Inhaltsverzeichnis

1 Introduction

In this experiment we will determine the grating constant of an simple amplitude grating and an aperture-function of another grid. A wave in liquid, created by an ultrasound, creates a phase grid.

2 Scope of work

1. Determine the grating constant of a sinus grid, from the distance of the order of diffraction.
2. Determine the grating constant and the resolution of five amplitude grids which are full illuminated.
3. Compute the aperture-function of the grid with the biggest grating constant. Also a period of the aperture-function should be drawn.
4. Determine the relation between the gap width and the grating constant through the aperture-function.
5. a) Measurement of the distribution from the intensity, in dependence of the voltage on the piezoelectric crystal.
   b) Comparison of the measurement results with the Raman-Nath-theory.
   c) Determine of the sonic-wavelength in Isooktan through measure the diffraction order and compare it with a calculated value.

All our references, including images, are from the 'Ültraschall Versuchsanleitung", Fortgeschrittenen Praktikum, Albertludwigsunversität Freiburg' [1].

3 Theoretical principles

3.1 Diffraction

Diffraction is called a process of changing the way of an electro-magnetic wave, without refraction or reflection of the wave. Instead there is a barrier on the way of the light, which changes the direction of propagation from the wave. This you can explain with the Huygen's concept, which says that a wave will expand in a spherical from every point of a wave front. These waves are interfere with each other and it will be generated constructive interference or destructive interference. It holds:

$$g = m \cdot \lambda \tag{1}$$

$$\sin(\theta) = \frac{m \cdot \lambda}{K} \tag{2}$$

3.1.1 Amplitude grid

An amplitude grid shows the effect of diffraction through changing the amplitude of the incoming wave. The phase wont be changed. The aperture function describes the transmission of light in dependency of the location.

3.2 Aperture-function

Each grid can be allocate a aperture-function. These describes the characteristics of the grid, by collate a value to each point in the slit-plane. With the Kirchhoff integral theorem can be show, that the intensity distribution I of the diffraction image is the Fourier transformation from the aperture-function g of the grid.

$$I = |\psi(x,y)|^2 = |\int_{slit} g(\vec{k}) \cdot e^{i\vec{k}\vec{r}} dA|^2 \tag{3}$$

So backwards must be follow that the aperture-function can be written as a Fourier transformation of the intensity distribution.
The Fourier transformation of an function $g(\vec{x})$ is define by:

$$g(\vec{x}) = \int_{slit} g(\vec{x}) \cdot e^{i\vec{k}\vec{x}} d\vec{x} \tag{4}$$

For cases, in which the intensity distribution is not known in detail, we can approximate the aperture-function with a Fourier series. In this the root of the intensity maxima appear as coefficients:

$$g(\vec{x}) = \sum_{j=0}^{\infty} \pm \sqrt{I_j} cos(\frac{\vec{x}}{K} 2\pi j) \tag{5}$$

For example a sinus grid has the function: $g(x) = \sqrt{I_0} + \sqrt{I_1} cos(\frac{x}{K} 2\pi)$, because there are only maxima zeroth or first order.

From the graph of the aperture-function can be read out the proportion between the width of slit and the distance of the slits.

3.3 Angular resolution

The angular resolution a is define by:

$$a = \frac{\lambda}{\Delta\lambda} \tag{6}$$

where λ the wavelength and $\Delta\lambda$ the wavelength distance is, for the case that the diffraction can be see. Also can be showed that:

$$a = Nm \tag{7}$$

So the angular resolution is the number of light-flooded grid lines N multiply by the maximum number of diffraction m, which we can see.

3.4 Phase grid

In this experiment the phase grid is be created by an ultrasound in isooctane. The ultra-wave produce a periodical ducal fluctuation in the liquid and so differential refractive indexes. This is the reason for different angular phase shift in the grid. For the refractive index n in the medium follow:

$$n(x) = n_0 + \Delta n sin(\frac{2\pi}{\Lambda}x) \tag{8}$$

where the ultra-wave length Λ the analogon to the lattice parameter is. The intensity of the wave S is proportional to the square of the voltage U, which is invested at the cell, and also to the square of the relative density: $S \propto (\frac{\Delta\rho}{\rho_0})^2$. With $\frac{\Delta n}{n-1} = \frac{\Delta\rho}{\rho_0}$ follows $\Delta n \propto S$ and so we can change the refractive index with the amplitude of the sound-wave field.

3.5 Raman-Nath-theory

- It is necessary for the angular of the intensity maxima from the diffraction image of m^{th} order, that:

$$sin(\Theta) = \pm\frac{\lambda}{\Lambda}m \tag{9}$$

- The Bessel-function I shows the proportion between the intensity of the maxima from two different order m and m':

$$\frac{I_m}{I_{m'}} = \frac{J_m^2(\Delta n D\frac{2\pi}{\lambda})}{J_{m'}^2(\Delta n\frac{2\pi}{\lambda})} \tag{10}$$

And for our experiment follows:

$$I_m = J_m^2(\Delta n D\frac{2\pi}{\lambda}) = J_m^2(\alpha U) \tag{11}$$

4 Setup And Execution

This experiment is composed of a He-Ne-Laser, two mirrors to redirect the laser beam, a rotating mirror, three optical lenses, two photo-diodes and a beam splitter. Between the blend and the third lens we applied several gratings or the ultrasonic-oscillating-quartz. The mirror which rotating reflects the laser beams on the photo diodes, which are justified in different height.

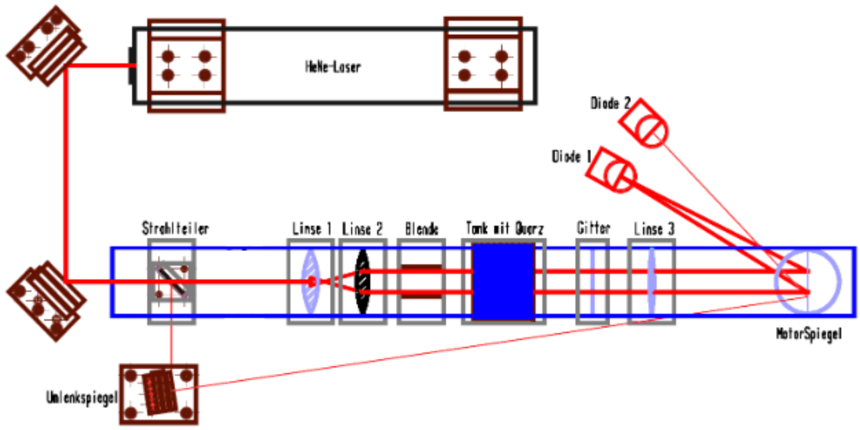


Abbildung 1: Setup of the experiment [1]

5 Execution

At first we had established the experiment. We focussed the laser on the center of the optical axis. Then we placed the sinus grid and a white screen behind and measured the distance of the first orders of the diffraction. After this we measured the five amplitude gratings. Therefor we have to setting the lenses, so that the beam can impact on the diode. We can then use the computer to save the data of the maxima. Then we change the amplitude gratings, into the phase grating. We measured then the intensity of the diffraction from the maxima.

6 Analysis

6.1 Grating Constant Of The Sinus-Grid

To determine the grating constant of the sinus-grid we measured the distance a between the grid and the screen and the distance x between the zero-maxima and the first maxima. For our different values on the left and the right side of the zero-maxima we get the following values for the first maxima:

$$x_1 = (4.7 \pm 0.1)cm \tag{12}$$
$$x_2 = (4.5 \pm 0.1)cm \tag{13}$$

So we get the mean value with

$$\bar{x} = (4.6 \pm 0.1)cm \tag{14}$$

Now we using the formulas:

$$\sin(\theta) = \frac{\bar{x}}{\sqrt{\bar{x}^2 + a^2}} \tag{15}$$

$$K = \frac{\lambda}{\sin(\theta)} = \lambda \cdot \frac{\sqrt{\bar{x}^2 + a^2}}{\bar{x}} \tag{16}$$

$$s_K = \frac{a\lambda^2}{K\bar{x}^2} \cdot \sqrt{s_a^2 + \frac{a^2 s_x^2}{4\bar{x}^2}} \tag{17}$$

So we get, with $\lambda = 632.8nm$, for our grating-constant:

$$K = (1.04 \pm 0.05) \cdot 10^{-6} m \tag{18}$$

6.2 Amplitude-Gratings

At first we show a reference grid to calibrate the time axis. We know that our grid has a grating-constant of $K = 0.125mm$. With the following context we can oak our time line. We use the formula:

$$\sin(\theta_m) = m\lambda/K \tag{19}$$

$$at + b = m\lambda/K \tag{20}$$

$$\tag{21}$$

Now we plot the sinus against Δt, shown in figure (6.2), and get the slope. This is now our factor to oak.

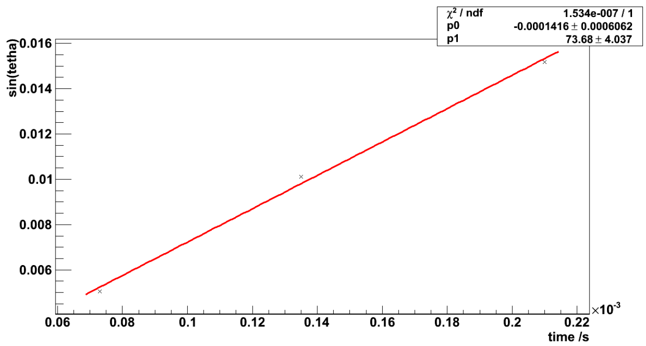


Abbildung 2: Adjustment measurement

So we get

$$a = (74 \pm 4) \tag{22}$$

$$b = (0.00014 \pm 0.00061) \tag{23}$$

Now we can calculate the grating constants of the different grids. We get our results in tab(6.2) with:

$$K = \frac{m\lambda}{at+b} \tag{24}$$

$$s_K = \frac{K}{a\cdot\Delta t+b}\sqrt{\Delta t^2 s_a^2 + s_b^2 + a^2 s_{\Delta t}^2} \tag{25}$$

We compute K for every peak we see and calculate then the mean value. This gets our final K's.

Tabelle 1: grating-constants

Grating	$K/10^{-6}m$	$s_K/10^{-6}m$
1	133	13
2	34	2
3	103	10
4	91	9
5	54	4

We can also compute the resolution of the gratings now. We use the following equation for that.

$$a = \frac{\lambda}{\Delta\lambda} = m\cdot N = m\frac{D_{Laser}}{K} \tag{26}$$

$$s_a = a\sqrt{\left(\frac{s_D}{D_{Laser}}\right)^2 + \left(\frac{s_K}{K}\right)^2} \tag{27}$$

With m = Number of visible order and $D_{Laser} = (4\pm0.5)mm$ So we get the values:

Tabelle 2: resolution of gratings

Grating	a	s_a
1	90	9
2	236	15
3	78	8
4	88	9
5	145	10

6.3 Aperture-Function

Now we use grid one to determine the aperture function. To compute these we get the approximation of the Fourier series:

$$g(x) = \sum_{j=0}^{m}\sqrt{I_j}\cos\left(\frac{x}{K}2\pi j\right) \tag{28}$$

For the intensity I_j we use the mean values of the measured maxima of the deflection.

Tabelle 3: Intensities

Order	Intensity /V	s_I /V
I_0	3,85	0,08
I_1	0,185	0,004
I_2	0,146	0,003
I_3	0,845	0,0017

Now we can plot the aperture-function, see in fig(3)

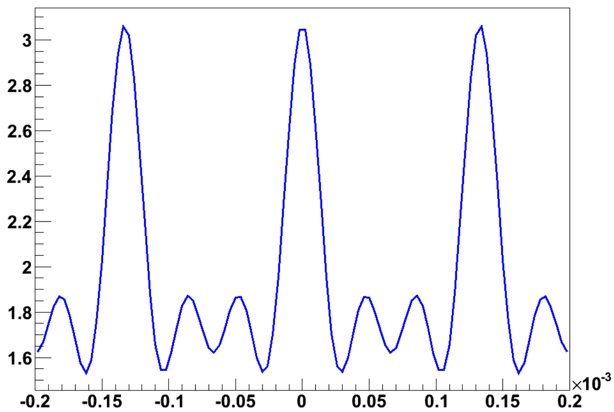


Abbildung 3: Aperture-Function

6.4 Relation between the width of slit to the grating constant

In this part we determine the relation of width of the gap and the grating constant from our aperture function. At first we designate the full width of the half of the maximum, from the aperture function and get so the width of the slit $b = (27.8 \pm 0.9) \cdot 10^{-6}$.

With the following formulas we calculate the rate v.

$$v = b/d \qquad d = K - b \tag{29}$$

$$\Rightarrow v = \frac{b}{K-b} = (0.25 \pm 0.03) \tag{30}$$

The error of v we calculated with:

$$s_v = v \cdot \sqrt{\left(\frac{s_b}{b}\right)^2 + \frac{s_K^2 + s_b^2}{d^2}} \tag{31}$$

6.5 Phase grating

We expect for the phase grating, that the amplitude of the differnt maxima to be related to the Bessel functions(equation 11). So first we have to determine the parameter α. We can find α if we compare the first minimum of the Bessel function J_0 with the first minimum of our zeroth order and also we compare the first maximum of J_1 with the first maximum of our first order. In our case follow:$\alpha_0 = \frac{2,4}{9} = 0,267$ and $\alpha_1 = \frac{1,84}{6,5} = 0,283$. For our fit we will use the mean of this values: $\alpha = 0,275$. For our fit we normalized the mean of the two measured values with the intensity of the zeroth order maximum at 0 volt:

$$I_m = \frac{I_{left} + I_{right}}{2I_0} = \frac{I}{2I_0} \tag{32}$$

For our error we calculate with a uncertainties of $\frac{1}{10}division$ for locating the peak in our spectrum and a relative error of 3% from the resolution of the oscilloscope. So for our errors follows:

$$s_{I_{left,right}} = \sqrt{(0,03I_{left,right})^2 + \frac{1}{10}div} \tag{33}$$

$$\tag{34}$$

$$s_I = \sqrt{(s_{I_{right}})^2 + (s_{I_{left}})^2} \tag{35}$$

$$s_{I_m} = I_m\sqrt{(\frac{s_I}{I})^2 + (\frac{s_{I_0}}{I_0})^2} \tag{36}$$

So now can plot our data (Fig. 6.5). We see in our plot that we can confirm the Raman-Nath-thoery with our data.

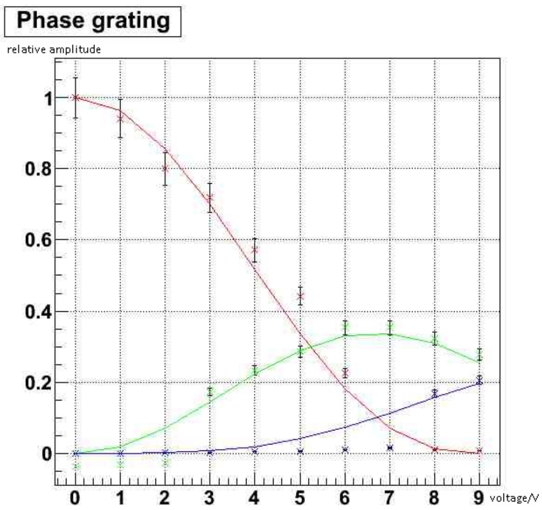


Abbildung 4: red: zeroth order, green: first order, blue: second order

6.6 Sonic Wavelength

Now we would compute the sonic wavelength in Isooktan. Therefor we measured a reference grating, like before, see in fig(5).

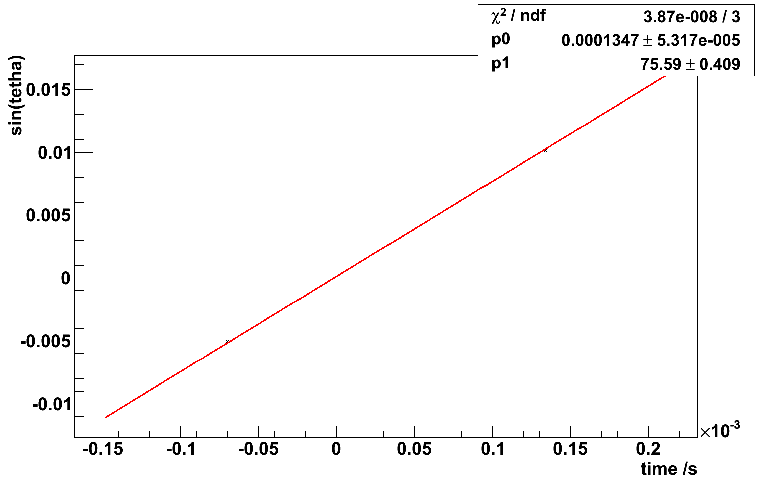


Abbildung 5: Reference Grating; p0 = b, p1 = a

Like before, our K is now the wavelength Λ, it holds:

$$sin\theta = \frac{m\lambda}{\Lambda} \tag{37}$$

$$\Rightarrow \Lambda = \frac{m\lambda}{at+b} \tag{38}$$

Averaged we get for the sonic wavelength in Isooktan:

$$\Lambda = (534 \pm 51)\mu m \tag{39}$$

Comparison With The Theory With the literature value of the speed of the sound in Isooktan, $v = 1111\frac{m}{s}$, and the frequency which was applied, $f = (2100 \pm 1)kHz$ we get:

$$\Lambda_0 = \frac{v}{f} = (529 \pm 3)\mu m \tag{40}$$

So we can see, that our Λ has just one standard deviation difference to Λ_0.

7 Conclusion

In our first measurement we test a sinus grid. We compute the grating constant with:

$$K = (1.04 \pm 0.05) \cdot 10^{-6} m \tag{41}$$

Compared with the theoretical value $K_t = 0.98 \cdot 10^{-6} m$, we can see, that our value is just one standard deviation away.

For our five amplitude gratings we get the grating-constants:

$$K_1 = (133 \pm 13) \mu m \tag{42}$$
$$K_2 = (34 \pm 2) \mu m \tag{43}$$
$$K_3 = (103 \pm 10) \mu m \tag{44}$$
$$K_4 = (91 \pm 9) \mu m \tag{45}$$
$$K_5 = (54 \pm 4) \mu m \tag{46}$$

And, also for the five gratings, we get the resolution:

$$a_1 = (90 \pm 9) \mu m \tag{47}$$
$$a_2 = (236 \pm 15) \mu m \tag{48}$$
$$a_3 = (78 \pm 8) \mu m \tag{49}$$
$$a_4 = (88 \pm 9) \mu m \tag{50}$$
$$a_5 = (145 \pm 10) \mu m \tag{51}$$
$$\tag{52}$$

With the aperture-function we have created fig(3), and measured with this the relation between the width of the slit and the grating constant:

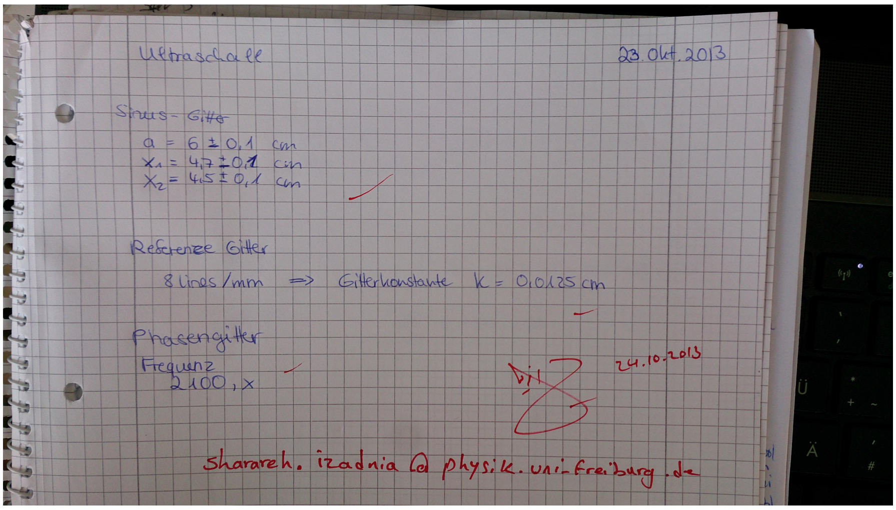
$$v = (0.25 \pm 0.03) \tag{53}$$

At last we have probed the phase grating. With the phase grid we can confirm the qualitative behaviour of the Raman-Nath-Theory. Moreover we compute the ultrasonic wavelength in Isooktan.

$$\Lambda = (534 \pm 51) \mu m \tag{54}$$

Which is just one standard deviation away from the calculated value with $\Lambda_0 = (529 \pm 3) \mu m$

8 Addendum


Ultraschall
23. Okt. 2013
Spalt-Gitter
a = 6 ± 0,1 cm
x1 = 4,7 ± 0,1 cm
x2 = 4,5 ± 0,1 cm
Referenz Gitter
8 Lines / mm ⇒ Gitterkonstante k = 0,0125 cm
Phasengitter
Frequenz
2100, x
24.10.2013
sharareh.izadnia@physik.uni-freiburg.de

Literatur

[1] M.Kohli. Versuchsanleitung Ultraschall. http://wwwhep.physik.uni-freiburg.de, 2011.