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1 Abstract

In this experiment we have a closer look at a sinus grid and calculate its grid constant. Then we examine several different amplitude grids, calculate their grid constants and resolution power. Besides, we define the aperture function of a grid by investigating the intensity distribution. Finally, we examine light diffraction in an acousto-optic modulator and compare it to the predictions of the RAMAN-NATH-theory.

2 Theory

2.1 Diffraction

We are talking about diffraction if a wave is bent around the corner of an obstacle. This could be a grid, a simple slit or similar constructions. Because of the bending, the wave is entering the area of geometrical shadow and can interfere with other bent wave parts. Taking a closer look at this phenomenon, we divide diffraction into two kinds, the first one is called FRAUENHOFER-diffraction and describes wave diffraction viewed from a wide perspective. The single wave packages are seen as parallel and coherent and each single package is bent by an angle θ around the corner. The second one is the FRESNEL-diffraction where the different wave packages have a range of diffraction angles between $\theta \pm \Delta\theta$.

2.1.1 Aperture function

The light emitted by a laser is almost parallel and coherent. As a consequence, it can be refracted when it hits a grid. The resulting intensity distribution is depending on the geometric circumference of the grid which is described by the aperture function $g(x,y)$. x and y are coordinates in the plane of the grid which are used to describe all locations on the grid. g gives each point of the grid a transmission probability between 0 and 100%.
The resulting intensity distribution is the square of the absolute value of the Fourier transform of the aperture function:

$$I = |y|^2(\vec{x}) = |\int_{\text{split}} g(\vec{k}) \cdot e^{i\vec{k}\cdot\vec{x}} d\vec{k}|^2 \tag{1}$$

2.1.2 Single Slit

As an example the aperture function g of a **single split** with a width b is given by:

$$g(x) = \begin{cases} 0 \ \text{for} |x| > b/2 \\ 1 \ \text{for} |x| \leq b/2 \end{cases} \tag{2}$$

.

The Amplitude A at an angle θ behind the split can then be calculated with the Fourier transformation

$$A(\Theta) \sim \int_{-b/2}^{b/2} e^{ikx\sin(\Theta)}dx = \frac{\sin(kb\sin(\Theta)/2)}{kb\sin(\Theta)} \tag{3}$$

By using $\beta(\Theta) = kb\sin(\Theta)$ we get the intensity distribution as

$$I(\Theta) = A(\Theta)^2 \sim \left(\frac{\sin(\beta(\Theta))}{\beta(\Theta)}\right)^2 \tag{4}$$

Being a symmetrical transformation, the Fourier transformation can also be used to transform an intensity distribution back to an aperture function. Like that, the refraction of light can tell us how the surface of a small object looks like.

2.1.3 Grid

For a **grid with N lines** we can make a similar approach. The grid constant K is given as the distance between the centre of two adjacent splits.
The aperture function is now given as:

$$g(x) = \begin{cases} 1 \text{ for } j\cdot K \leq x \leq j\cdot K + b \text{ with } j = 0, ..., N-1 \\ 0 \text{ otherwise} \end{cases} \tag{5}$$

Thus, the amplitude A at a given angle is the sum over N integrals:

$$A(\theta) = \sum_{j=0}^{N-1} \int_0^{j\cdot K+b} e^{ikx\sin\theta}dx \tag{6}$$

And for the intensity distribution we receive:

$$I(\theta) = A(\theta)^2 \sim \left(\frac{\sin\beta(\theta)}{\beta(\theta)}\right)^2 \cdot \left(\frac{\sin N\gamma}{N\gamma}\right)^2 \tag{7}$$

with the same $\beta(\theta)$ as above and $\gamma(\theta) = kK\sin\theta/2$.
Here you can see that the intensity distribution is dependent on two main parts, the first one is surrounding the peaks and the second one gives their exact position.

2.1.4 Experimental determination

The relation between the angle θ and the order m of the resulting peaks is given by:

$$m\lambda = K \sin\theta. \tag{8}$$

λ is the wavelength of the light we used and K is the grid constant.

Now we want to receive the aperture function through the Fourier transformation of the intensity distribution. Thus, we approximate the transform with a Fourier series:

$$g(x) = \sum_{j=0}^{\infty} \pm\sqrt{I_j} \cos\left(\frac{x}{K} 2\pi j\right) \tag{9}$$

Because the higher the diffraction order is, the smaller are the amplitudes, we can approximate the Fourier series and for a sinus grid we get:

$$g(x) = \sqrt{I_0} + \sqrt{I_1} \cdot \cos\left(\frac{x}{K} 2\pi\right) \tag{10}$$

Here the transmission is harmonic periodic.

2.2 Grids

An optical obstacle with various columns at which the incoming light beam is diffracted is called grids. We can differentiate between grids that modulate the amplitude and those that change the phase of the incoming light.

2.2.1 Resolution power

In general the resolution power of a grid is defined by

$$a = \frac{\lambda}{\Delta\lambda} \tag{11}$$

where λ is again the used wave length and $\Delta\lambda$ is the smallest distance between two lines which we can differentiate.
If we have the special case of an amplitude grid we can also define the resolution power by

$$a = N \cdot m \tag{12}$$

with N being the grid lines which were illuminated and m the number of peaks seen. Thanks to this equation it becomes possible to determine the resolution power in our experiment.

2.2.2 Amplitude Grid

The refraction index of an amplitude grid is the same everywhere, thus it only changes the amplitude of the incoming light beam. Its aperture function is real-valued and 1 for every x with 100% transmission and 0 for every x without any transmission.

2.2.3 Phase grid

Phase grids allow us to have a constant transmission at different refection indices, that's how diffracted light gets phase modulated. Because now the different beams take different paths we receive an interference pattern at the end and the aperture function becomes complex.

2.3 Acustooptical Effect

Quite a common method to preserve a phase grid is by sending ultrasound waves with a wavelength Λ through a medium which results in a change of the refraction index by Δn and a change in the density $\Delta\rho$.

$$\frac{\Delta n}{n-1} = \frac{\Delta\rho}{\rho_0} \tag{13}$$

Here ρ_0 is the density of the medium without the ultrasound waves. The intensity of the sound is $S \propto (\frac{\Delta\rho}{\rho_0})^2$, thus $\Delta n \propto \sqrt{S}$, with $\sqrt{S}$ being the amplitude of the ultrasound field.
The local changes of the refraction indices are given by:

$$n(x) = n_0 + \Delta n \sin\left(\frac{2\pi x}{\Lambda}\right) \tag{14}$$

2.3.1 RAMAN-NATH-Effect

The relation between the angle at which we find a maximum of m. order and the sound wavelength Λ is given by

$$\sin\theta = \pm m\frac{\lambda}{\Lambda}. \tag{15}$$

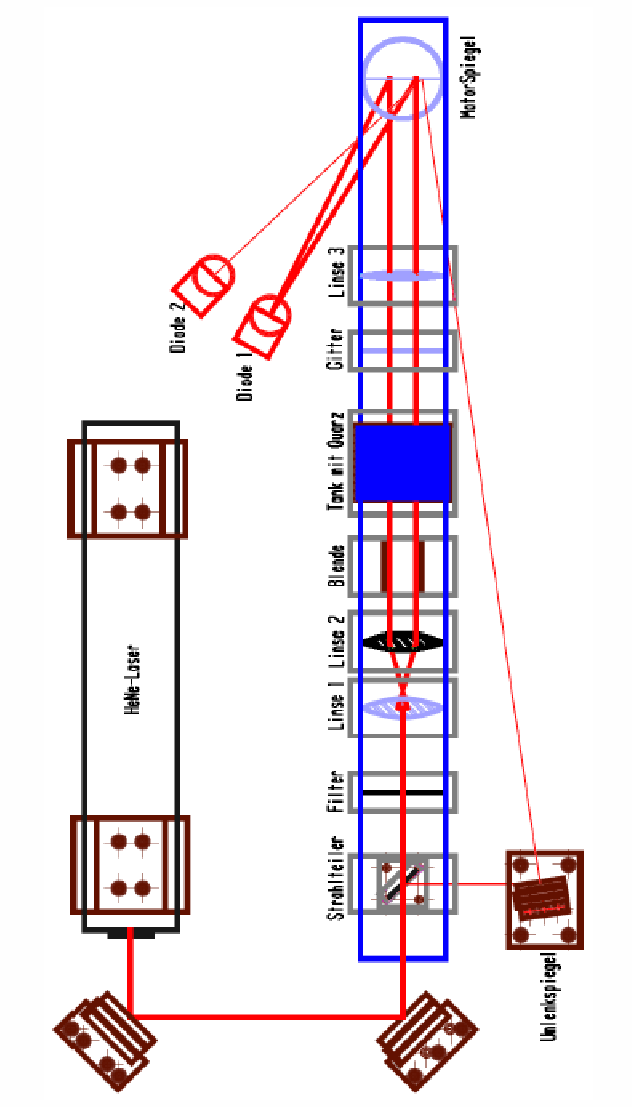
Besides, the link between the maximum of m. order and the Bessel functions J is:

$$I_m = {J_m}^2(D\frac{2\pi}{\lambda}) = {J_m}^2(\alpha U) \tag{16}$$

where D is the thickness of the medium where the sound goes through.

3 Experimental setup

Figure 1: Schematic setup of the experiment [1].



The used laser is an 632,8 nm He-Ne-Laser.
First, the ray is split up into two parts. The main part goes straight through the beam splitter, gets filtered and then widened up by two convex lenses (f= 50 mm, f= 100 mm). The resulting light beam is parallel and adjusted with an aperture. Based on the part of the experiment, different grids can be used to refract the beam. After refraction, a third convex lens (f= 300 mm) focuses the beam in the direction of a spinning mirror. The focus of the lens is behind the mirror so that the reflected ray is thrown on a first

diode that measures the intensity of the beam. This arrangement of lenses and grids is also called "Frauenhofersche Anordnung"
The second part of the split beam is adjusted to be reflected by the spinning mirror as well. A second diode measures the intensity of the incoming light and is used to send a trigger signal as soon as the un-refracted ray hits it.
For the last part of the experiment, a continuous ultrasonic wave in a tank filled with isooktan is used instead of a classical grid. Because of the ultrasound in the medium there is a change in the density of the material and this setup can be used as a phase grid.

4 Execution

1. At first we observe a sinus grid. Due to the fact that the angle between the 0. and 1. main maximum is quite large, we dom't widen up the beam by removing the first two lenses. Then we measure the distance between the 0. main maximum and the two maxima of first order on a screen directly behind the grid. Since we don't measure the angle, but an amplitude on the screen, we measure the distance between screen and grid as well.

2. To measure the angle of the maxima for common amplitude grids, we use the spinning mirror and the two diodes. The two lenses are put back into the ray to widen up the beam.
   By measuring the intensity at the first diode as a function of time, we get an intensity distribution. Since the mirror is spinning with a speed around 12.5 turns per second, the trigger signal of the second diode is needed to mark the centre of the distribution.
   To be able to calculate grid constants, a reference grid is used to find a function for transforming the intensity over time into an intensity over angle. The grid constant of the calibration grid was given as $K = 80$ lines / cm.
   After the calibration, we take measurements for 5 different grids by determining the position of their maxima. Afterwards we remove the cover from the beam and put the screen in front of the grid. On the screen, we were able to measure the diameter of the beam which we will need later to find the number of slits which are illuminated.

3. For the first grid, we additionally measure the heights of peaks to get information about the relative intensities of the peaks.

4. Now we use the sonic wave in the isooctan tank as a phase grid.
   The two lenses, used to widen up the beam, are not exactly centred and their position has to be adjusted. As we modify our objects in the beam, we re-calibrate our experiment by putting the calibration grid behind the tank. With a Voltage U of 0 V on our wave generator, we determine the position of the maxima again.
   After calibrating, the sonic frequency was set to (2322,54 $\pm$ 1) kHz. The intensity distribution was measured 11 times for different voltages between 0 V and 11 V.

5 Data analysis

Remark: All figures can be found in a larger scale in the attachment

5.1 Sinus grid

With the sinus grid as a refraction object, the maxima of 0. and 1. orders are visible as dots on the screen. Maxima of higher orders are not visible. For both first order maxima, we measure their position in relation to the main maximum. As they are moved in both the x- and the y-direction, we calculate the distance d to the origin with Pythagoras.

$$d = \sqrt{x^2 + y^2} \tag{17}$$

The error on both x and y is estimated at $s_x = s_y = 1$ mm. The resulting error on d therefor is

$$s_d = \sqrt{\left(\frac{x}{d}s_x\right)^2 + \left(\frac{y}{d}s_y\right)^2} \tag{18}$$

This resulted in the following values

Table 1: Values for the sinus grid

Maximum	x	y	d	s_d
right	5,2	0,6	5,23	0,10
left	-5,2	-0,6	5,23	0,10

As we expected the two points are equally far away, we can calculate the arithmetic middle. This reduces our error by a factor of $1/\sqrt{2}$

$$d = (5,23 \pm 0,07) \text{ cm} \tag{19}$$

With the amplitude d calculated and the distance between the the screen and the grid measured ($l = 6,5 \pm 0,1$ cm), we can calculate the angle of deviation.

$$\Theta = \arctan\left(\frac{d}{l}\right) \tag{20}$$

Using equation 8, we get the grid constant as

$$K = \frac{\lambda}{\sin\left(\arctan\left(\frac{d}{l}\right)\right)} = \lambda \cdot \sqrt{1 + \left(\frac{l}{d}\right)^2} = 1,0089\mu\text{m} \tag{21}$$

Therefor we use that $m = 1$, and the wavelength is given as $\lambda = 632,8$ nm. For calculating the total error, the error on λ is irrelevant, because the relative error is much less than the relative error raised by l/d. The resulting error is

$$s_K = \frac{\lambda l}{d^2\sqrt{1+\left(\frac{l}{d}\right)^2}} \cdot \sqrt{s_l^2 + \left(\frac{l}{d} s_d\right)^2} = 0,0013\mu\text{m} \tag{22}$$

So our grid constant for the sinus grid is

$$\boxed{K = (1,0089 \pm 0,0013)\mu\text{m}} \tag{23}$$

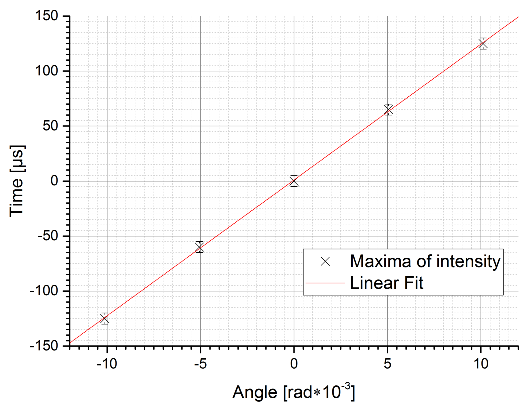
5.2 Amplitude grids

At first, we have to calculate the constant of proportionality with which we can convert our intensity distribution into a distribution over angle instead of time. For each main intensity maximum we know the exact angle because the grid constant was given by $K = 80$ lines/cm. Now we use

$$m\lambda = K \sin\theta \tag{24}$$

and we can find the angles at which we should find the different peaks. We can plot the calculated angle and the measured time in a diagram and use a linear fit to find the constant of probability we were looking for.

Figure 2: First calibration [1].



Because we expect the motor to spin the mirror at a constant angular speed, a linear regression is justified. The used data is shown in tabular 7

Table 2: Data used for linear regression for first calibration

number	angle / rad $\cdot 10^{-3}$	t / μs	s_t / μs
0	0	0	5
1	5,062	65	5
-1	-5,062	-60	5
2	10,125	125	5
-2	-10,125	-125	5

With the inclination of

$$a = (12346 \pm 99)\frac{\mu\text{s}}{\text{rad}} \tag{25}$$

found in the plot we already have our constant of proportionality. Furthermore, with the inclination found, we can determine the angular frequency of the mirror we used. It is the reciprocal of the inclination and therefore given by

$$\boxed{\omega = (81,0 \pm 0,6)Hz} \tag{26}$$

The error was calculated by

$$\omega = \frac{1}{a} \Rightarrow \quad s_\omega = \frac{1}{a^2} \cdot s_a = \omega \cdot \frac{s_a}{a} \tag{27}$$

where ω is the angular frequency and a the calculated inclination. With the angular frequency we can check how many turns the mirror did per second by dividing by 2π.
The original angular frequency is an information about how fast the intensity distribution passes the diode. To reach the same point in the distribution, a whole turn of the mirror is needed. That's why we can directly use the frequency to describe the mirror.
We receive

$$\boxed{f = (12,89 \pm 0,10)\text{Hz}} \tag{28}$$

which equals the turns per second.

After the calibration process we can now start measuring the 5 given grids. Therefore we transform our measured time into an angle.

$$\theta = \omega \cdot t \tag{29}$$

The error on θ is given as

$$s_\theta = \theta \cdot \sqrt{\left(\frac{\text{s}_\omega}{\omega}\right)^2 + \left(\frac{s_t}{t}\right)^2} \tag{30}$$

Afterwards, we draw for each maximum $sin(\theta)$ on the y-axis and $m \cdot \lambda$ on the x-axis. The error on $sin(\theta)$ is given by

$$s_{\text{sin}} = \cos(\theta) \cdot s_\theta \tag{31}$$

We now make a linear regression. According to formula 8, the inclination is exactly the inverse grid constant.

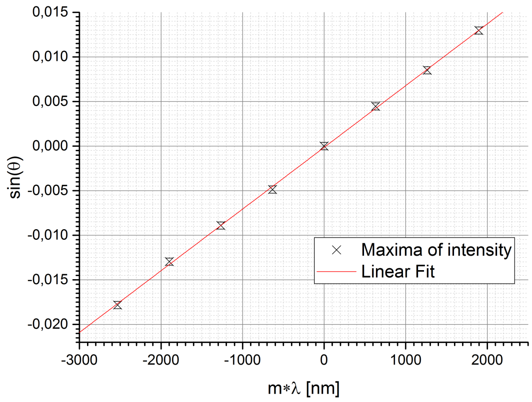
$$K = \frac{1}{a} \tag{32}$$

The data used for the linear regressions can be found in the tables 3 - 7

Table 3: Data used for linear regression of grid 1

number	$\sin\theta$	$s_{\sin\theta}$	m·λ in nm
0	0,0000	0,0004	0
1	0,0045	0,0004	632,8
-1	-0,0049	0,0004	-632,8
2	0,0085	0,0004	1265,6
-2	-0,0089	0,0004	-1265,6
3	0,0130	0,0004	1898,4
-3	-0,0130	0,0004	-1898,4
-4	-0,0178	0,0004	-2531,2

Figure 3: Grid 1 [1].



For the first grid, we find an inclination of $\boxed{a_1 = (6{,}92 \pm 0{,}06)\frac{1}{\mu\text{m}}}$ and therefore get a grid constant of

$$\boxed{K_1 = (144{,}6 \pm 1{,}2)\mu\text{m}} \tag{33}$$

Afterwards we measure the area which was illuminated by the laser to calculate the resolution power of each grid. Therefor we remove the grid and put a screen at its place.

$$\boxed{d = 0,45 \pm 0,05} \tag{34}$$

d is the diameter of our beam.
If we divide the illuminated area by the grid constant calculated in equation 33 we get the number of slits which are actually illuminated.

$$N_1 = \frac{d}{K_1} = 31 \tag{35}$$

The error is calculated by

$$s_{N_1} = N_1 \cdot \sqrt{\left(\frac{s_K}{K}\right)^2 + \left(\frac{s_d}{d}\right)^2} = 3 \tag{36}$$

$$\boxed{N_1 = 31 \pm 3} \tag{37}$$

With these results we can determine the resolution power of the grid using $A = N \cdot m$ (see formula 12), where N is the number of slits illuminated and m is the number of received maxima.
To avoid confusion, we use A for the resolution power instead of a which already is the inclination.

$$\boxed{A_1 = 280 \pm 31} \tag{38}$$

The calculations for the other 4 grids were analogue.

Table 4: Data used for linear regression of grid 2

number	$\sin\theta$	$s_{\sin\theta}$	m·λ in nm
0	0,0000	0,0004	0
1	0,0162	0,0004	632,8
-1	-0,0162	0,0004	-632,8

Here we find an inclination of $\boxed{a_2 = (25,60 \pm 0,06)\frac{1}{\mu\text{m}}}$ and a grid constant

$$\boxed{K_2 = (39,06 \pm 0,09)\mu\text{m}} \tag{39}$$

With the same method as for grid 1 we find the number of illuminated slits $\boxed{N_2 = 115 \pm 13}$ and therefor a resolution power of

$$\boxed{A_2 = 346 \pm 38} \tag{40}$$

Table 5: Data used for linear regression of grid 3

number	$\sin\theta$	$s_{\sin\theta}$	m·λ in nm
0	0,0000	0,0004	0
1	0,0053	0,0004	632,8
2	0,0109	0,0004	1265,6
-1	-0,0057	0,0004	-632,8
-2	-0,0109	0,0004	-1265,6

The inclination for grid 3 is $\boxed{a_3 = (8,64 \pm 0,06)\frac{1}{\mu\text{m}}}$ and the grid constant

$$\boxed{K_3 = (115,74 \pm 0,86)\mu\text{m}} \tag{41}$$

Therefor we have $\boxed{N_3 = 39 \pm 4}$ and

$$\boxed{A_3 = 194 \pm 22} \tag{42}$$

Table 6: Data used for linear regression of grid 4

number	$\sin\theta$	$s_{\sin\theta}$	m·λ in nm
0	0,0000	0,0003	0
1	0,0055	0,0003	632,8
-1	-0,0053	0,0003	-632,8

The inclination is $\boxed{(a_4 = 8,58 \pm 0,07)\frac{1}{\mu\text{m}}}$ and the grid constant

$$\boxed{K_4 = (116,6 \pm 1,0)\mu\text{m}} \tag{43}$$

We find $\boxed{N_4 = 39 \pm 4}$ and

$$\boxed{A_4 = 116 \pm 13} \tag{44}$$

The inclination is $\boxed{a_5 = (16,96 \pm 0,18)\frac{1}{\mu\text{m}}}$ and the grid constant

$$\boxed{K_5 = (59,0 \pm 0,6)\mu\text{m}} \tag{45}$$

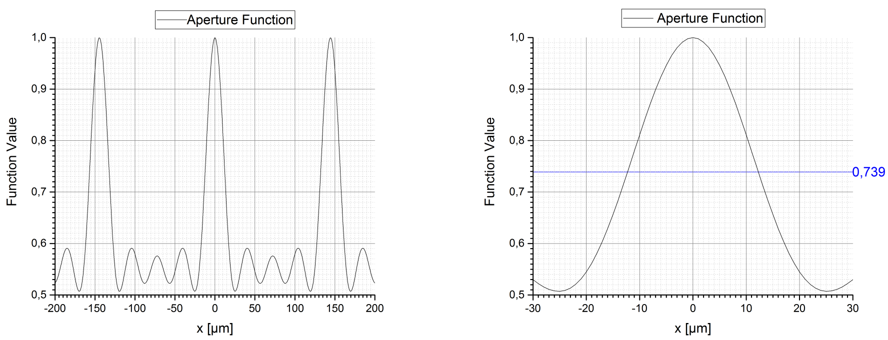
We find $\boxed{N_5 = 76 \pm 9}$ and

$$\boxed{A_5 = 229 \pm 26} \tag{46}$$

number	$\sin\theta$	$s_{\sin\theta}$	m$\cdot\lambda$ in nm
0	0	0,0004	0
1	0,0105	0,0004	632,8
-1	-0,0109	0,0004	-632,8

Table 7: Data used for linear regression of grid 5

5.3 Aperture Function of Grid 1



(a) Aperture Function (b) Aperture Function Detail

Figure 4: Aperture Function Grid 1

We want to calculate the aperture function of grid 1 (G1) by using a Fourier series (see 9). That's why we measure the heights of our peaks. Since we are able to detect maxima up to the 4th order, we had to develop our infinite sum up to n = 4.

$$
\begin{aligned}
g(x) = \sqrt{I_0} + \sqrt{I_1} \cdot \cos\left(\frac{x}{K}2\pi\right) + \sqrt{I_2} \cdot \cos\left(\frac{x}{K}4\pi\right) \\
+ \sqrt{I_3} \cdot \cos\left(\frac{x}{K}6\pi\right) + \sqrt{I_4} \cdot \cos\left(\frac{x}{K}8\pi\right)
\end{aligned}
\tag{47}
$$

The detected intensities of peaks are shown in tabular 8. For orders with both a peak on the positive and the negative side of the main maximum, we calculate the arithmetic middle and divided the error by $\sqrt{2}$. For each peak except the main maximum at 0. order the error is 0,2 DIV at a scale of 50 mV/DIV. The error for the 0. Order is 0,05 DIV at a scale of 2 V/DIV.

To normalise our function, we divide all values of the resulting function by the highest value of the function which can be found at $x = 0$ since all

Order	I / mV	s_I / mV
0	7200	100
1	295	7
2	207,5	7
3	142,5	7
4	80	10

Table 8: Values used to calculate the aperture function

cos terms become one. The resulting value to normalise our function is

$$g(x) = \frac{\sqrt{I_0} + \sqrt{I_1} \cdot \cos\left(\frac{x}{K} 2\pi\right) + \sqrt{I_2} \cdot \cos\left(\frac{x}{K} 4\pi\right) + \sqrt{I_3} \cdot \cos\left(\frac{x}{K} 6\pi\right) + \sqrt{I_4} \cdot \cos\left(\frac{x}{K} 8\pi\right)}{\sqrt{I_0} + \sqrt{I_1} + \sqrt{I_2} + \sqrt{I_3} + \sqrt{I_4}} \tag{48}$$

Using the intensities of table 8 and the calculated grid constant of $K_1 = (144,6 \pm 1,2)\mu$m (see 33), we get the following aperture function

$$g(x) = 0,6179 + 0,1251 \cos(0,0435 \cdot x) + 0,1049 \cos(0,0869 \cdot x) \tag{49}$$

$$+0,0869 \cos(0,1304 \cdot x) + 0,0651 \cos(0,1739 \cdot x) \tag{50}$$

Two periods of this function are shown in figure 11a.

5.4 Slit width and proportion to grid constant

We now want to know the width b of our slits in the grid. In the aperture function this is exactly the width at half height of the main peak. The minimum of the aperture function is a lot higher than 0. That's why we calculated the height h in which we measure the width as

$$h = \frac{\text{MAX - MIN}}{2} + \text{MIN} = 0,739 \tag{51}$$

This height is marked in figure 11b with a blue line. On the cross-section with the aperture function we can read the width in positive x-direction as $b/2 = 12,2 \pm 2\mu$m. As we know that our function is symmetric, the width is

$$\boxed{b = (24,2 \pm 4)\mu\text{m}} \tag{52}$$

The proportion to our grid constant can be calculated by

$$V = \frac{b}{K_1} = 0,169 \tag{53}$$

The error follows from the relative errors on b and K_1 as

$$s_V = V \cdot \sqrt{\left(\frac{s_b}{b}\right)^2 + \left(\frac{s_K}{K}\right)^2} = 0,003 \tag{54}$$

$$\Rightarrow \boxed{V = 0,169 \pm 0,003} \tag{55}$$

5.5 Intensity Distribution with Ultrasonic Phase Grid

To find the dependence between the intensity distribution and the used voltage in our experiment, we detect the intensity distribution at eleven different voltages. The distributions of intensity are shown in the figures below.

To get our data of the distributions, we used the built-in measuring program to take a set of 2000 data points from the oscilloscope. The picture on the oscilloscope flickered because it is not triggered perfectly. So in the original set of data, for each voltage, the data was shifted by a different amount to the right. Furthermore the point, where $t = 0$s is on the very left side of our distribution. To fix this issue, we took the time component for each voltage of the main maximum of 0. order and shifted the time-axis for this amount in negative direction. As a consequence, the 0. main maximum is always exactly at $t = 0$ or $\Theta = 0$.

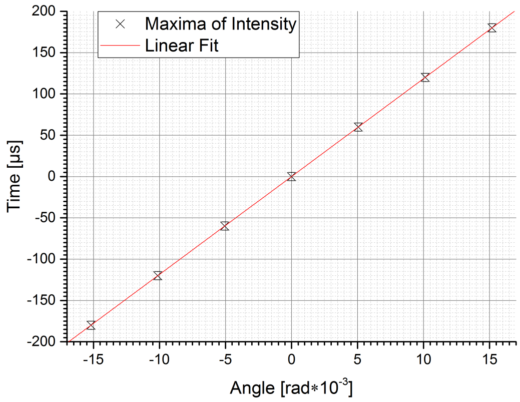
The original signal displayed on the y-axis is a voltage signal. To minimise reading errors, we make the picture on the oscilloscope as big as possible. This results in negative voltages for low intensities. We are only interested in the relative intensities. That's why we shift all Voltages up until the smallest value becomes zero. For normalisation, we divide all values of all distributions by the maximum value of the distribution without soundwave ($U = 0$V).

At last, we want to make our distributions similar to what we did in part 5.2 a function of angle instead of time. A new calibration is necessary because we modified the objects in the beam and had to refocus it with the lenses. The data used for linear regression is shown in table 9. The linear regression is shown in figure 12

Table 9: Data used for linear regression of second calibration

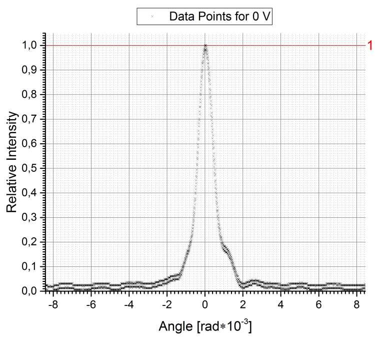
number	angle / rad $\cdot 10^{-3}$	t / μs	s_t / μs
0	0	0	5
1	5,0624	60	5
-1	-5,0624	-60	5
2	10,1250	120	5
-2	-10,1250	-120	5
3	15,1878	180	5
-3	-15,1878	-180	5

Figure 5: Second calibration [1].

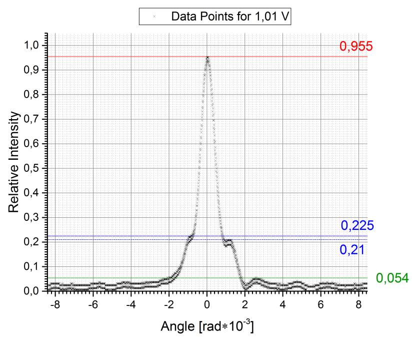


The inclination of the fit is $a = (11851,73 \pm 0,06)\frac{\mu s}{\text{rad}}$
This led to

$$\omega = (84,3759 \pm 0,0004)\text{Hz} \tag{56}$$
$$f = (13,42883 \pm 0,00007)\text{Hz} \tag{57}$$



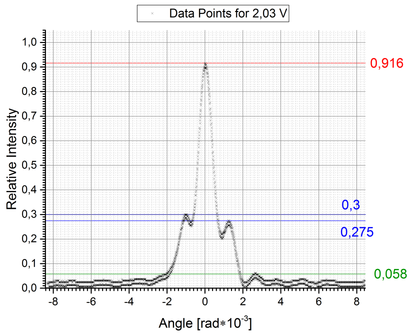
(a) U=0V



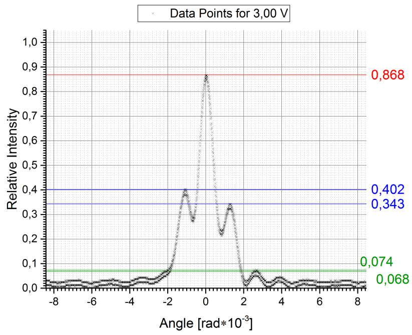
(b) U=1.01V

To verify RAMAN-NATH theory, we read the relative height of the peaks for all different voltages. They are shown in tabular 10. In the figures they are marked with horizontal lines. 0. order maxima are marked red, 1. order maxima blue and 2. order maxima green.
Due to the symmetry of the experiment we expect the peaks of same orders to be equally high in both the positive and the negative direction. That's why we calculate the arithmetic middle.

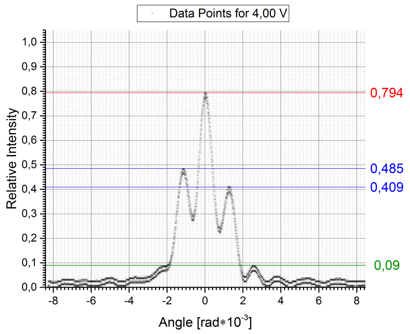
With the arithmetic means calculated, we fit a Bessel-function for each order of the maxima. According to the RAMAN-NATH theory, we use the



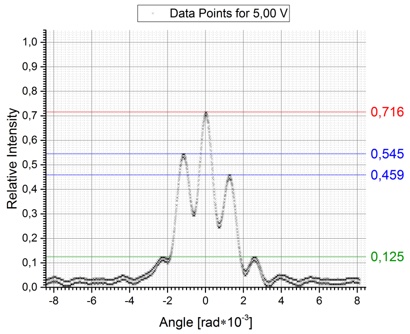
(c) U=2.03V



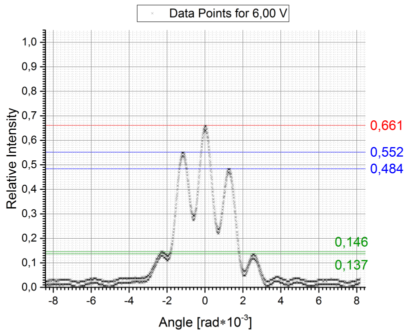
(d) U=3.00V



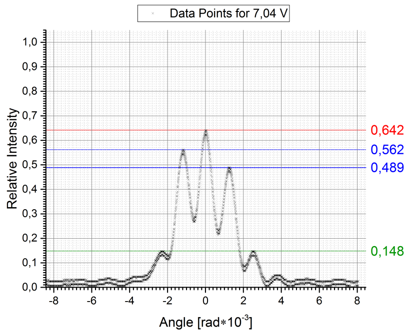
(e) U=4.00V



(f) U=5.00V

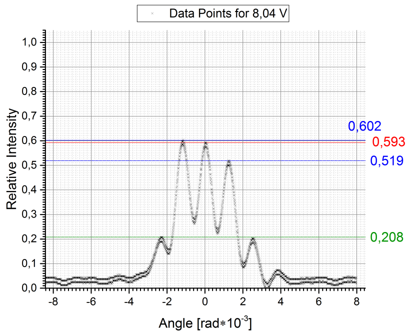


(g) U=6.00V

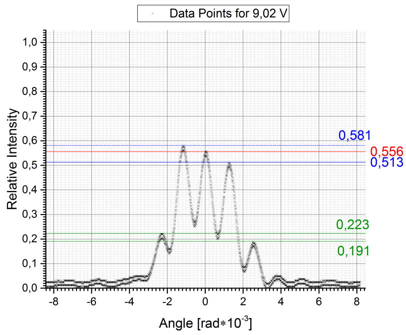


(h) U=7.04V

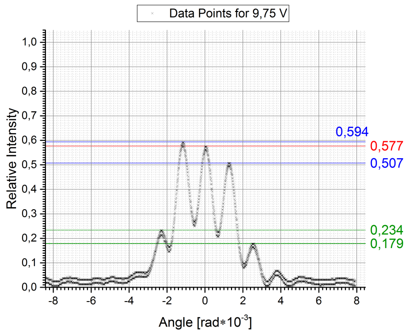
Bessel-functions of the first kind. The order of the used Bessel-function was always the same as the order of the maximum we are looking at.
Since our first fitting results don't match with our measured data, we expect a systematic error. To compensate this error we use the following fit function



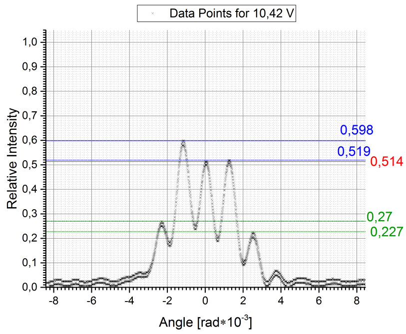
(i) U=8.04V



(j) U=9.02V



(k) U=9.75V



(l) U=10.42V

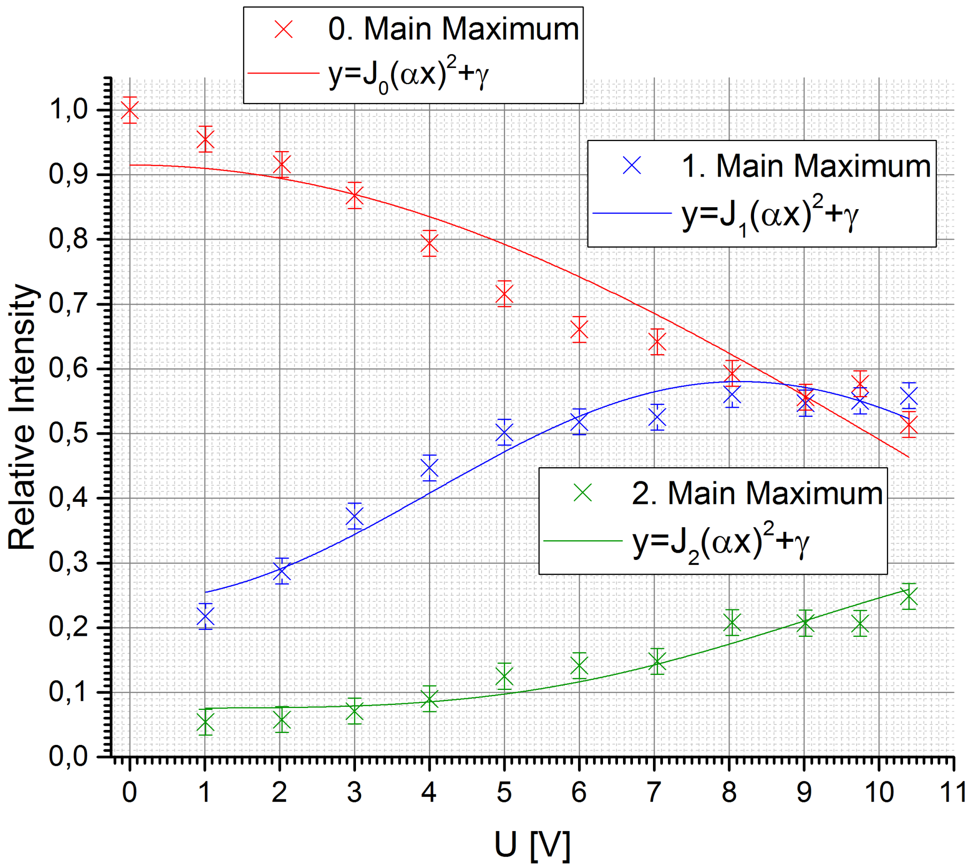
	order of maximum						
U / V	0	1. left	1. right	middle 1	2. left	2. right	middle 2
0,00	1	-	-	-	-	-	-
1,01	0,955	0,225	0,21	0,2175	0,054	0,054	0,054
2,03	0,916	0,3	0,275	0,2875	0,058	0,058	0,058
3,00	0,868	0,402	0,343	0,3725	0,068	0,074	0,071
4,00	0,794	0,485	0,409	0,447	0,09	0,09	0,09
5,00	0,716	0,545	0,459	0,502	0,125	0,125	0,125
6,00	0,661	0,552	0,484	0,518	0,146	0,137	0,1415
7,04	0,642	0,562	0,489	0,5255	0,148	0,148	0,148
8,04	0,593	0,602	0,519	0,5605	0,208	0,208	0,208
9,02	0,556	0,581	0,513	0,547	0,223	0,191	0,207
9,75	0,577	0,594	0,507	0,5505	0,234	0,179	0,2065
10,40	0,514	0,598	0,519	0,5585	0,27	0,227	0,2485

Table 10: Relative intensities of all maxima at different voltages

$$I = (J_n(\alpha \cdot x))^2 + \gamma \tag{58}$$

n is the order of the Bessel-function and γ the systematic offset in y-direction.
A systematic offset in x-direction doesn't make sense because the systematic error of the power supply was given as 2% +2 digits and was 2 magnitudes lower than our values. The fits are shown in figure 25. For the y-error we use the reading error of 0,2. This error is justified because the original signal vibrated. The values for the parameters α, β and γ can be found in table 11

Figure 6: Besselfits



parameter	J_0	J_1	J_2
α	0,101 ± 0,007	0,20 ± 0,02	0,0229 ± 0,011
γ	-0,08 ± 0,02	0,242 ± 0,009	0,076 ± 0,010

Table 11: calculated parameters for besselfits

5.6 Sonic wavelength

To calculate the sonic wavelength, we read the angle for each main maximum at each voltage and calculate the arithmetic mean.
Knowing the wavelength of our laser, we can make a linear regression using formula 15.
On the y-axis is $\sin(\theta)$, on the x-axis $m \cdot \lambda$. The inclination is the inverse wavelength of our sonic wave. The error on $\sin(\theta)$ follows due to the error propagation as

$$s_{\sin} = \cos(\theta) \cdot s_\theta \tag{59}$$

whereas s_θ is the error on the mean of all angles of this peak.The reading error on one angle is valued as $0,5 \cdot 10^{-3}$ rad

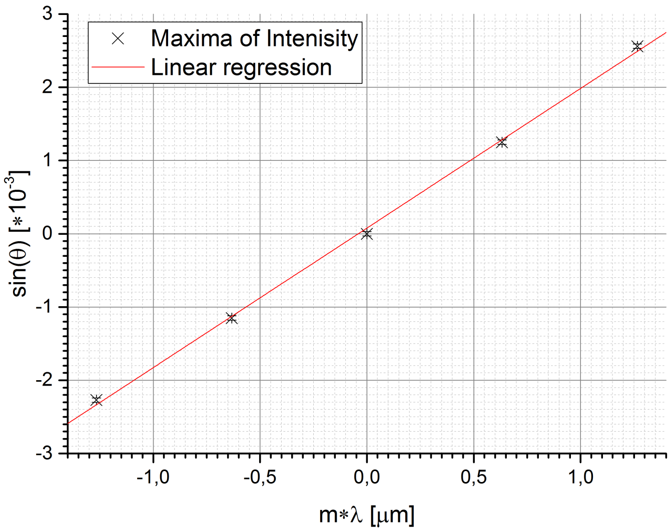
$$s_\theta = \frac{1}{\sqrt{n}} \cdot 0,5 \cdot 10^{-3} \text{ rad} \tag{60}$$

with n being the number of angles used for the main. The calculated values can be found in table 12

Maximum	1 left / rad$\cdot 10^{-3}$	1 right / rad$\cdot 10^{-3}$	2 left / rad$\cdot 10^{-3}$	2 right / rad$\cdot 10^{-3}$
	0,9	1,2		2,6
	1	1,2		2,6
	1,1	1,3	2,1	2,7
	1,2	1,3	2,2	2,6
	1,2	1,3	2,3	2,6
	1,2	1,2	2,3	2,5
	1,2	1,3	2,3	2,5
	1,2	1,2	2,3	2,5
	1,2	1,2	2,3	2,6
	1,2	1,3	2,3	2,5
	1,2	1,2	2,3	2,5
	arithmetic middle			
	1,15	1,25	2,27	2,56
Error	0,03	0,03	0,03	0,03

Table 12: Data used for linear regression to calculate the sonic wavelength

Figure 7: Linear regression used to calculate the sonic wavelength



With the calculated inclination a we are able to define the wavelength Λ

$$a = (1906 \pm 37)\frac{1}{\mu\mathrm{m}} \tag{61}$$

$$\boxed{\Lambda = (525 \pm 10)\mu\mathrm{m}} \tag{62}$$

6 Discussion of the final results

6.1 Amplitude grids

For the given **sinus grid** we receive a grid constant of

$$\boxed{K = (1{,}0089 \pm 0{,}0013)\mu\text{m}} \tag{63}$$

This seems to be an acceptable value for our grid since the detected angle of deflection is relatively high.

The first calibration measurement gives us the following angular frequency

$$\boxed{\omega = (81{,}0 \pm 0{,}6)Hz} \tag{64}$$

The frequency of the mirror was

$$\boxed{f = (12{,}89 \pm 0{,}10)\text{Hz}} \tag{65}$$

This fits with our given information about the mirror (12.5 times per second [1]). Since the information in the instruction is not given with an insecurity, a direct comparison in respect of the errors is not useful.

By taking a closer look at **five different amplitude grids**, we determine the following grid constants K and resolution powers a:

grid	K / μm	s_K / μm	a	s_a
K_1	144,6	1,2	280	31
K_2	39,06	0,09	346	38
K_3	115,7	0,9	194	22
K_4	116,60	1,00	116	13
K_5	59,0	0,6	229	26

Table 13: Summary of calculated grid constants and resolution powers

All calculated grid constants are in the expected magnitude.

For grid 1 we are able to calculate the slit width b from the **aperture function**

$$\boxed{b = (24{,}2 \pm 4)\mu\text{m}} \tag{66}$$

The proportion to the grid constant K_1 followed as

$$\boxed{V = 0{,}169 \pm 0{,}003} \tag{67}$$

6.2 Second calibration

At the second calibration we receive the following angular frequency

$$\omega = (84,3759 \pm 0,0004)\text{Hz} \tag{68}$$
$$f = (13,42883 \pm 0,00007)\text{Hz} \tag{69}$$

Even though we use the same grid as in the first calibration, we measure a significantly higher frequency. Furthermore we are able to see one higher order of maxima.
Either the two maxima of 3. order were not seen in the first calibration due to scaling issues, or a different grid was used by accident for the second calibration. However, a closer look at the original data shows that the measured times are for both calibration measurements in an deviation of 1 σ. For all other grids we measured, the deviation to our 2. calibration grid was higher.
This leaves us with the conclusion, that the right grid was used.

Another fact worth mentioning is the preposterous small errors on the second angular frequency. Probably this comes from an issue with origin, where the given error for our measured data was only used to weight the different data points. With the data being randomly on a straight line, the error became too small.

6.3 Ultrasonic phase grid

For the ultrasonic phase grid, we measure the expected intensity distributions. However, the intensity distribution has a skewness which was probably caused by a not perfectly centred beam. Peaks on the left side are usually higher than their counterparts on the right side. Another thing we have to mentione is that peaks on the right side appear much earlier than on the left side. Even without a voltage, there is a slight peak on the right side of the 0. main maximum. Adjusting the lenses changes the height of this peak which leads to the conclusion that the beam was not perfectly centred. Effects like spherical aberration can then lead to diffraction effects even without a phase grid.
Another thing we realise is that there were usually two lines of data points above each other in die intensity distributions.
We don't think that this can be only blamed on the noise of the signal, because there are no data points between the two lines. Noise is creating the up and down of each line.
What the measuring program actually did, is saving the voltages detected in a period of time into a file. Probably in this period of time, the intensity distribution got measured twice. Due to the trigger signal the time got reset

at the trigger.

Looking at the development of the peaks as a function of voltage, we see RAMAN-NATHs theorie confirmed.
For a peak of n-th order, a first kind Bessel-function of n-th order describes the behaviour of our data points very well.
However, the systematic offset in y-direction was of various heights for the different Bessel-functions. This could be because of ambient light, that got also detected by the diode when measuring the intensity. Moreover, the peak of the 0. main maximum is wide enough to add some intensity to the angle where the 1. main maximum is detected. This can be seen in the figure for 0 V.
The fit parameter α and the systematic errors are shown in the following tabular

	0. main max.	1. main max.	2. main max.
α	0,101 ± 0,007	0,20 ± 0,02	0,0229 ± 0,011
systematic y-error	-0,08	0,242	0,076

Table 14: calculated parameters for besselfits

With a frequency of (2322,54 ± 1) kHz, we measure a wavelength of

$$\boxed{\Lambda = (525 \pm 10)\mu\text{m}} \tag{70}$$

This value also seems quite reasonable. To get some numerical intuition, weather the value is realistic, we can calculate the speed of sound using

$$c = \Lambda \cdot f \approx 1219\text{m/s}. \tag{71}$$

If we compare this value to the speed of sound in other fluid media we see, that it should be in the right scale.
For example the speed of sound in water is $c_w = 1407$ m/s and in ethyl alcohol $c_e = 1168$ m/s [2]

References

[1] [Instruction paper]
Instruction paper published at http://hacol13.physik.uni-freiburg.de/fp/Versuche/FP1/FP1-2-Ultraschall/ 24.08.2017 14:51

[2] [Wikipedia]
https://de.wikipedia.org/wiki/Schallgeschwindigkeit
02.09.2017 10:42

7 Attachement

Figure 8: First calibration [1].

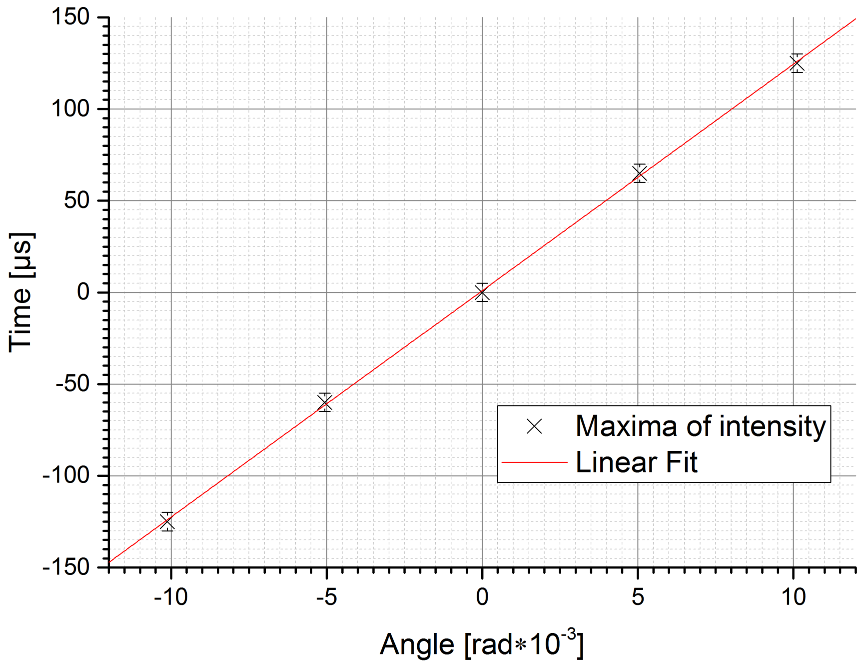


Figure 9: Grid 1 [1].

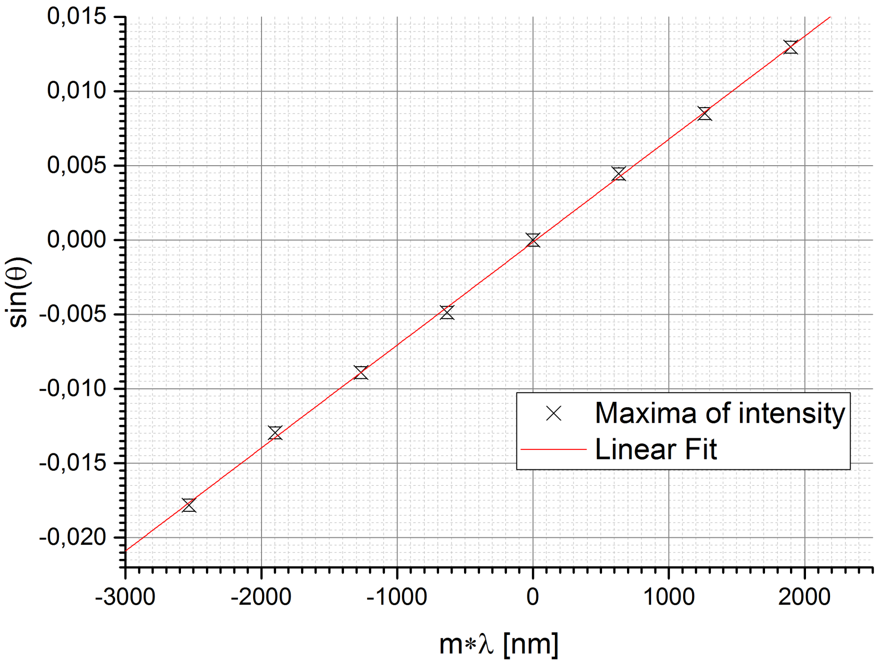


Figure 10: Aperture Function

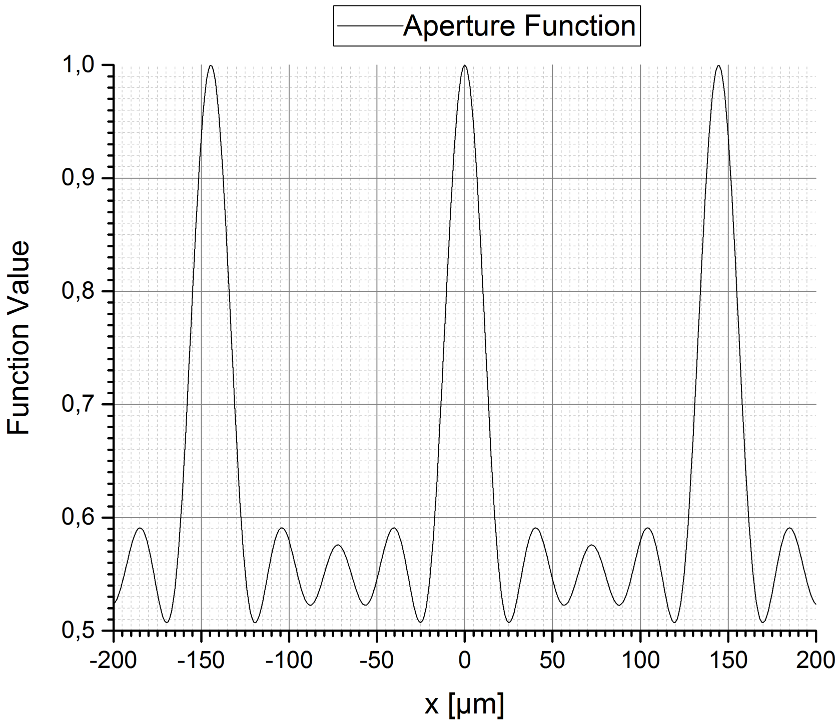


Figure 11: Aperture Function Detail

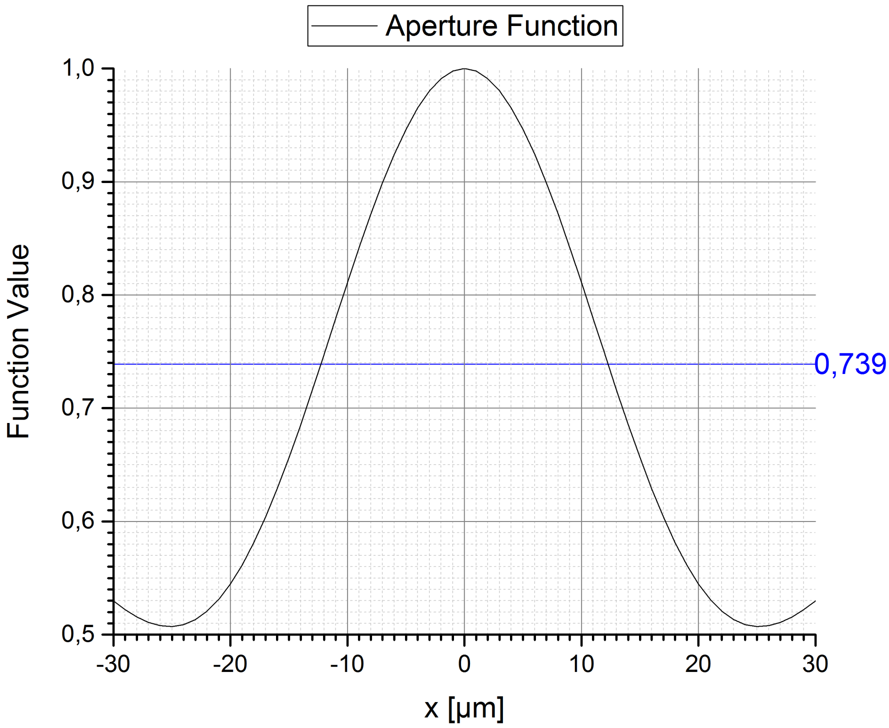


Figure 12: Second calibration [1]

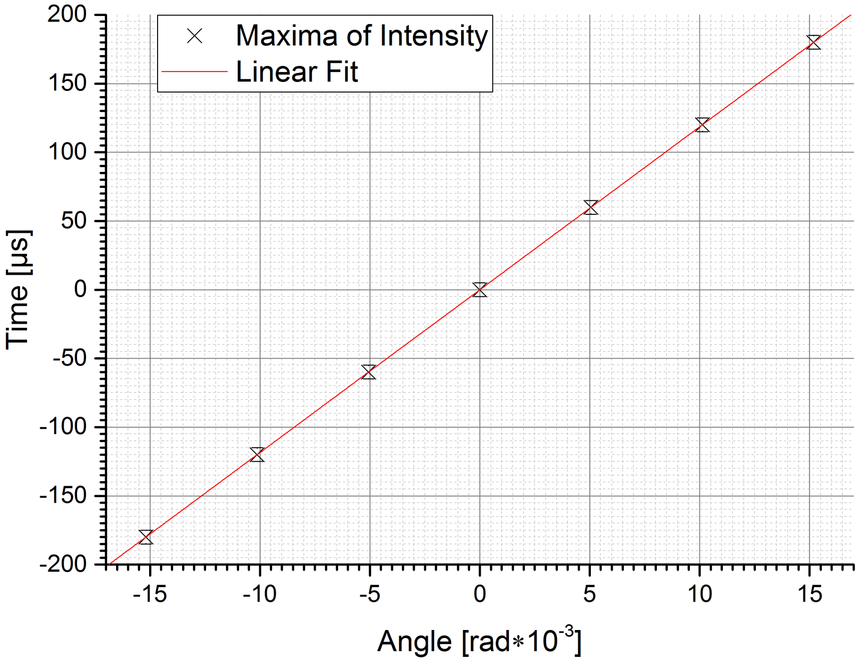


Figure 13: U=0V

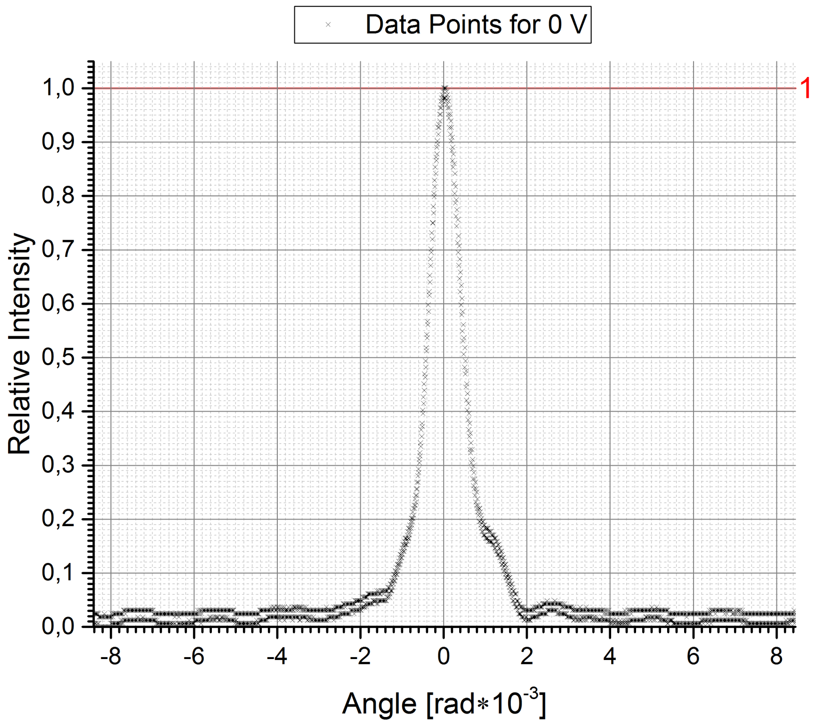


Figure 14: U=1.01V

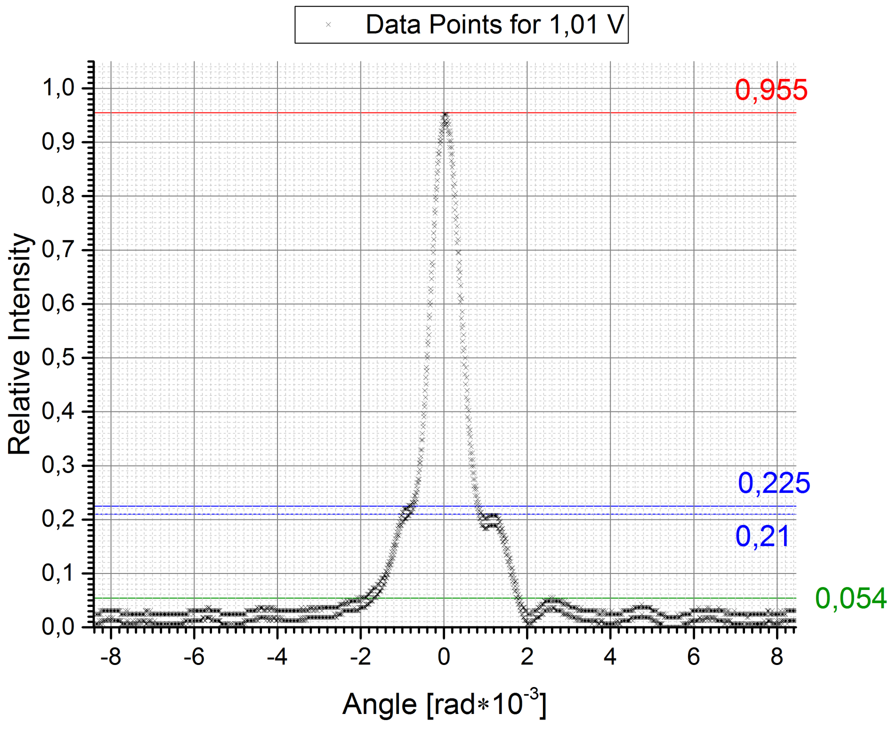


Figure 15: U=2.03V

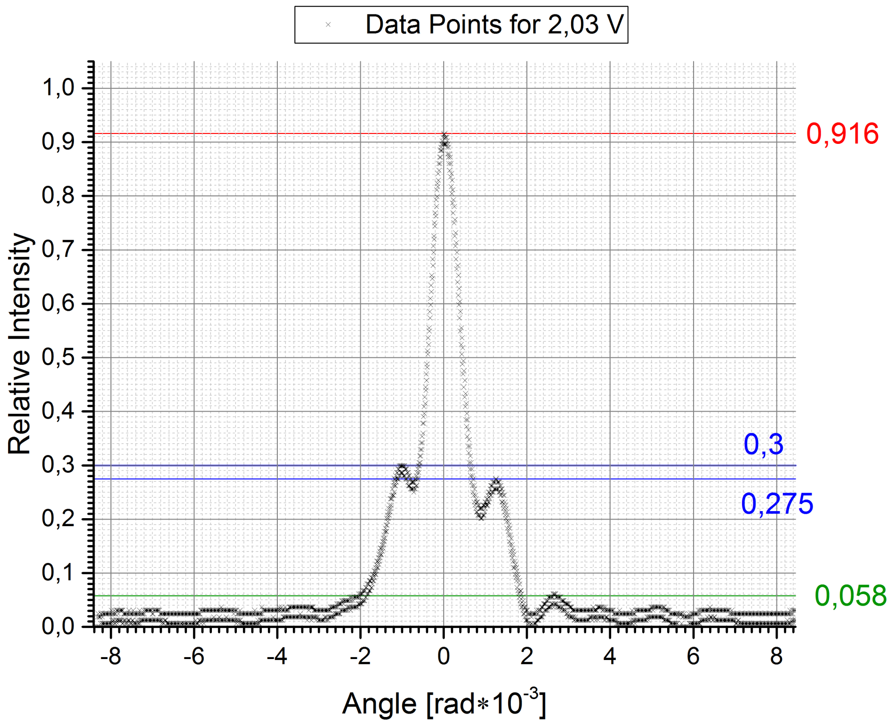


Figure 16: U=3.00V

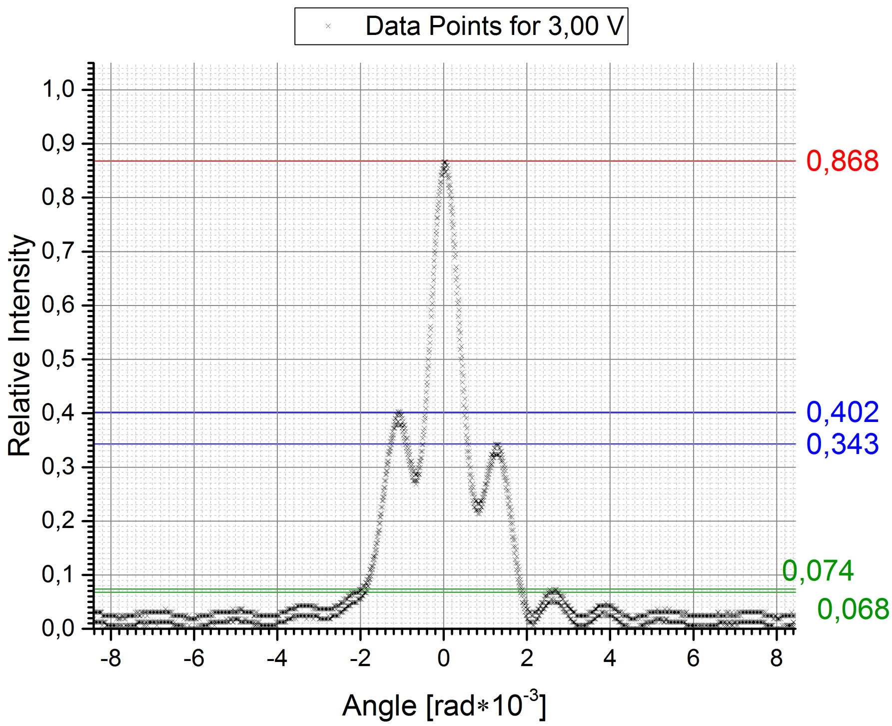


Figure 17: U=4.00V

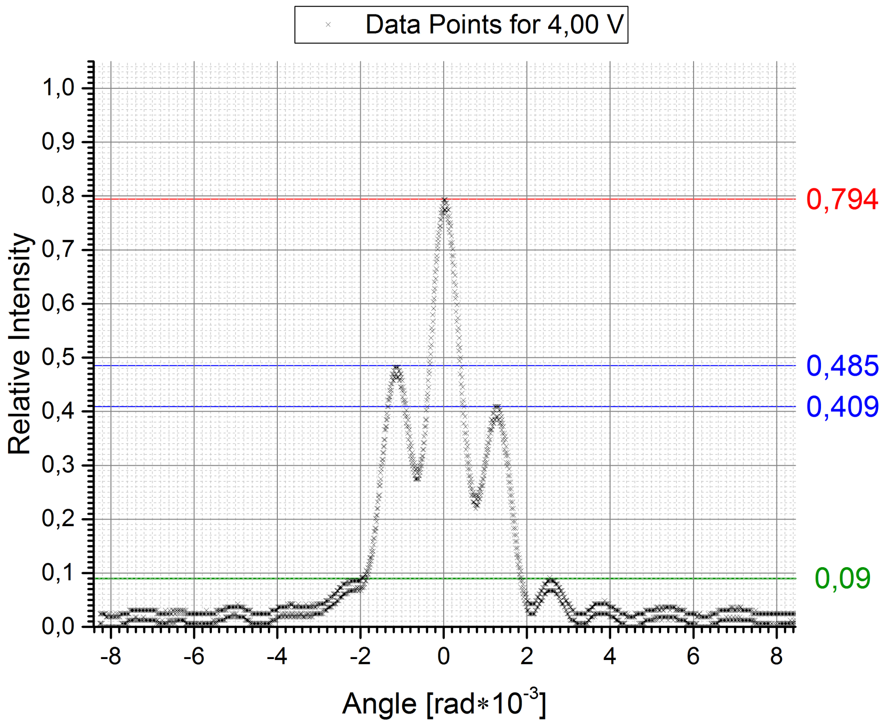


Figure 18: U=5.00V

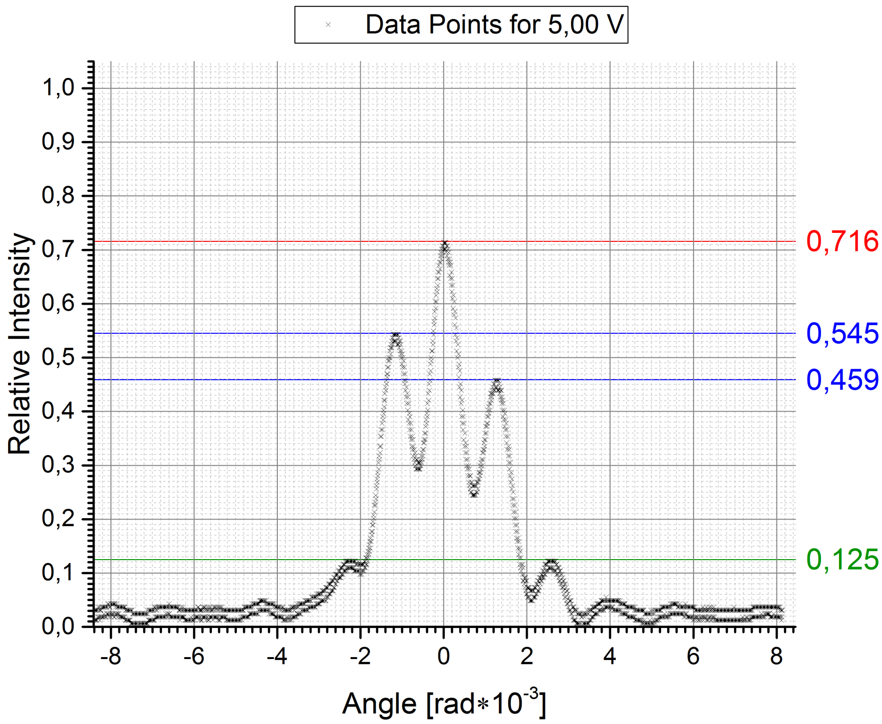


Figure 19: U=6.00V

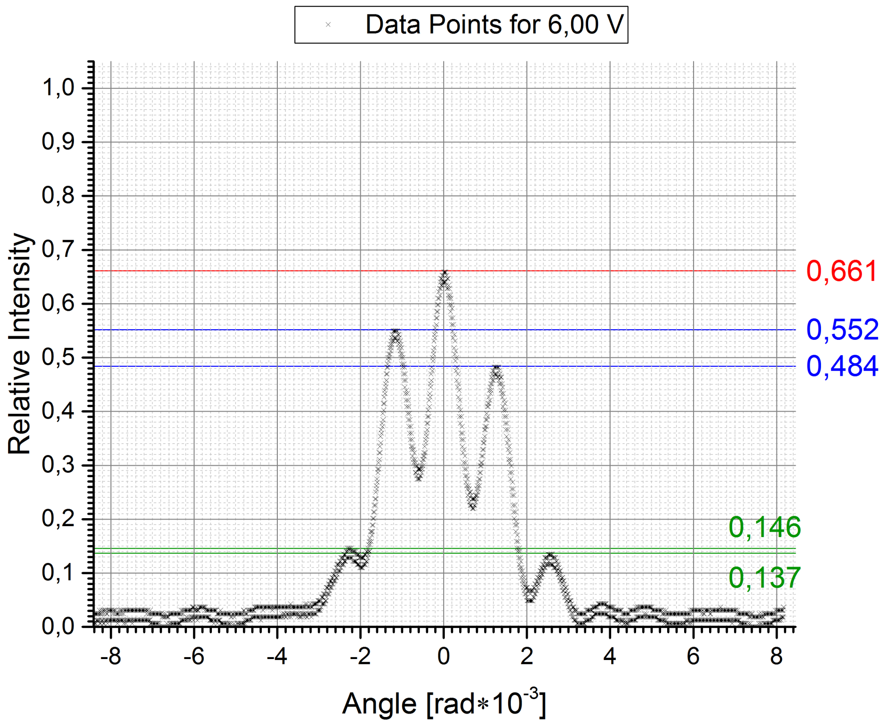


Figure 20: U=7.04V

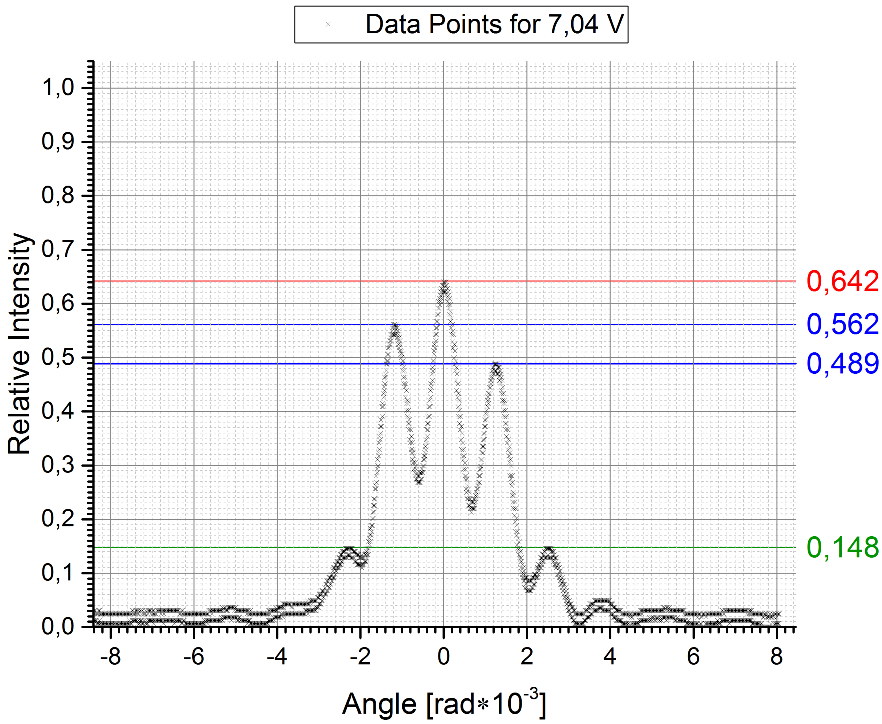


Figure 21: U=8.04V

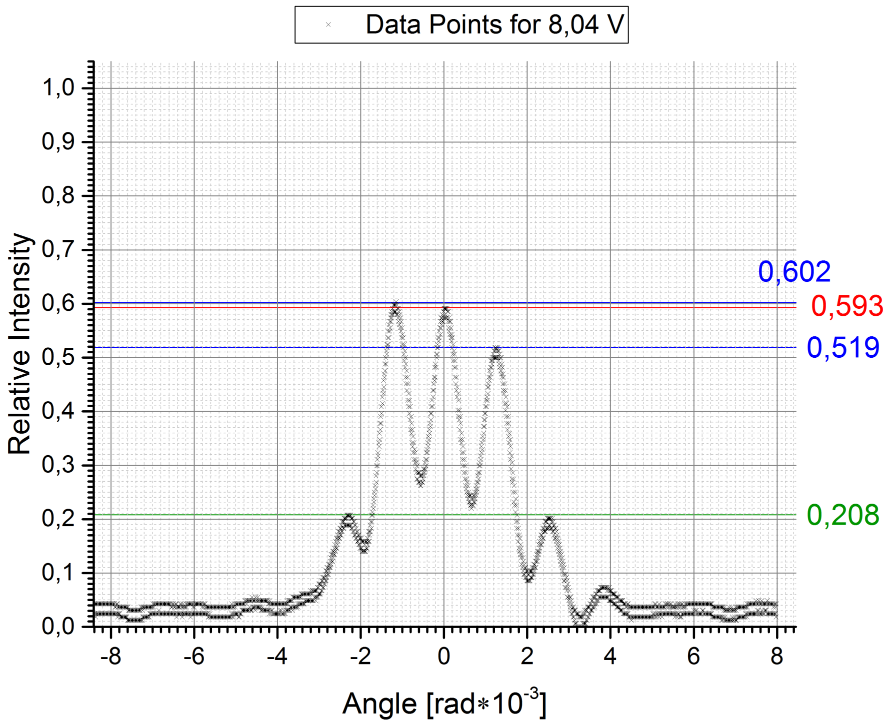


Figure 22: U=9.02V

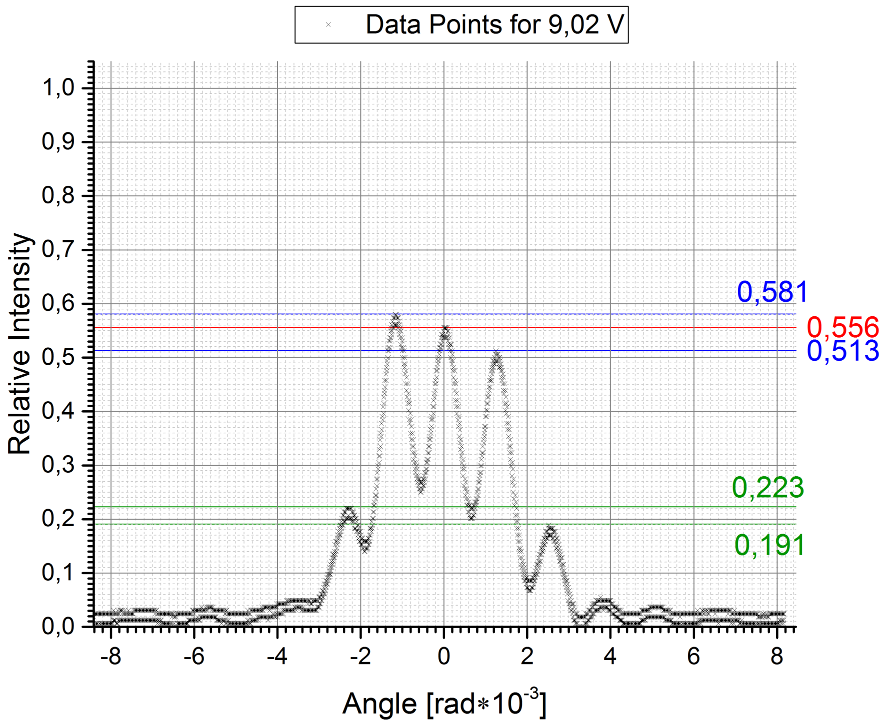


Figure 23: U=9.75V

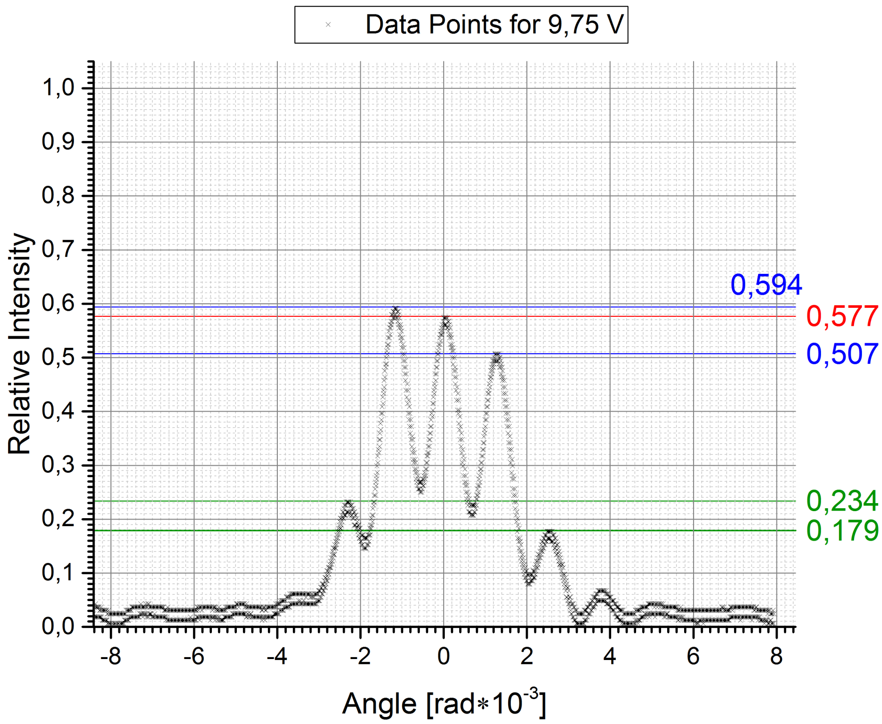


Figure 24: U=10.42V

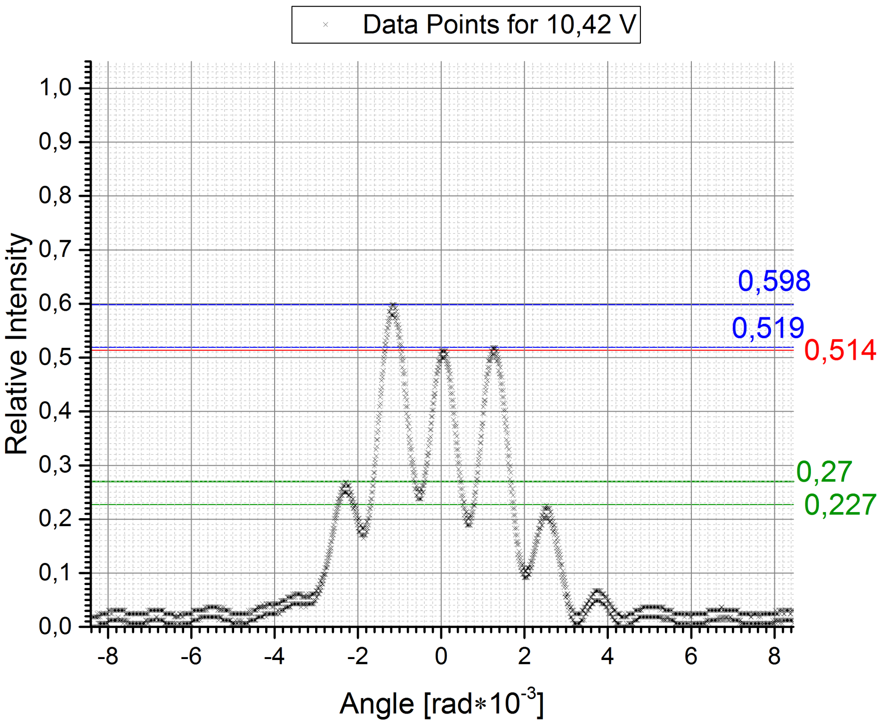


Figure 25: Besselfits

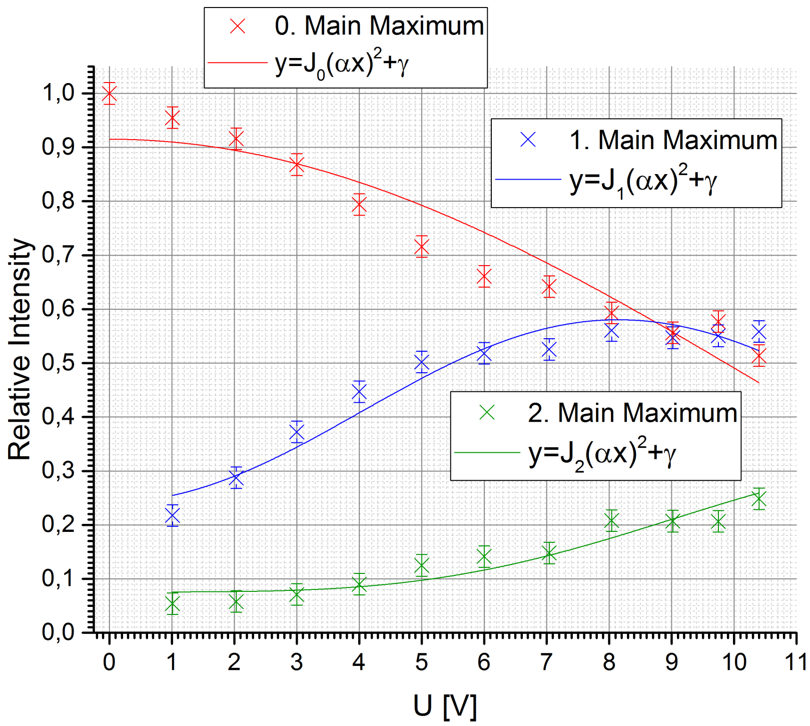


Figure 26: Linear regression used to calculate the sonic wavelength