Universität Freiburg
Advanced physics lab, part 1
Holiday internship in the summer semester 2024

Experiment 7
Michelson Interferometer

[redacted]
(Group 11)

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Dates of experimental execution: September 26th & 27th, 2024
Tutor: [redacted]

Contents

1 Abstract

Michelson interferometers can be used for various scientific experiments such as gravitational wave measurements or spectroscopy. In this experiment it is used to determine the wavelengths λ of three different lasers. Furthermore the thermal expansion coefficient α of aluminium is measured by connecting one mirror to the aluminium rod.
The final results are all tolerable by the literature values. The measured laser wavelengths are

$$\lambda_{\text{green}} = (536 \pm 10)\,\text{nm}$$

for the green laser,

$$\lambda_{\text{red}} = (639 \pm 14)\,\text{nm}$$

for the red laser and

$$\lambda_{\text{blue}} = (415 \pm 9)\,\text{nm}$$

for the blue laser.
The thermal expansion coefficient is determined to be

$$\alpha = (23.9 \pm 1.0) \times 10^{-6}\,\text{K}^{-1}.$$

2 Introduction

The Michelson interferometer was invented and developed by Albert Abraham Michelson in the 1880s [1]. His interferometer was used in the famous Michelson-Morley experiment. There the goal was to measure the speed of the earth relative to the ether and check whether there is such a thing like ether at all. This ether was considered to be a medium for light in space. As a result they found out, that the ether wind is very small or zero compared to the speed of light. Because of that negative outcome, the theory of the existence of an ether was discarded. Today, Michelson interferometers are used for various measurements. For example as a spectrometer or for gravitational wave detection.
The interferometer itself uses a beam splitter to split a light beam into two paths (s_1 and s_2) which are reflected at mirrors and recombined again at the beam splitter. The difference in travel length for the light Δl between the two paths lead to a phase difference $\Delta\varphi$ between them. The recombined beam hits a screen where an interference pattern is visible.
Depending on the phase difference $\Delta\varphi = \frac{2\pi}{\lambda} \cdot \Delta l$ constructive interference $\Delta l = m\lambda$ or destructive interference $\Delta l = (2m+1)\frac{\lambda}{2}$ can occur. On the screen, this leads to rings with high and low intensity next to each other. One bright-dark-bright transition happens for a length difference $2\Delta s = \Delta l = \lambda$ in the light arms of the interferometer. Because the light travels from the beam splitter to the mirrors and the same way back, the wavelength can be calculated by

$$\begin{aligned} N \cdot \lambda &= 2 \cdot \Delta s \\ \longrightarrow \quad \lambda &= \frac{2\Delta s}{N}. \end{aligned} \tag{1}$$

where N is the number of bright-dark transitions on the screen in the zero order spot. These transitions can be generated with one mirror movable. A change in the position of one mirror by the amount of Δs changes the length between the interferometers arms, therefore it creates depending on the wavelength a specific number of bright-dark transitions which can be counted.
If an object is heated, it expands. With one of the mirrors of the interferometer connected to this object, the interference changes while heating. This effect is described by the thermal expansion coefficient α with

$$\alpha = \frac{1}{L}\frac{dL}{dT}. \tag{2}$$

The change in length is dL and the change in temperature dT. For an object with known length L_0, the thermal expansion coefficient can be calculated by equation (3). The change in length follows in first approximation in the second part of this equation.

$$\begin{aligned} L &= L_0 \exp\left(\alpha \cdot \Delta T\right) \\ \longrightarrow \quad \Delta L &\approx \alpha \cdot L_0 \cdot \Delta T \end{aligned} \tag{3}$$

If the arms of the interferometer are aligned perfectly at the beginning, ΔL is now equal to Δs. The thermal expansion coefficient α can be calculated trough combining equation (3)with equation (1).

$$\begin{aligned} \Delta s &= \Delta L \\ \frac{1}{2}N\lambda &= \alpha L_0 \Delta T \\ \longrightarrow \quad \alpha &= \frac{N\lambda}{2L_0\Delta T} \end{aligned} \tag{4}$$

3 Methods

3.1 Laser Wavelength

The setup used in this experiment, is a rather conventional Michelson interferometer. It consists of two mirrors, one beam splitter (BS), one laser, one lens and one screen. Those parts are arranged like in figure 1 depicted.

The mirror on the left is attached to a micrometer screw which can move the mirror in the laser beam direction to one micron precision. There were five measurements taken, for each measurement the micrometer screw was slowly turned for different lengths and the number of light dark transitions on the screen was counted. This process was done for every of those three lasers available.

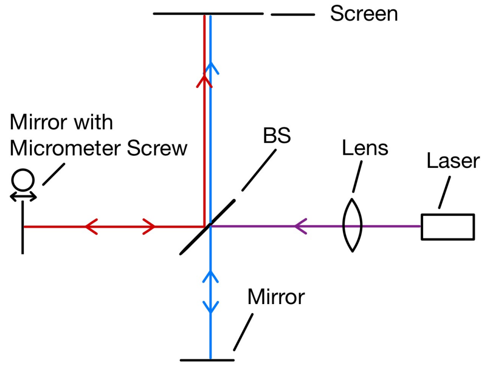


Figure 1: Setup for measuring the wavelength from the lasers

3.2 Thermal Expansion of Aluminium

This setup shown in figure 2 is essentially the same as the first setup, just the micrometer screw with mirror is replaced by an aluminium rod with an mirror attached to its end. The aluminium rod was heatable so that the temperature could be monitored with an thermometer inside the rod.

Analog to the laser wavelength measurements the number of light dark transition on the screen was counted, but in dependence on some temperature difference of the aluminium rod. This was done for several temperature ranges.

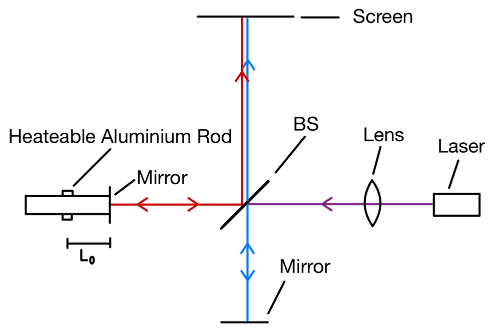


Figure 2: Setup for measuring the thermal expansion coefficient of aluminium

4 Data Analysis and Result

The uncertainty for a variable x with triangular distributed error a_x is calculated by the formula

$$\Delta x = \frac{a_x}{\sqrt{6}}. \tag{5}$$

All the uncertainties come from the errors of the used values through the Gaussian error propagation

$$\Delta A = \sqrt{\left(\frac{\partial A}{\partial x} \cdot \Delta x\right)^2 + \left(\frac{\partial A}{\partial y} \cdot \Delta y\right)^2}. \tag{6}$$

This is an example for a function A dependent on the variables x and y and their uncertainties.

4.1 Characterisation of the Laser Wavelength

For this measurements several errors were important. First of all the error on the travel length of the mirror s attached to the micrometer screw was estimated by

$$\Delta s = 0.4\,\mu\mathrm{m}.$$

Another important error was the error on the number of light dark transitions N, this was very difficult to count, because the vibrations on the table were pretty severe and the transitions occurred very fast. This is the reason behind the conservative error of

$$\Delta N = N \cdot 5\%.$$

Those two errors resulted in the uncertainties depicted in the following diagrams. The observed rings were recorded and played in slow motion to count everything. Both of those errors were statistical errors, and could be compensated by many measurements.

4.1.1 Green Laser

the measurement with the green laser is illustrated in figure 3. In the plot, the measured number of observed periods N is pictured in dependency of the difference in length Δs between the two interferometer arms. This comes through the change in the length x of the arm with the micrometer screw. In the plot the data points are shown with their uncertainty calculated with equation (6). On top of this, a linear regression is made which leads to the fit with uncertainty band. The wavelength of the green laser is calculated from the slope of the fit by using equation (1). Finally the wavelength is

$$\lambda_{\mathrm{green}} = (536 \pm 10)\,\mathrm{nm}.$$

The fit has a reasonable good quality with a reduced $\chi^2/\mathrm{ndf} = 1.45$.

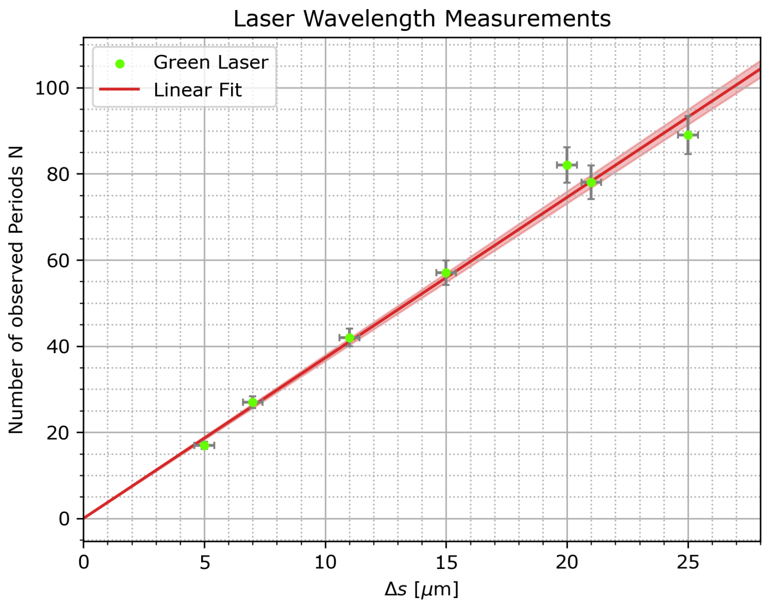


Figure 3: Number of observed rings N dependent on the difference in length Δs for the green laser.

4.1.2 Red Laser

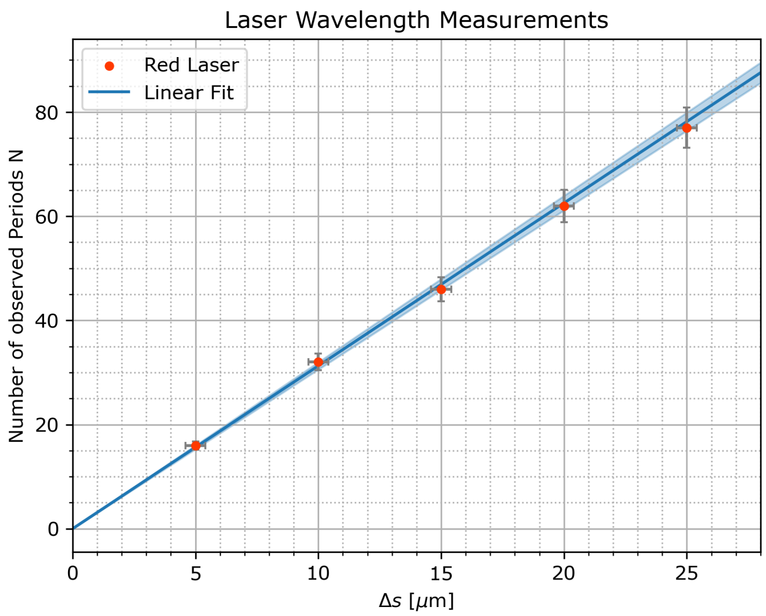


Figure 4: Number of observed rings N dependent on the difference in length Δs for the red Laser.

The analysis to calculate the wavelength of the red laser is done equally to section 4.1.1. From figure 4, λ_{red} is calculated from the slope to

$$\lambda_{\text{red}} = (639 \pm 14)\,\text{nm}.$$

The fit has a very good quality with a reduced $\chi^2/\text{ndf} = 0.17$.

4.1.3 Blue Laser

For the blue laser the same analysis is done again by using the fit from figure 5. This leads to the wavelength

$$\lambda_{\text{blue}} = (415 \pm 9)\,\text{nm}.$$

The fit has also a very good quality with a reduced $\chi^2/\text{ndf} = 0.15$.

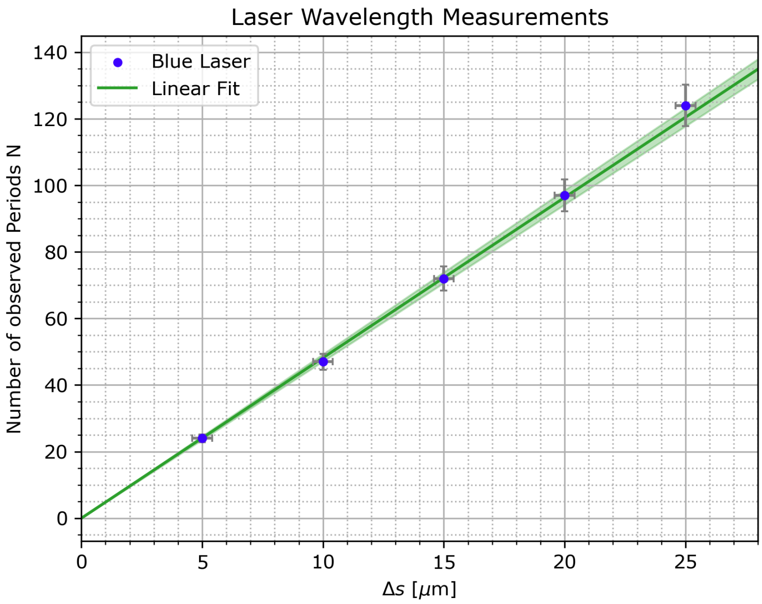


Figure 5: Number of observed rings N dependent on the difference in length Δs for the blue Laser.

4.2 Thermal Expansion Coefficient

For the thermal expansion coefficient two different variables were taken, one was like in the measurements before the number of light and dark transitions N but this time it was easier to count so the error was decreased to

$$\Delta N = 1.$$

The other variable measured was the difference in temperature at the start and at the end of the measurement. Because the absolute temperature was given with an uncertainty of $\Delta T = 1\,°\mathrm{C}$ [2], this yields for the temperature difference ΔT an error of

$$\Delta(\Delta T) = 1.4\,°\mathrm{C}.$$

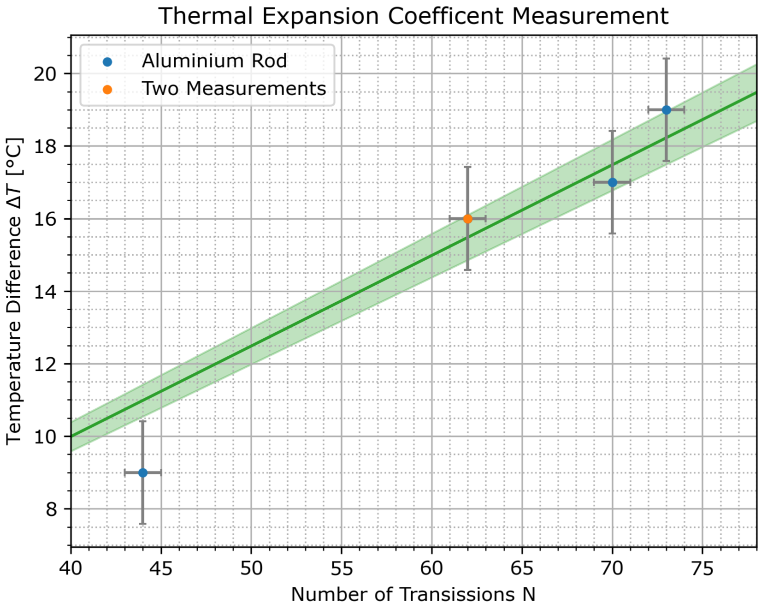


Figure 6: Measured temperature difference ΔT for the amount of counted rings N.

In figure 6 the measured difference in temperature ΔT of the aluminium rod is shown in dependency on the number of observed light-dark transitions N. The values are illustrated with their uncertainties. Furthermore, a fit (in green with uncertainty) is made by using linear regression. The one measurement point is orange, because the exact same value was measured twice. The blue values were all measured once. For this experiment the thermal expansion of aluminium α is calculated with equation (2) and the slope of the fit. A value of

$$\alpha = (23.9 \pm 1.0) \times 10^{-6}\,\mathrm{K}^{-1}$$

is achieved. The quality of the fit is good even with a reduced $\chi^2/ndf = 2.66$. But this is okay in this case because there are just five data points and with such a low sample rate the reduced χ^2 is not that significant, the one outlier at $N = 44$ is reasonable okay.

5 Discussion

5.1 Final Results

the final results for the wavelength measurements with the three lasers are presented in table 1.

Laser	Wavelength λ
Green	$(536 \pm 10)\,\mathrm{nm}$
Red	$(639 \pm 14)\,\mathrm{nm}$
Blue	$(415 \pm 9)\,\mathrm{nm}$

Table 1: Results for the measured laser wavelengths

The thermal expansion coefficient is determined to be

$$\alpha = (23.9 \pm 1.0) \times 10^{-6}\,\mathrm{K}^{-1}.$$

5.2 Comparison with Expected Results

Whether a measured value x is tolerable with the literature value y or not can be determined with a t-test

$$t = \frac{\mid \hat{x} - \hat{y} \mid}{\sqrt{(\Delta x)^2 + (\Delta y)^2}}. \tag{7}$$

With a significance level of $\alpha = 0.05$ a divergence of $t \geq 2$ is considered significant. Is the t-value smaller, the result is compatible with the literature value.

5.2.1 Characterisation of the Laser Wavelength

Literature Wavelengths and uncertainties				
Color	λ_{min}	λ_{typical}	λ_{max}	$\Delta\lambda$
Green[3]	531 nm	532 nm	533 nm	± 0.4 nm
Red[4]	630 nm	635 nm	643 nm	± 3 nm
Blue[5]	400 nm	405 nm	410 nm	± 2 nm

Table 2: Literature Values for the lasers with minimal, typical and maximum wavelengths λ and estimated uncertainty $\Delta\lambda$.

The uncertainty on the literature value is calculated with $a_\lambda = \frac{\lambda_{\text{max}} - \lambda_{\text{min}}}{2}$ triangular distributed. This leads to the uncertainties shown in table 2. A t-test is made for all measured values (table 3). Because the t-values are < 2 for all lasers, the measured wavelengths of the lasers are all tolerable by their literature values. This measurement refers as a check on our setup for the determination of the thermal expansion coefficient. With that result we can say, our setup was good and the lasers worked as expected.

Literature Wavelengths λ and t-Tests		
Green	$(532.0 \pm 0.4)\,\mathrm{nm}$	$t = 0.46$
Red	$(635 \pm 3)\,\mathrm{nm}$	$t = 0.31$
Blue	$(405 \pm 2)\,\mathrm{nm}$	$t = 1.05$

Table 3: Literature Values of the lasers wavelength λ and t-tests for the measured values compared to the literature values.

5.2.2 Thermal Expansion Coefficient

The literature value for the thermal expansion coefficient of aluminium is

$$\alpha_{\mathrm{lit}} = 23.1 \times 10^{-6}\,\mathrm{K}^{-1}[6].$$

With a t-test for a literature value without uncertainty, this yields a t-value of

$$t = \frac{\mid \hat{\alpha} - \alpha_{\mathrm{lit}} \mid}{\Delta\alpha} = 0.87.$$

This shows, that our measurement was very close to the literature value and is compatible with it.

5.3 Improved Methods

First of all the setup was really prone to vibration, because the table was not that stable. This is a major problem for counting the light dark transitions. This could be improved by mounting the table on a vibration damping table. Another really helpful device would be a counter connected to a photo diode, this would not only improve the precision of the counting, but also speed up the measuring speed. It would help to reduce the error from counting and it would reduce the statistical error from the performed fits, because it would be possible to do more measurements in the same time.

6 Conclusion

The experiment yields some very precise measurements for the wavelength and the thermal expansion coefficient, which are fully in line with the expected values. Those measurements were limited by the vibrations of the table, not by the setup in general. With some longer measurements, less vibrations and a better way of counting the number of transitions the precision could be increased significantly.

A List of figures

B List of tables

C Literature

[1] NobelPrize.org. "Albert a. michelson – facts." (2024), [Online]. Available: https://www.nobelprize.org/prizes/physics/1907/michelson/facts/ (visited on 09/30/2024).

[2] Greisinger. "Gmh 1150." (2024), [Online]. Available: https://www.greisinger.de/files/upload/de/produkte/bda/GMH1150_DE.pdf (visited on 09/27/2024).

[3] Thorlabs. "Collimated dpss laser module 532 nm, 0.9 mw, round beam." (Mar. 28, 2014), [Online]. Available: https://www.thorlabs.com/drawings/4959347149275281-9F932B2E-056D-BD07-D542910282FA5965/CPS532-C2-SpecSheet.pdf (visited on 09/30/2024).

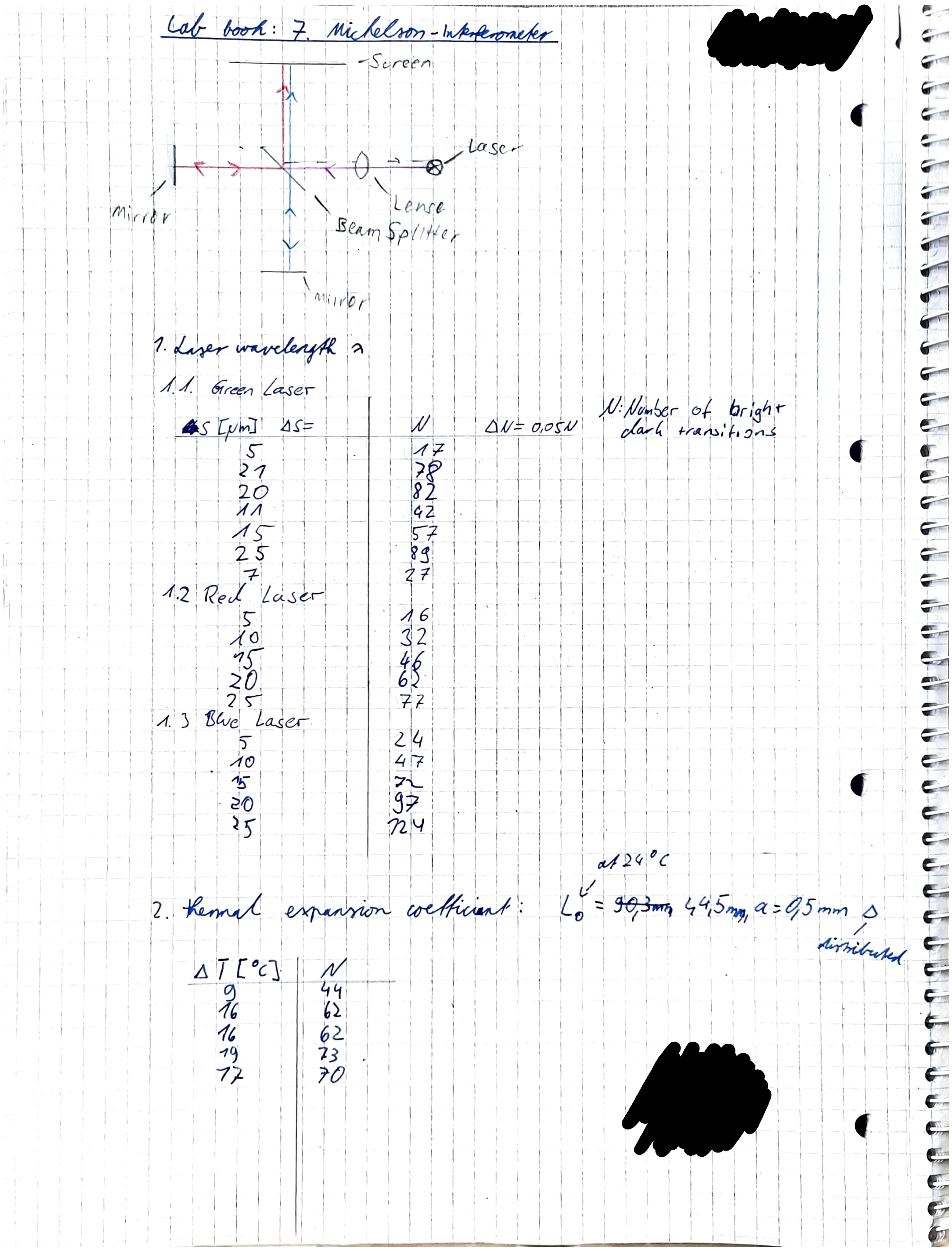
[4] Thorlabs. "Compact laser module with usb, 635 nm." (Jan. 30, 2024), [Online]. Available: https://www.thorlabs.com/drawings/4959347149275281-9F932B2E-056D-BD07-D542910282FA5965/PL202-SpecSheet.pdf (visited on 09/30/2024).

[5] Thorlabs. "Compact laser module with usb, 405 nm." (Jan. 23, 2024), [Online]. Available: https://www.thorlabs.com/drawings/4959347149275281-9F932B2E-056D-BD07-D542910282FA5965/PL205-SpecSheet.pdf (visited on 09/30/2024).

[6] W. M. Haynes, "Crc handbook of chemistry and physics 95th edition," in CRC Press, 2014, ch. 12.

D Attachment

D.1 Lab Book



Lab book: 7. Michelson-Interferometer [redacted]

1. Laser wavelength λ

1.1. Green Laser

N: Number of bright dark transitions

ΔS [μm] ΔS=	N ΔN = 0,05N
5	17
21	78
20	82
11	42
15	57
25	89
7	27

1.2 Red Laser

ΔS [μm]	N
5	16
10	32
15	46
20	62
25	77

1.3 Blue Laser

ΔS [μm]	N
5	24
10	47
15	72
20	97
25	124

2. thermal expansion coefficient: L_0 = ~~90,3mm~~ 44,5mm, a = 0,5mm Δ

at 24°C

distributed

ΔT [°C]	N
9	44
16	62
16	62
19	73
17	70

[redacted]

Figure 7: Lab book