.05 * N # error on N: 5 % von N

x = np.array([4.5,7,10.5,14,18, 21.5,25.5,3,6]) # in µm (11.5,15.5)
xerr = np.array([1 / np.sqrt(6)] * len(x))
# triangular distribution (2a = 2µm) read out + reaction time

# potential error in last two measurements


# performing a linear fit y = ax + b
def Linear(x, a, b):
    return a*x + b

p0 = (0.5, 0)

popt, pcov = scipy.optimize.curve_fit(Linear, N, x, p0, xerr)

# best fit values and their uncertainties
# uncertainties are square roots of diagonal of covariance matrix

print("Optimal parameters:")

print("a = %g +- %g" %(popt[0], np.sqrt(pcov[0][0])))
print("b = %g +- %g" %(popt[1], np.sqrt(pcov[1][1])))

values = np.linspace(0, 75) # xvalues for fit
fit1 = Linear(values, *popt)
phi = np.array([values, 1], dtype=object)
fit1err = np.sqrt(phi @ pcov @ phi.T)

# performing a chisquare test to assess goodness of fit
# degrees of freedom = # data points - # fitted parameters
dof = len(x) - 2

print("\nManual chi^2 test:")
chi2m = ((x-Linear(N,*popt))**2 / xerr**2).sum()
p = scipy.stats.distributions.chi2.sf(chi2m, dof)
print("chi2/dof = %g / %d = %g" %(chi2m, dof, chi2m/dof))
print("p-value = %g" %p)

print("\nCalculated wavelength:")
lambda_calc = popt[0] * 2
lambda_calc_err = np.sqrt(pcov[0][0]) * 2
t_value = (lambda_calc - lambda_ref)/lambda_calc_err
print("lambda_calc = %g +- %g µm" %(lambda_calc, lambda_calc_err))
print("lambda_ref = %g µm" %(lambda_ref))
```

```
print("t-value: %g" %(t_value))
```

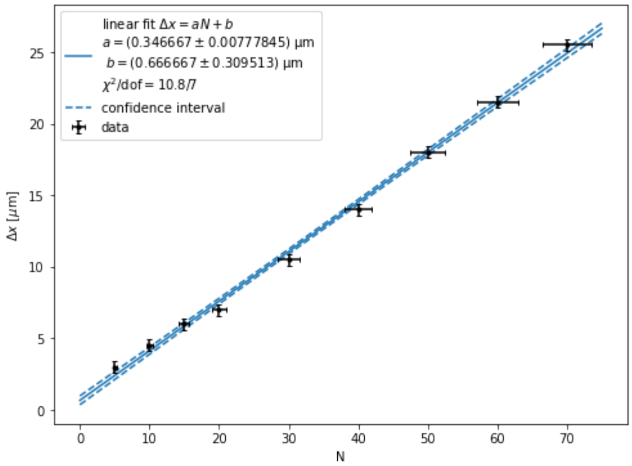
```
Optimal parameters:
a = 0.346667 +- 0.00777845
b = 0.666667 +- 0.309513

Manual chi^2 test:
chi2/dof = 10.8 / 7 = 1.54286
p-value = 0.147584

Calculated wavelength:
lambda_calc = 0.693333 +- 0.0155569 µm
lambda_ref = 0.532 µm
t-value: 10.3705
```

In [13]:

```
plt.figure(figsize=(8,6))
plt.errorbar(N, x, xerr=Nerr, yerr=xerr, fmt="k.", capsize=2,
             label="data", zorder=5)
plt.plot(values, fit1, label=f"linear fit $\Delta x = a N + b$\n"
         "$a = (%g \pm %g)$ µm\n $b = (%g \pm %g)$ µm\n"
         "$\chi^2/$dof$ = %g / %d$" %(popt[0], np.sqrt(pcov[0][0]),
         popt[1], np.sqrt(pcov[1][1]), chi2m, dof))
plt.plot(values, fit1 + fit1err, 'C0--', label="confidence interval")
plt.plot(values, fit1 - fit1err, 'C0--')
plt.xlabel("N")
plt.ylabel(f"$\Delta x$ [$\mu$m]")
plt.legend()
plt.savefig("plotPart1.png")
```



In [14]:

```python
# part 2: determining the thermal expansion coefficient of Al

L0_ref = 90 #mm original length of aluminium rod according to manual
alpha_ref = 2.31e-5 #1/K Al expansion coefficient according to manual

N2 = np.array([9,18,26,35,43,52,61,69,78,88,101,112,124,139,157,176,
               193,210,228,246])
N2err= N2*0.05 # 5% on N-value

Tstart = 26 #° celsius
Tstarterr = 0.5/np.sqrt(3) # rectangular distribution, 2a = 1°C

Tend = np.array([27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,
                 44,45,46])
Tenderr = 0.5/np.sqrt(3)

DeltaT = Tend - Tstart
DeltaTerr = np.array([1/2 * np.sqrt(2) / (np.sqrt(3))] * len(DeltaT))
# gaussian error propagation of Tend-Tstart

DeltaL = lambda_ref * N2 / 2000 #mm
DeltaLerr = lambda_ref * N2err / 2000

# exponential fit
def Exponential(x, alpha):
    return L0_ref * np.exp(alpha*x) - L0_ref
p02 = 0.1

popt2, pcov2 = scipy.optimize.curve_fit(Exponential, DeltaT, DeltaL,
                                        p02, DeltaLerr)

# best fit values and their uncertainties
# uncertainties are square roots of diagonal of covariance matrix

print("Optimal parameters:")

print("alpha = %g +- %g" %(popt2[0], np.sqrt(pcov2[0][0])))
print("t = %g" %((popt2[0] - alpha_ref)/np.sqrt(pcov2[0][0])))
print("rel = %g" %((popt2[0] - alpha_ref)/alpha_ref))

values2 = np.linspace(0, 20)
fit2 = Exponential(values2, *popt2)
fit2max = Exponential(values2, popt2[0] + np.sqrt(pcov2[0][0]))
# upper border of confidence interval
fit2min = Exponential(values2, popt2[0] - np.sqrt(pcov2[0][0]))
# upper border of confidence interval

# performing a chisquare test to assess goodness of fit
# degrees of freedom = # data points - # fitted parameters
dof = len(DeltaT) - 1

print("\nManual chi^2 test:")

chi2m2 = ((DeltaL-Exponential(DeltaT,*popt2))**2 / DeltaLerr**2).sum()
p2 = scipy.stats.distributions.chi2.sf(chi2m2, dof)
print("chi2/dof = %g / %d = %g" %(chi2m2, dof, chi2m2/dof))
print("p-value = %g" %p2)
```

```
Optimal parameters:
alpha = 2.79105e-05 +- 7.38834e-07
t = 6.51099
rel = 0.208249

Manual chi^2 test:
chi2/dof = 105.145 / 19 = 5.53392
p-value = 6.20526e-14
```

In [15]:

```python
plt.errorbar(DeltaT, DeltaL, xerr=DeltaTerr, yerr=DeltaLerr, fmt="k.",
             capsize=2, label="data", zorder=5)
plt.plot(values2, fit2, label="exponential fit "
        "$\Delta L = L_0 $exp$(a \Delta T) - L_0$\n"
        "$a = (%g \pm %g)\cdot 10^{-5}$ 1/K\n $\chi^2/$dof$ = %g / %d$"
        %(popt2[0]*10**5, np.sqrt(pcov2[0][0])*10**5, chi2m2, dof))
plt.plot(values2, fit2max, "C0--", label="confidence interval")
plt.plot(values2, fit2min, "C0--")
plt.xlabel(f"$\Delta T$ [°C]")
plt.ylabel(f"$\Delta$ L [mm]")
plt.legend()
plt.savefig("plotPart2.png")
```