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Ultrasound

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Assistant:

Abstract

In the experiment “ultrasound phase grating“, essential effects of diffraction are examined by analysing the different behaviour of amplitude and phase gratings. In a first step, different amplitude gratings are evaluated in respect of their grating constant, resolution and aperture function. In a second step a phase grating, produced by an ultrasound wave generator in a tank of isooctane, can be analysed and its interference pattern can be compared to the Raman-Nath-Theory.

Whereas the amplitude gratings were all successfully analysed with the results being on a realistic scale, the analysed phase grating showed roughly the correct trend, but couldn’t match the Raman-Nath-Theory perfectly. In contrast, the calculated ultrasound wave length provided a very realistic value.

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1 Introduction

In this experiment the fundamental effects of diffraction are studied by evaluating the behaviour of different gratings on an optical bench. As the first part of the experiment focuses on amplitude gratings, that vary the transmission of the light beams going through the grating, the second part mainly studies a phase grating that shifts the phase of the beam and is realised by an ultrasound wave.

In total, the experiment is divided into three main parts. The first part focuses on the determination of the grating constants and resolution of different amplitude gratings. The second part deepens the first part by finding the aperture function of one of the gratings. Finally the phase grating is studied and the results are compared to the Raman-Nath-Theory.

2 Theory

The following sections introducing the theory and methodology necessary for the experiment are mainly based on the experiment description, which is provided by the advanced physics lab team [1] and the diploma thesis by Lutz Lefèvre [2].

2.1 Table of the used variables

Tab. 1: Table of the used symbols for the parameters used in the protocol.

Symbol	Parameter
$g(x, y)$	aperture function
I	Intensity
K	grating constant
m	order of the maximum
$m_{\max}$	highest order
N	number of lightened slits
θ_m	angle to maximum m on the screen
A	resolution
λ	wavelength (light)
Λ	wavelength (ultrasound)
$\Delta\lambda$	difference in wavelength (light)
U	voltage on the ultrasound generator
ρ	density
n	refractive index
α	unknown factor in the Bessel function
a, x	distances on the optical bench

2.2 Basic theory about diffraction

The experiment investigates the fundamental effects of diffraction. In a diffraction experiment a light source that transmits parallel beams (for example by using a collimating lens) sends light through a slit or a grating, generating an interference pattern on a screen.

Generally there a two fundamentally different sorts of gratings – the amplitude and the phase grating [1]. Gratings are typically described by their aperture function $g(x, y)$, which depends on the geometry of the grating in the xy-plane. An amplitude grating varies the transmission T of the light, which means that the aperture function $g(x, y) = T$ is completely real, whereas the phase grating has a complex aperture function and therefore generates phase shifted beams.

The connection between intensity and aperture function is described by the absolute square of the Fourier transformation [1]:

$$I = |y|^2(\vec{x}) = \left| \int_{\text{aperture}} g(\vec{k}) e^{i\vec{k}\cdot\vec{x}} \mathrm{d}\vec{k} \right|^2. \tag{1}$$

2.3 Amplitude gratings: grating constant and resolution

For the first parts of the experiment the gratings that have to be examined are all amplitude gratings. A characteristic quantity of the gratings is the grating constant K that describes the distance between two neighbouring slits. For an amplitude grating the following formula gives the correspondence between the interference maximum m, the wave length of the light λ and the deflection angle θ_m [1]:

$$m\lambda = K \sin(\theta_m). \tag{2}$$

Alternatively the grating constant can be displayed as the number of slits per length $1/K$. In the following protocol this definition is the preferred one.

An estimation of the quality of a given grating can be calculated by the resolution A which is defined as the quotient of wavelength λ and the difference in wavelength $\Delta\lambda$ that still generates two separable interference patterns on the screen. In the case of an amplitude grating one can also use an alternative formula derived from the Rayleigh criteria with N being the number of lightened slits and $m_{\max}$ being the order of the outermost maximum detected [1]:

$$A = \frac{\lambda}{\Delta\lambda} = N m_{\max}. \tag{3}$$

To get the aperture function of an amplitude grating an approximation can be useful. Because intensity and aperture function are connected by Fourier transformation and the image on the screen is mainly made up of discrete maxima, a good approximation can be achieved by using the Fourier series with the square roots of the intensities being the coefficients of the Fourier series [1]:

$$g(x) = \sum_{j=0}^{\infty} \pm\sqrt{I_j} \cos\left(\frac{x}{K} 2\pi j\right), \tag{4}$$

$$\Rightarrow g(x) = \sqrt{I_0} + \sqrt{I_1} \cdot \cos\left(\frac{x}{K} 2\pi\right). \tag{5}$$

Equation 5 shows the specific aperture function of a sine grating that only has the first order of the Fourier series and therefore only leads to first order maxima on the screen. This specific grating will also later be discussed in the experiment.

2.4 Phase gratings: generation and Raman-Nath-Theory

In the other parts of the experiment a phase grating is studied. This grating is realised by applying a voltage U to a piezo crystal, which produces an ultrasound wave in a tank of isooctane. The wave produces fluctuations in the density $\Delta\rho$ of the isooctane and thereby fluctuations in the refractive index Δn. These fluctuation cause different optical path lengths and therefore different phases in the exiting beams [1]:

$$n(x) = n_0 + \Delta n \sin\left(\frac{2\pi x}{\Lambda}\right) \quad \text{with} \quad \Delta n \propto \Delta\rho \propto U. \tag{6}$$

The description of the intensity distribution after a phase grating is described by the Raman-Nath-Theory. In analogy to Equation 2 it provides an expression to find the angle for the maxima θ_m in dependence of their order m, the wavelength of the light λ and the ultrasound wavelength Λ [1]. In addition the Raman-Nath-Theory provides a formula to find the intensity I_m of the maximum of order m [1]:

$$\sin(\theta_m) = \pm m \frac{\lambda}{\Lambda}, \tag{7}$$

$$I_m = J_m^2(\alpha U). \tag{8}$$

J_m is a Bessel-function and α is a constant that depends on multiple outer factors [1]. In our experiment the concrete composition of α is irrelevant, because it can later be found by fitting the Bessel-function to the data.

3 Setup and measurements

To perform the different measurements described in the introduction, three different setups are needed. First the grating constant of a sine grating should be found by directly measuring the distance between the two first maxima. The setup for this part can be seen in Figure 1.

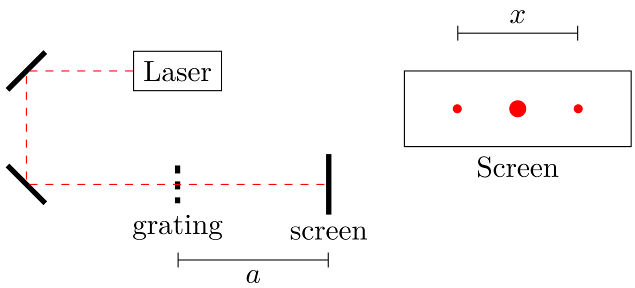


Fig. 1: Sketch of the setup for the first measurement on a sine grating with all the relevant parameters for the measurements. The distance a between grating and screen and the distance x between the two maxima are also shown.

The laser beam with a wavelength of $\lambda = 632.8\,\mathrm{nm}$ (no uncertainty found in the data sheet [3]) goes through the sine grating and hits the screen. The grating constant of the sine grating $1/K_S = 1016\,\mathrm{mm}^{-1}$ is recorded for comparison with the calculated value. For the distance a between screen and grating and the distance x between the two first maxima the following values are measured with the uncertainty approximated with a triangular distribution [4]:

$$a = (6.40 \pm 0.12)\,\mathrm{cm}, \tag{9}$$
$$x = (10.60 \pm 0.16)\,\mathrm{cm}. \tag{10}$$

For the measurements of the other amplitude gratings another setup, which is displayed in Figure 2, is used.

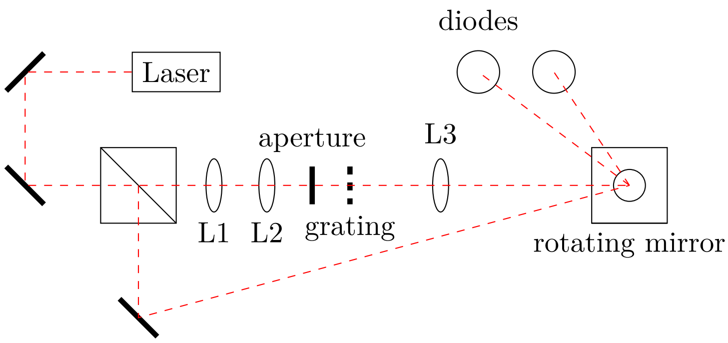


Fig. 2: Sketch of the setup for the other measurement of amplitude gratings with all the relevant parameters and components for the measurements.

On this setup the laser first passes two lenses that widen and then collimate the beam and afterwards it travels through an aperture. The focal lengths of the lenses can be found in Figure 30. It then hits the given amplitude grating before passing another lens and hitting a rotational mirror that sends the light to a diode. With a beam splitter the beam is divided in two parts to get a trigger signal on another diode. With an oscilloscope the interference patterns can be analysed digitally.

For calibration reasons the first measurement is taken with a grating R with the well-known grating constant $1/K_R = 80\,\mathrm{cm}^{-1}$. Afterwards five other gratings are evaluated. To get the resolution of the gratings, the length L with lightened slits on the grating has to be measured. The uncertainty is again approximated with formulas of a triangular distribution from [4]:

$$L = (4.0 \pm 0.4)\,\mathrm{mm}. \tag{11}$$

For the last part of the measurements the only change in the setup is the exchange of the amplitude gratings by the ultrasound cell to measure a phase grating (Figure 3). To calibrate the new setup another measurement with grating R is made. In the following, the ultrasound generator is turned on and a frequency is found that gives a stable signal ($f \approx 2070\,\mathrm{kHz}$, the exact values vary with the voltage and can be found in Figure 31). By varying the voltage in 21 steps between 0 V and 10 V the phase grating can by analysed.

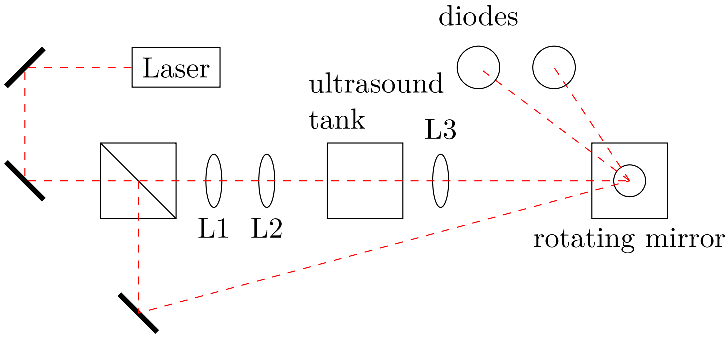


Fig. 3: Sketch of the setup for the last measurements on a phase grating realised by a ultrasound cell with all the relevant parameters and components for the measurements.

4 Data analysis and discussion of uncertainties

4.1 Analysis of the measurements of amplitude gratings

4.1.1 Determining the grating constant of a sine grating

With Equation 2 the grating constant of the sine grating can be directly found by measuring geometrical parameters:

$$m\lambda = K\sin(\theta_m), \tag{12}$$

$$\Rightarrow \frac{1}{K} = \frac{\sin(\arctan(x/(2a)))}{m\lambda}, \tag{13}$$

with $m = 1$, $\lambda = 632.8\,\mathrm{nm}$, x the distance between the maxima and a the distance between the grating and the screen. The uncertainty can be found by Gaussian propagation of uncertainty [5]:

$$\Delta\left(\frac{1}{K}\right) = \frac{1}{m\lambda}\sqrt{\left(\frac{\Delta x}{2a(\frac{x^2}{4a^2}+1)^{(3/2)}}\right)^2 + \left(\frac{x\Delta a}{2a^2(\frac{x^2}{4a^2}+1)^{(3/2)}}\right)^2}. \tag{14}$$

By plugging in the values one can get the following grating constant:

$$\frac{1}{K} = (1008 \pm 15)\,\mathrm{mm}^{-1},$$
$$\Rightarrow K = (0.992 \pm 0.014)\,\mathrm{\mu m}.$$

4.1.2 Finding the grating constants of other amplitude gratings

To find the grating constant of the other gratings the data from the oscilloscope can be used. First there has to be a calibration measurement with grating R that has a well-known grating constant of $1/K_R = 80\,\mathrm{cm}^{-1}$. For every measurement three sets of data are taken to reduce noise by averaging the data. The interference pattern of the calibration measurement can be found in Figure 4. As one can see, there is still a lot of noise left. Reasons for this remaining noise are debated in the discussion part in subsection 5.3.

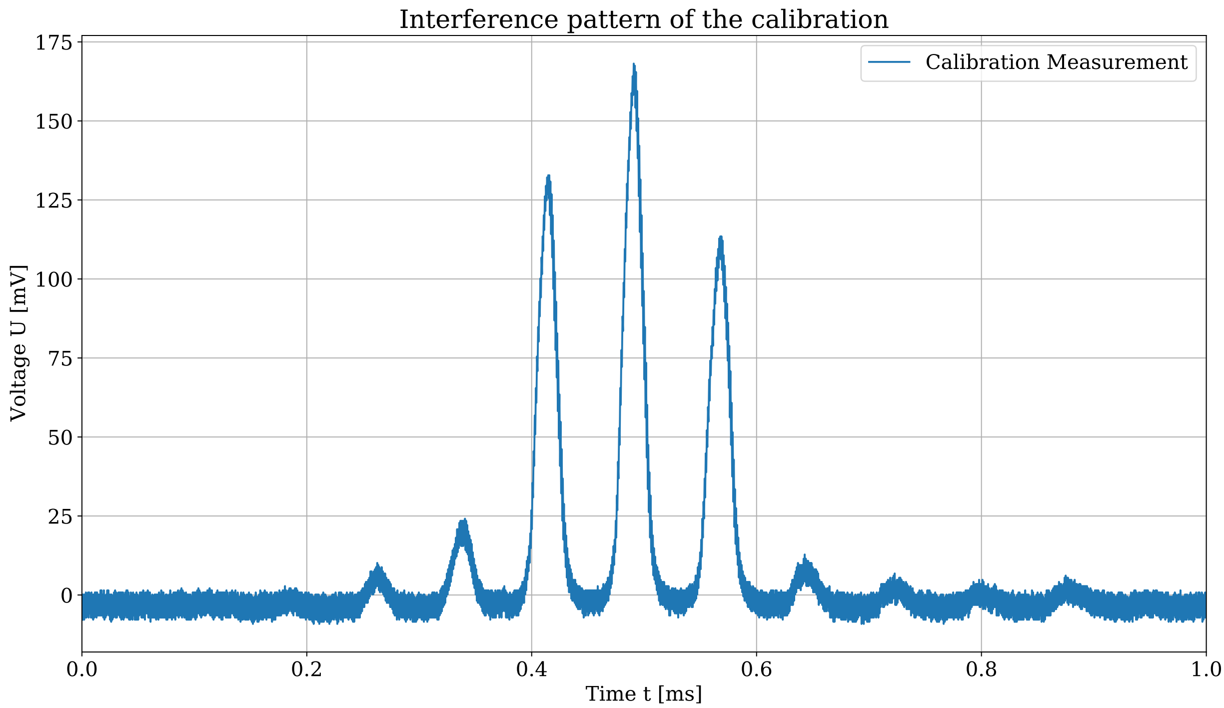


Fig. 4: In this graphic the Voltage U in mV is plotted against the time t in ms for the calibration grating. You can see 9 peaks in the interference pattern.

Because the oscilloscope has an arbitrary time axis as the x-axis we need to find a formula to convert the time into an angle by comparing the expected peaks calculated with Equation 2 with the peaks in the interference pattern in Figure 4. The first step is to calculate the expectation for the angle. Because later the sine of the angle is used, we can also find a conversion formula between $\sin(\theta_m)$ and t. From Equation 2 we get:

$$\sin(\theta_m) = \frac{m\lambda}{K}. \tag{15}$$

Because for none of this parameters an uncertainty is known, we do not find an uncertainty for $\sin(\theta_m)$. In a second step the peaks in the interference pattern for the calibration measurement are located. The peak position is found by fitting a Gauß-curve in the neighbourhood of the peak:

$$G(x) = \frac{A}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{16}$$

For the time t of the peak we take the parameter μ and its uncertainty as returned by the `scipy.optimize.curve_fit` function. Together with the expected sines, the peak position can be found in Table 9 in the appendix. In Figure 5 the interference pattern is displayed with the given peaks:

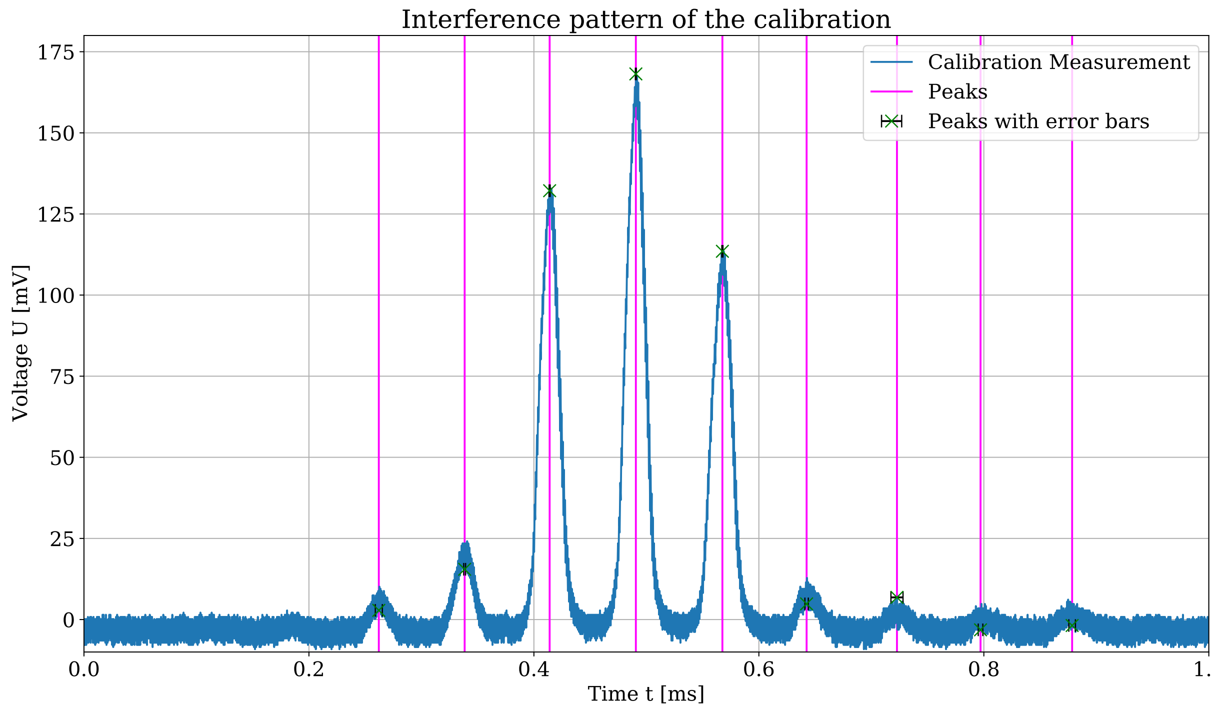


Fig. 5: In this graphic the Voltage U in mV is plotted against the time t in ms for the calibration grating. You can also see the 9 marked peaks in the interference pattern. Additionally the position of the peaks is marked by a cross with error bars.

To find the conversion factor a linear regression with the found data is proceeded:

$$t = a \cdot \sin(\theta_m) + b. \tag{17}$$

While a is the conversion factor between $\sin(\theta_m)$ and t, b is the time offset of the principal peak. For the values of a and b we get with `scipy.optimize.curve_fit`:

$$a = (15.13 \pm 0.04)\,\text{ms},$$
$$b = (0.4908 \pm 0.0002)\,\text{ms}.$$

For later conversion of data only a is relevant, because the time-offset differs from measurement to measurement. The linear regression can be found in Figure 6.

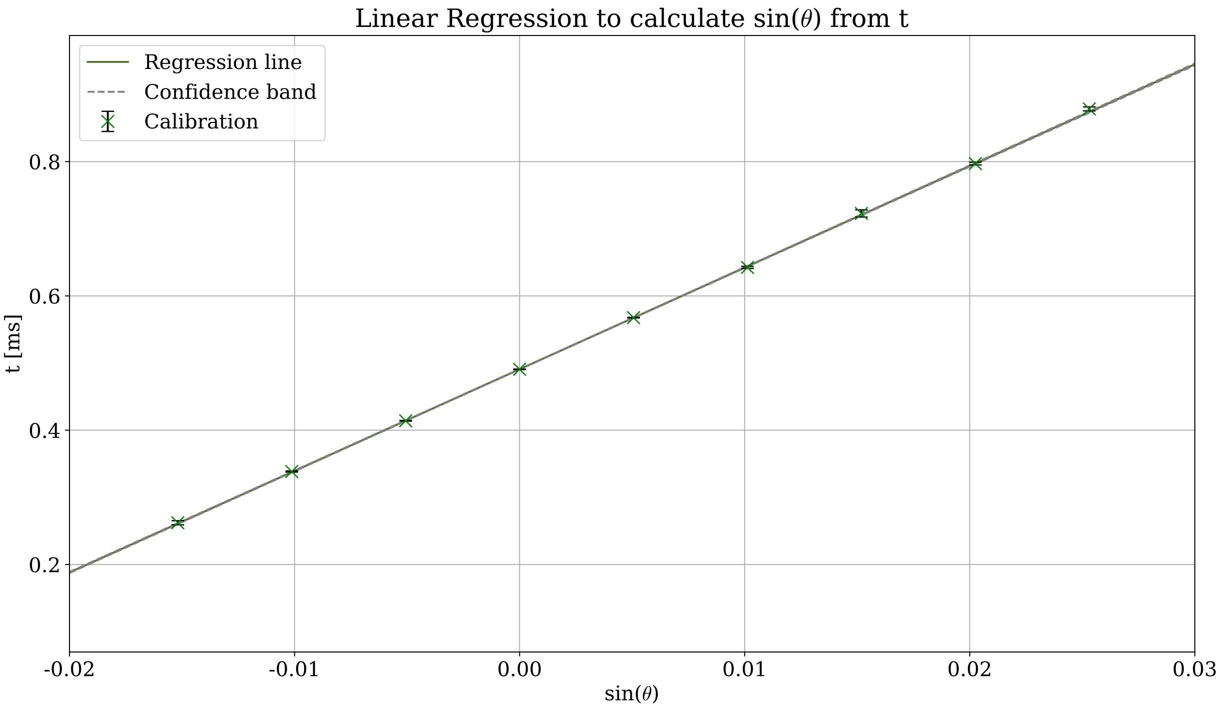


Fig. 6: In this graphic the calculated times t in ms of the peaks are plotted against the expected $\sin(\theta)$ for the calibration grating. There is also a regression line with a confidence band to find the conversion factor. The confidence band is too small to be visible.

For the analysis of the grating constants we only look at one of the gratings, the analogue discussion is made with the other gratings and the graphics can be found in the appendix (Figure 17 to Figure 20). Analogue to the calibration measurement the data is imported. After finding the time code of the main peak t_0 with Equation 16, the data is shifted so that the main peak is at $t = 0$. Than the data is converted into $\sin(\theta)$. The uncertainty is splitted into systematic uncertainty from the calibration and the main peak and statistical uncertainty from the measurement:

$$\sin(\theta) = \frac{t - t_0}{a}, \tag{18}$$

$$\Delta_{\text{stat}} \sin(\theta) = \frac{\Delta t}{a}, \tag{19}$$

$$\Delta_{\text{syst}} \sin(\theta) = \sqrt{\left(-\frac{\Delta t_0}{a}\right)^2 + \left(\frac{(t - t_0)\Delta a}{a^2}\right)^2}. \tag{20}$$

Analogue to the calibration we get the intensity distribution with the shown peaks portrait in Figure 7 and summarised in Table 2.

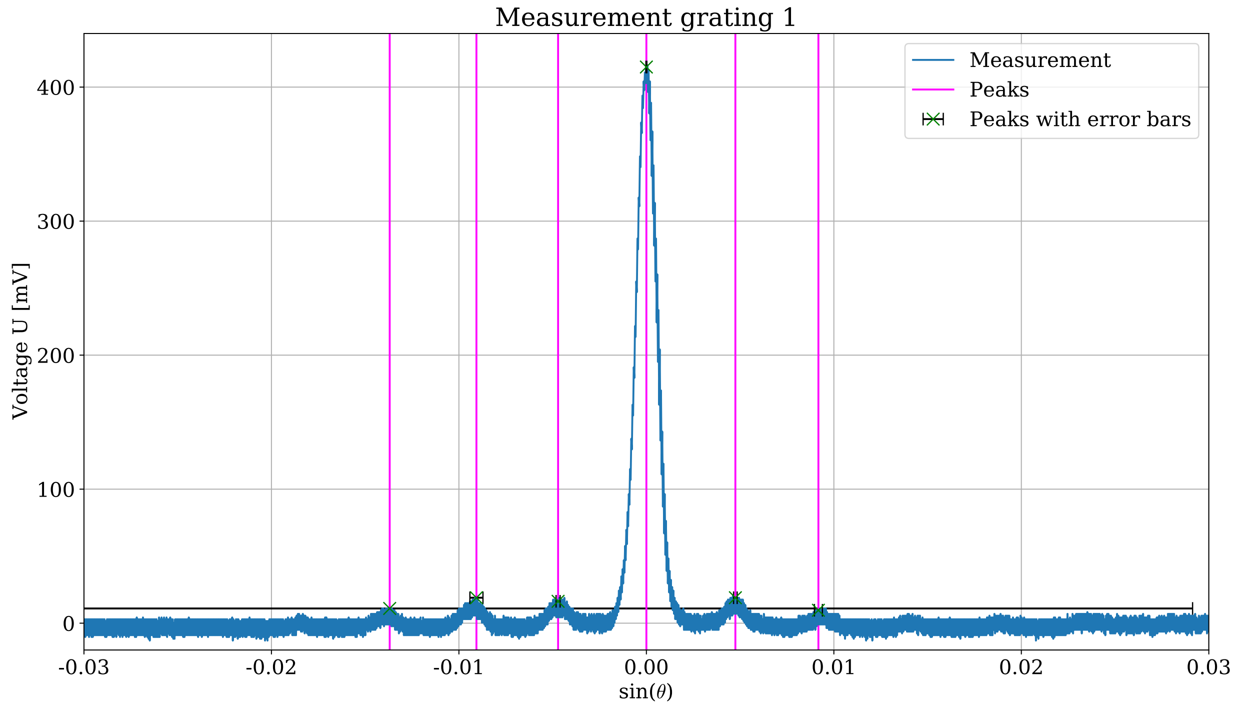


Fig. 7: In this graphic the Voltage U in mV is plotted against the converted $\sin(\theta)$ for the first grating. You can also see the 6 marked peaks in the interference pattern. Additionally the position of the peaks is marked by a cross with error bars.

The values for K can now be easily found with Equation 2:

$$\frac{1}{K} = \frac{\sin(\theta_m)}{m\lambda}, \tag{21}$$

$$\Delta_{\text{stat/syst}}\left(\frac{1}{K}\right) = \frac{\Delta_{\text{stat/syst}}\sin(\theta_m)}{m\lambda}. \tag{22}$$

All the values for K and the positions of the peaks are summarised in Table 2:

Tab. 2: Values for the grating constant $1/K$ in m^{-1} for the first grating. The first column shows the given order m of the maximum, the second column lists the peak positions $\sin(\theta_m)$ and in the last column the calculated grating constants $1/K$ of each peak are listed with the statistical and systematical error.

m	$\sin(\theta_m)\cdot 10^3$	grating constant $1/K$ in m^{-1}
-3	$(-13.69 \pm_{\text{stat}} 42.79 \pm_{\text{syst}} 0.04)$	$(7210 \pm_{\text{stat}} 22540 \pm_{\text{syst}} 20)$
-2	$(-9.07 \pm_{\text{stat}} 0.32 \pm_{\text{syst}} 0.03)$	$(7160 \pm_{\text{stat}} 250 \pm_{\text{syst}} 20)$
-1	$(-4.710 \pm_{\text{stat}} 0.090 \pm_{\text{syst}} 0.017)$	$(7440 \pm_{\text{stat}} 140 \pm_{\text{syst}} 30)$
0	$(0.000 \pm_{\text{stat}} 0.012 \pm_{\text{syst}} 0.012)$	–
1	$(4.747 \pm_{\text{stat}} 0.078 \pm_{\text{syst}} 0.018)$	$(7500 \pm_{\text{stat}} 120 \pm_{\text{syst}} 30)$
2	$(9.18 \pm_{\text{stat}} 0.18 \pm_{\text{syst}} 0.03)$	$(7250 \pm_{\text{stat}} 150 \pm_{\text{syst}} 20)$

The great statistical uncertainty in the first row will be discussed in subsection 5.3.

The resulting value for K can now be found by averaging all the values for K. The statistical uncertainty can be calculated by the standard deviation of the mean [4] and the systematical uncertainty is calculated by Gaussian propagation of uncertainties [1]:

$$\frac{1}{K} = (73.14 \pm_{\text{stat}} 0.67 \pm_{\text{syst}} 0.11)\,\text{cm}^{-1},$$
$$K = (136.7 \pm_{\text{stat}} 1.2 \pm_{\text{syst}} 0.2)\,\mu\text{m}.$$

All the values for K for the five different gratings are summarised in Table 3:

Tab. 3: Summary of the values for the grating constant $1/K$ in m^{-1} and K in µm for all gratings.

	$1/K$ in m^{-1}	K in µm
Grating 1	$(7314 \pm_{\text{stat}} 67 \pm_{\text{syst}} 11)$	$(136.7 \pm_{\text{stat}} 1.2 \pm_{\text{syst}} 0.2)$
Grating 2	$(27701 \pm_{\text{stat}} 16 \pm_{\text{syst}} 77)$	$(36.10 \pm_{\text{stat}} 0.02 \pm_{\text{syst}} 0.10)$
Grating 3	$(9240 \pm_{\text{stat}} 50 \pm_{\text{syst}} 50)$	$(108.2 \pm_{\text{stat}} 0.6 \pm_{\text{syst}} 0.6)$
Grating 4	$(9330 \pm_{\text{stat}} 120 \pm_{\text{syst}} 140)$	$(107.2 \pm_{\text{stat}} 1.3 \pm_{\text{syst}} 1.6)$
Grating 5	$(18460 \pm_{\text{stat}} 70 \pm_{\text{syst}} 20)$	$(54.17 \pm_{\text{stat}} 0.21 \pm_{\text{syst}} 0.07)$

4.1.3 Discussion of the resolution of the gratings

Similarly to the last part, the resolution is again discussed with the first grating. As described by Equation 3, for calculating the resolution the number of lightened slits N and the order $m_{\max}$ of the highest maximum are needed. To get N the width of the beam on the grating $L = (4.0 \pm 0.4)\,\text{mm}$ is measured and afterwards multiplied by the previously found grating constant $1/K$. To find $m_{\max}$ the number of clearly visible peaks in the interference pattern is counted. In Figure 8 a screenshot from the oscilloscope is taken and the number of peaks is counted.

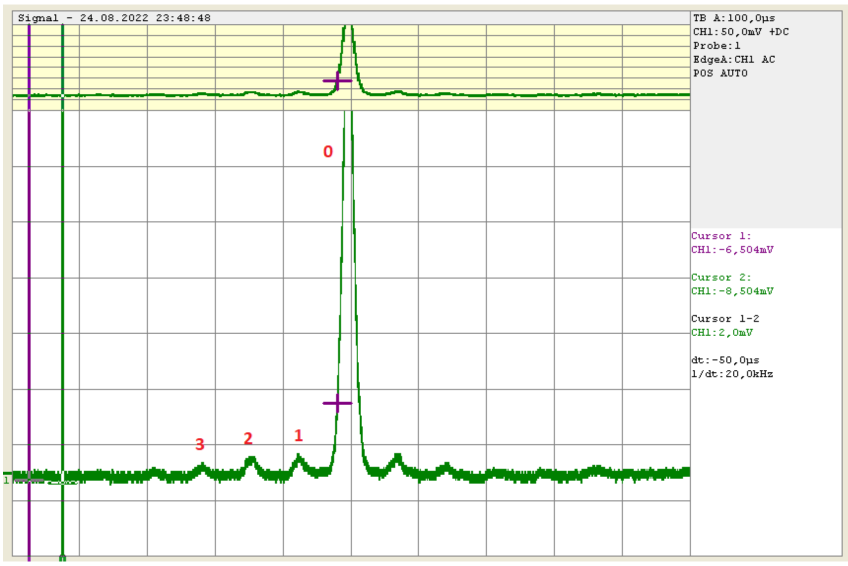


Fig. 8: Screenshot of the oscilloscope for counting the orders of the maxima for the first grating. The x-axis is the time-axis and the y-axis the intensity of the interference pattern. In red the three visible peaks are enumerated.

The resolution A can now be easily calculated:

$$A = \frac{Lm_{\text{max}}}{K}, \tag{23}$$

$$\Delta_{\text{syst}} A = m_{\text{max}} L \Delta_{\text{syst}} \left(\frac{1}{K} \right), \tag{24}$$

$$\Delta_{\text{stat}} A = m_{\text{max}} \sqrt{\left(L \Delta_{\text{stat}} \left(\frac{1}{K} \right) \right)^2 + \left(\frac{\Delta L}{K} \right)^2}. \tag{25}$$

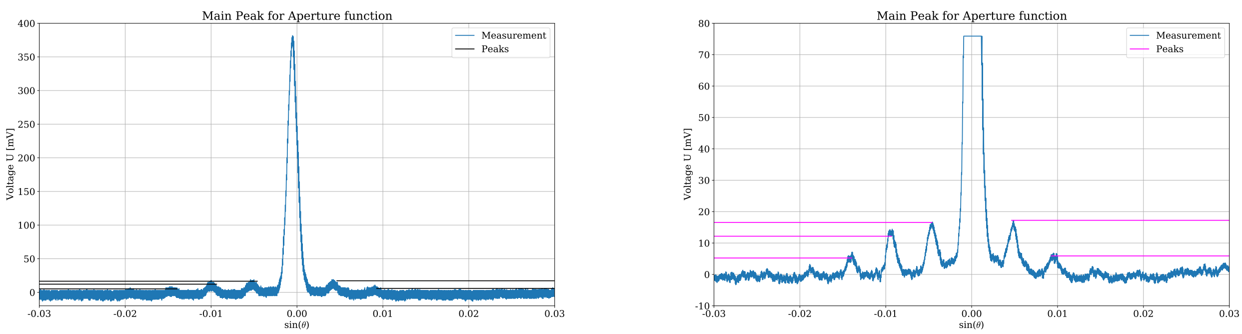
In Table 4 all the resolutions for the five gratings are summarised together with their grating constants and the number of visible maxima.

Tab. 4: Summary of the values for the grating constant $1/K$ in m^{-1} for all gratings. In addition the highest peak order m_{max} and resolution A are listed.

	$1/K$ in m^{-1}	m_{max}	resolution A
Grating 1	$(7314 \pm_{\text{stat}} 67 \pm_{\text{syst}} 11)$	3	$(87.8 \pm_{\text{stat}} 0.8 \pm_{\text{syst}} 9.0)$
Grating 2	$(27701 \pm_{\text{stat}} 16 \pm_{\text{syst}} 77)$	3	$(332.4 \pm_{\text{stat}} 0.2 \pm_{\text{syst}} 33.9)$
Grating 3	$(9240 \pm_{\text{stat}} 50 \pm_{\text{syst}} 50)$	2	$(73.9 \pm_{\text{stat}} 0.4 \pm_{\text{syst}} 7.6)$
Grating 4	$(9330 \pm_{\text{stat}} 120 \pm_{\text{syst}} 140)$	1	$(37.3 \pm_{\text{stat}} 0.5 \pm_{\text{syst}} 3.8)$
Grating 5	$(18460 \pm_{\text{stat}} 70 \pm_{\text{syst}} 20)$	4	$(295.4 \pm_{\text{stat}} 1.2 \pm_{\text{syst}} 30.1)$

4.1.4 Finding the aperture function of the first grating

For the first grating an approximation for the aperture function with Equation 4 should be found. Therefore all the intensities of the peaks are needed as coefficients in the Fourier series. Because the positions of the peaks were already used, the related height can be found easily. The uncertainty of the peak position is translated into an uncertainty of the height, represented by the voltage, by taking $\Delta U = U(\sin \theta_{\text{peak}}) - U(\sin \theta_{\text{peak}} \pm \Delta \sin \theta_{\text{peak}})$. For exacter values an extra measurement is made for the smaller peaks. Figure 9 shows the measurements for finding the aperture function with all the peak voltages marked.



(a) Main maximum marked

(b) Secondary maxima marked

Fig. 9: In the left graphic one can see the interference pattern of the first grating with the main maximum marked. On the right hand side one can see an exacter measurement with the smaller secondary maxima marked.

To get the intensity all the voltages have to be normalised by dividing them with the voltage of the main maximum:

$$I_i = \frac{U_i}{U_0}, \tag{26}$$

$$\Delta I_i = \sqrt{\left(\frac{\Delta U_i}{U_0}\right)^2 + \left(\frac{U_i \Delta U_0}{U_0^2}\right)^2}. \tag{27}$$

All the intensities are put together in Table 5:

Tab. 5: Normalised and averaged intensities for each order m of the maxima.

m	Intensity
3	0.0070 ± 0.0015
2	0.0241 ± 0.0013
1	0.045 ± 0.002
0	1.000 ± 0.016

By plugging all the intensities into $g(x) = \sum_{j=0}^{\infty} \pm\sqrt{I_j} \cos\left(\frac{x}{K} 2\pi j\right)$ we can find the aperture function which is portrait in Figure 10:

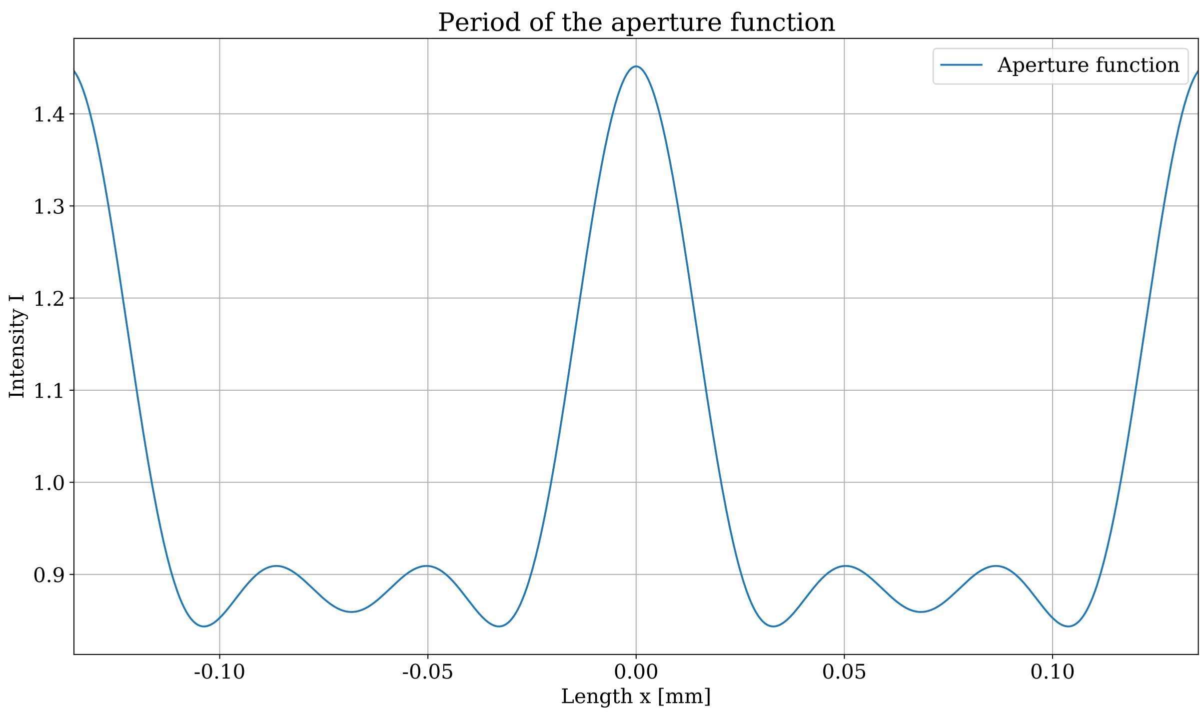


Fig. 10: This plot shows a period of the aperture function of the first grating. Therefore the intensity is plotted against the length on the grating in mm.

Some parts of the aperture function are greater than 1 which should not be possible. The reason for this is discussed later in subsection 5.3.

4.1.5 Finding the quotient of gap width and grating constant

For the last part of the analysis of amplitude gratings, the gap width b of the slits of the first grating should be found from the aperture function. Therefore we use the Full-Width-Half-Maximum of the middle peak of the aperture function as a guess for b. The FWHM is found by taking the average between maximum and minimum and then finding the width of the peak at this position. In Figure 11 the aperture function is plotted with a FWHM bar.

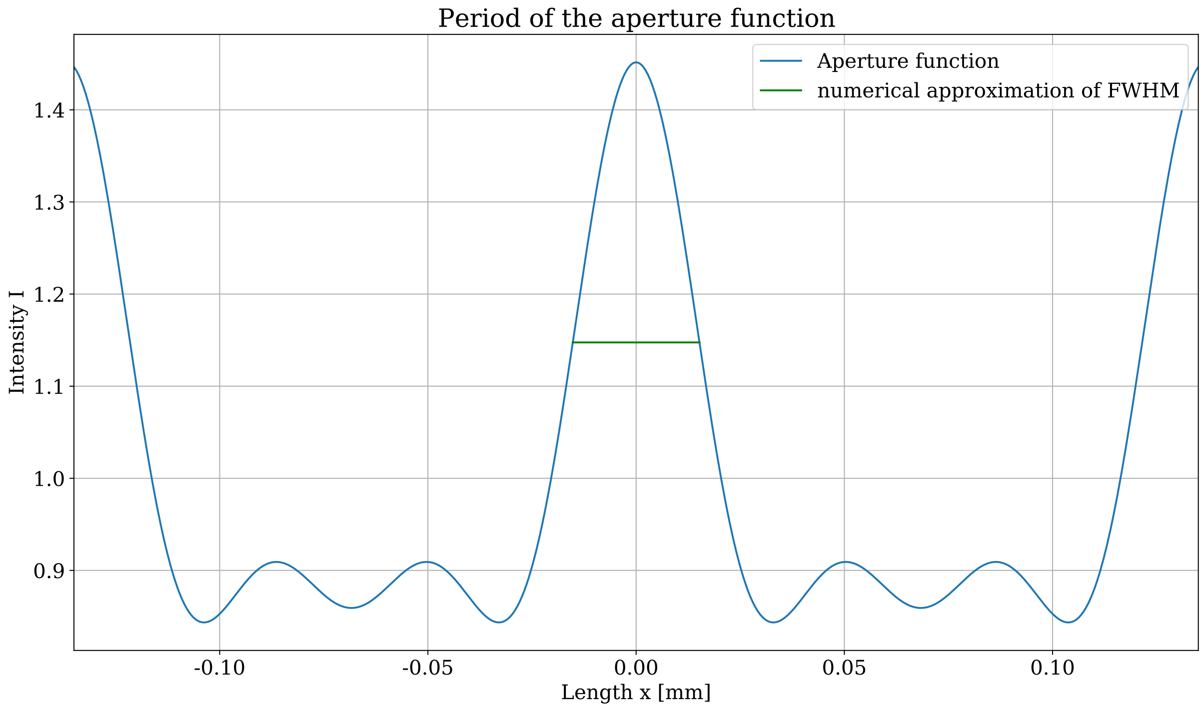


Fig. 11: This plot shows a period of the aperture function of the first grating. Therefore the intensity is plotted against the length on the grating in mm. Additionally the Full-Width-Half-Maximum (FWHM) of the function is shown as a green line.

As uncertainty we assume a statistical error of 5%. The quotient q of gap width and grating constant can now be directly calculated:

$$q = \frac{b}{K}, \tag{28}$$

$$\Delta_{\text{syst}} q = b \Delta_{\text{syst}} \left(\frac{1}{K} \right), \tag{29}$$

$$\Delta_{\text{stat}} q = \sqrt{\left(b \Delta_{\text{stat}} \left(\frac{1}{K} \right) \right)^2 + \left(\frac{\Delta b}{K} \right)^2}. \tag{30}$$

Plugging in the values gives the following value for q:

$$q = (0.222 \pm_{\text{stat}} 0.011 \pm_{\text{syst}} 0.003)$$

4.2 Analysis of the measurements of the phase grating

4.2.1 Intensity distribution of the ultrasound phase grating

Because the laser beam now travels through the tank of isooctane, another calibration is necessary. We again use grating R to calibrate the oscilloscope and do a linear regression completely analogue to subsubsection 4.1.2. In Figure 21 in the appendix one can find the marked peaks of the interference pattern of the calibration measurement, which are then used for the linear regression. In Figure 12 the linear regression is portrayed. For the regression parameters slope a and intercept b we get from `scipy.optimize.curve_fit`:

$$a = (15.40 \pm 0.06)\,\mathrm{ms},$$
$$b = (0.4967 \pm 0.0005)\,\mathrm{ms}.$$

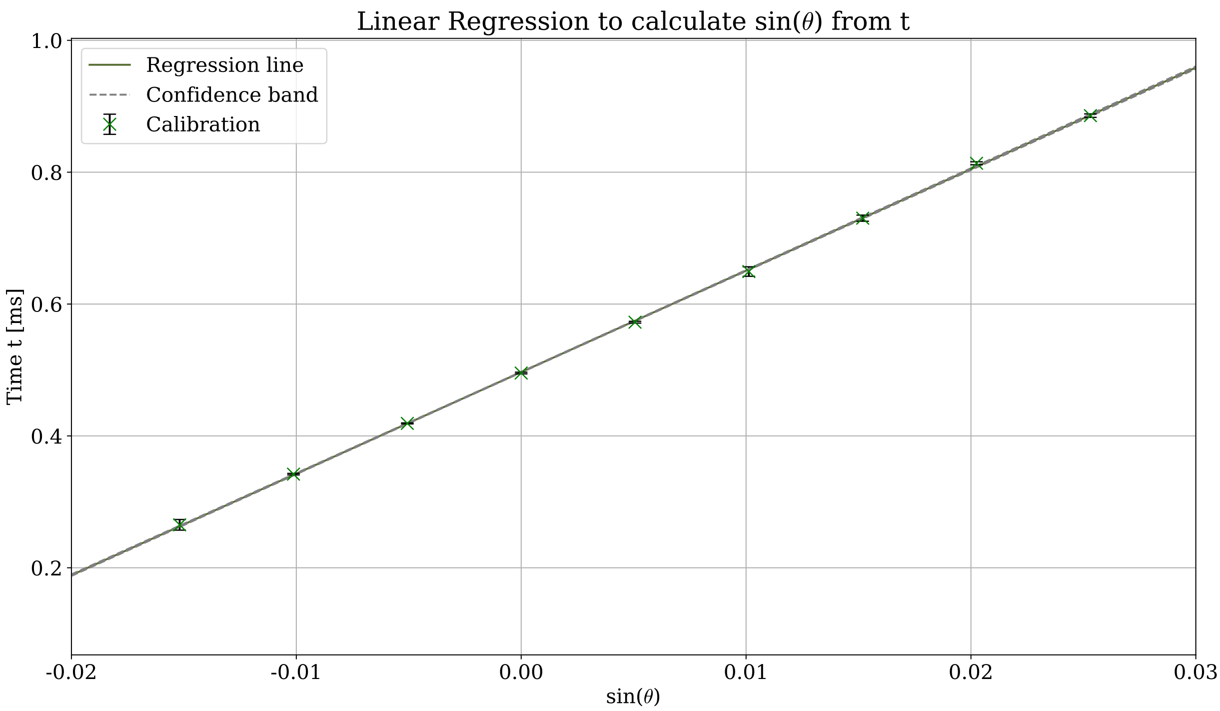


Fig. 12: In this graphic the calculated times of the peaks are plotted against the expected $\sin(\theta)$ for the calibration grating for the ultrasound tank. There is also a regression line with a confidence band to find the conversion factor. The confidence band is too small to be visible.

After importing the data, it is again shifted, so that the main maximum is at $t = 0$ and than converted to $\sin(\theta)$ by dividing with the conversion factor a (Equation 18 - Equation 20). After finding the amplitude of the main maximum at $0\,\mathrm{V}$ all the data can be normalised to get the intensity (analogue to Equation 26 - Equation 27). Plotting all the data in one plot gives us the following three dimensional Figure 13:

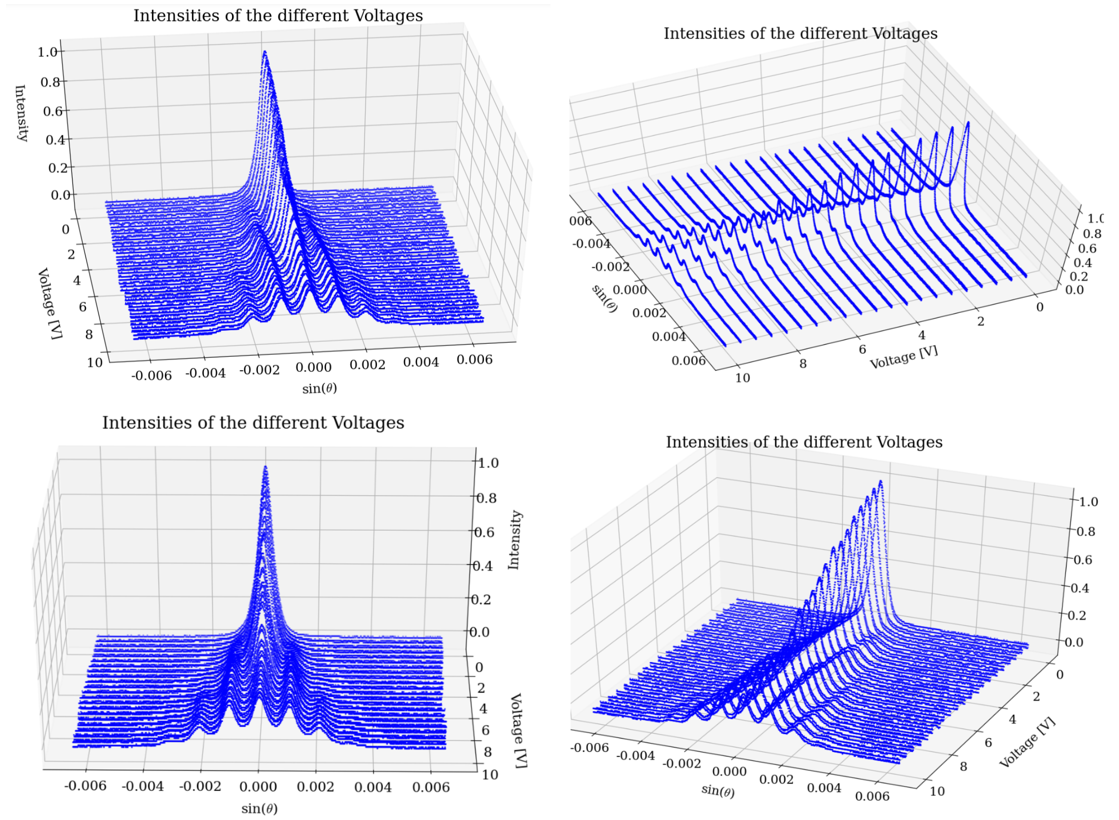


Fig. 13: In this graphic the $\sin(\theta)$ of the peaks are plotted against the normalised intensity for different voltages in V. In the four plots different perspectives are visualised.

4.2.2 Comparison with the Raman-Nath-Theory

To compare the measurements with the Raman-Nath-Theory (Equation 8) we have to separate all the peaks and create new plots that show the peak height in comparison to the voltage on the ultrasound generator. For doing this, first the position of the different peaks has to be found. To find the peaks, the measurement with the highest voltage is used, because here all the maximums can be seen. Again, a Gaussian curve is fitted to the peaks to find the best values for $\sin(\theta_m)$. In Figure 14 the peaks are shown and in Table 10 in the appendix they are all listed. The uncertainty on the intensity of the peaks is estimated by 10% because the noise of the measurement is quite high.

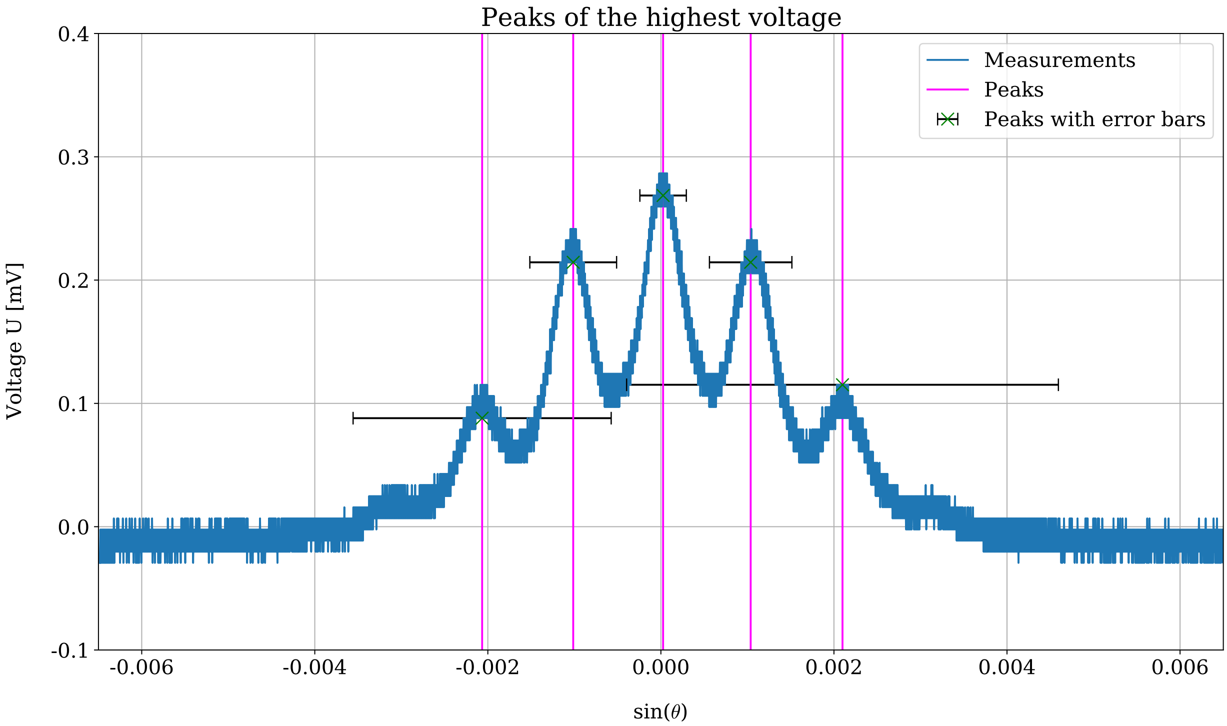


Fig. 14: In this graphic the Voltage U in mV is plotted against $\sin(\theta)$ for the highest voltage. The 5 peaks are marked.

After having found the position of the peaks the values for all the different voltages can be assembled. By using `scipy.optimize.curve_fit`, the theoretical trend of the Raman-Nath-Theory ($I_m = J_m^2(\alpha U)$, [1]) can be fitted with the data. Exemplary, the fitted Bessel function to the data of the main peak is shown in Figure 15, all the other fits can be found in the attachment in Figure 26 to Figure 29. Because for the lower voltage measurements not all the peaks are visible and the noise is dominating, only the data where clear peaks are recognisable are used for the fit. In the plots, all used data points are red and all the unused point are marked grey.

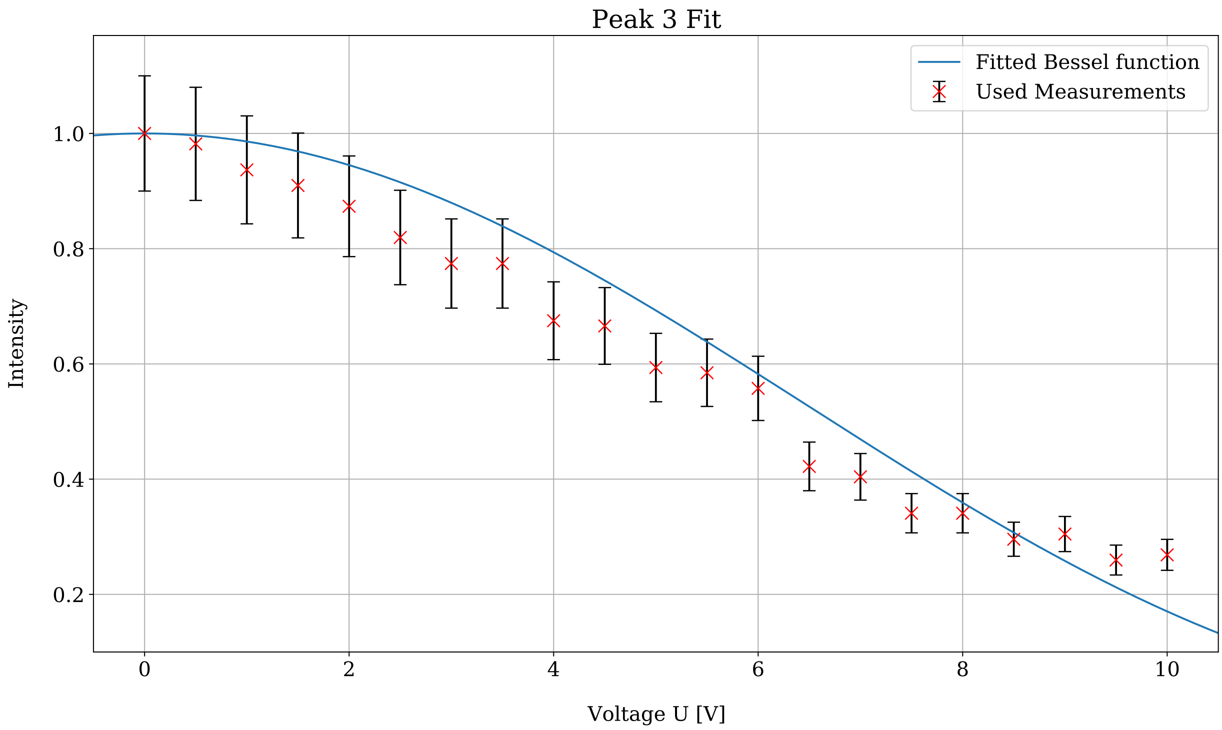


Fig. 15: In this graphic the intensities I of the main maxima are plotted against the voltage U in V. One can also see the fitted Bessel function.

For the fitting parameter α we get the values and uncertainties from `scipy.optimize.curve_fit`, all summarised in Table 6. As we can see in the plot, not all the data matches the theory, so a χ^2-value is calculated to give an approximation for the quality of the fit [5]. A χ^2-value of $\chi^2 \approx n - f$ where n is the number of data points and f is the number of fit parameters (in our case 1) characterises a good fit. If $\chi^2 \gg n - f$, the fit is not a good match for the data [5].

Tab. 6: Fit parameter values α in V^{-1} and χ^2-values for the fits with the squared Bessel-function $I_m = J_m^2(\alpha U)$. In addition, for comparison the value of $n - f$ where n is the data point number and f is the number of fit parameters is given.

m	α in V^{-1}	χ^2-values	$n - f$
-2	0.207 ± 0.002	103	11
-1	0.229 ± 0.004	387	20
0	0.167 ± 0.002	46	20
1	0.206 ± 0.004	403	20
2	0.191 ± 0.002	69	8

The differences in the values of the fit parameter α as well as the calculated values for χ^2 are later discussed in the subsection 5.3.

4.2.3 Finding the wavelength of the ultrasound wave

To find the wavelength of the ultrasound wave Λ Equation 7 can be used. Because the positions of the peaks of the interference pattern and their uncertainties have already been found and listed in Table 10 in the appendix, Λ can be simply found by a linear regression:

$$\sin(\theta_m) = \frac{1}{\Lambda} m\lambda, \tag{31}$$

$$\Rightarrow y = \frac{1}{\Lambda} x + b. \tag{32}$$

In a plot with $\sin(\theta_m)$ on the y- and $m\lambda$ on the x-axis, $1/\Lambda$ would be the slope and b would be the intercept. This plot can be found in Figure 16 with a linear regression and a confidence band.

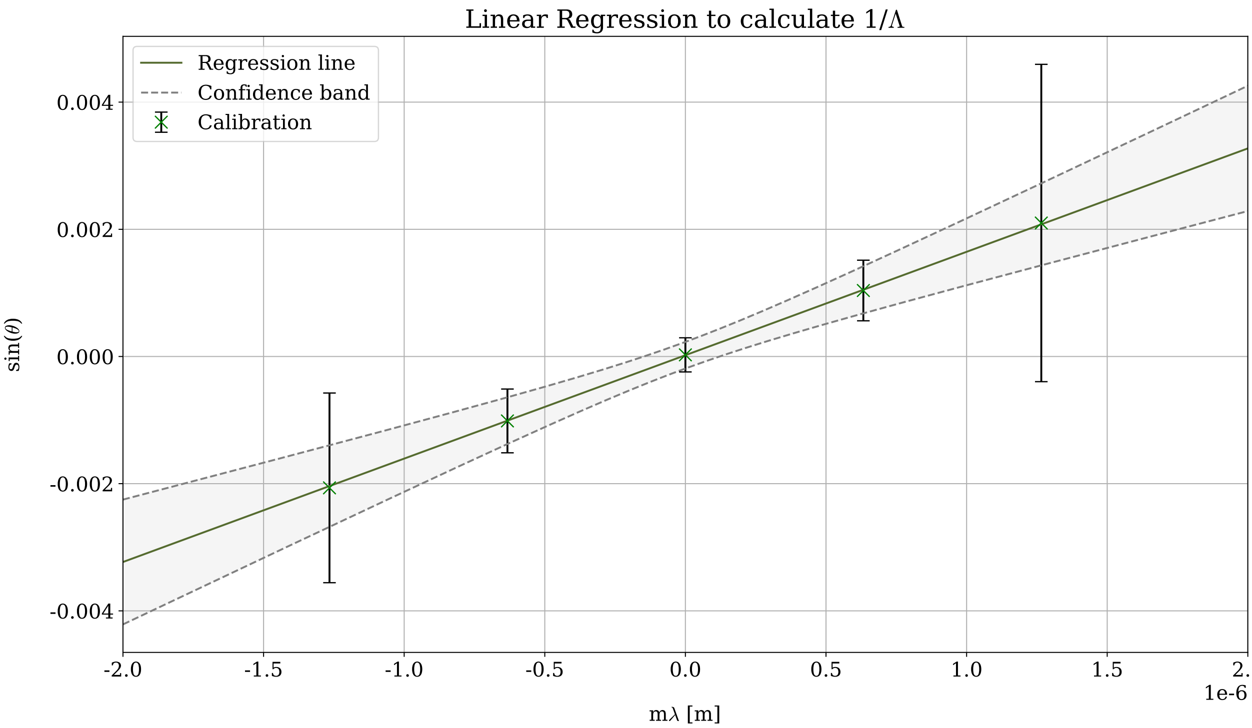


Fig. 16: In this graphic $\sin(\theta)$ is plotted against the order of the peak times the laser wave length λ in m. One can also see the regression line with the confidence band.

With `scipy.optimize.curve_fit`, the following parameters are found:

$$\frac{1}{\Lambda} = (1.6 \pm 0.5) \times 10^3\,\mathrm{m}^{-1},$$
$$b = (0.0 \pm 0.2) \times 10^{-3}.$$

As expected, b is nearly 0. The value for Λ can now be calculated, the uncertainty is calculated by Gaussian propagation of uncertainties. We get the following value:

$$\Lambda = (620 \pm 180)\,\mathrm{\mu m}.$$

5 Summary and discussion of the results

5.1 Summary of the results

In the first part of the experiment, different amplitude gratings were discussed. For the first grating, which was a sine grating, the following grating constant was found:

$$\frac{1}{K} = (1008 \pm 15)\,\mathrm{mm}^{-1},$$
$$\Rightarrow K = (0.992 \pm 0.014)\,\mu\mathrm{m}.$$

For five other amplitude gratings the grating constants and the resolution were calculated. All the values are summarised in Table 7:

Tab. 7: Summary of the values for the grating constant $1/K$ in m^{-1} for all gratings. In addition the highest peak order $m_{\max}$ and resolution A are listed.

	$1/K$ in m^{-1}	$m_{\max}$	resolution A
Grating 1	$(7314 \pm_{\mathrm{stat}} 67 \pm_{\mathrm{syst}} 11)$	3	$(87.8 \pm_{\mathrm{stat}} 0.8 \pm_{\mathrm{syst}} 9.0)$
Grating 2	$(27701 \pm_{\mathrm{stat}} 16 \pm_{\mathrm{syst}} 77)$	3	$(332.4 \pm_{\mathrm{stat}} 0.2 \pm_{\mathrm{syst}} 33.9)$
Grating 3	$(9240 \pm_{\mathrm{stat}} 50 \pm_{\mathrm{syst}} 50)$	2	$(73.9 \pm_{\mathrm{stat}} 0.4 \pm_{\mathrm{syst}} 7.6)$
Grating 4	$(9330 \pm_{\mathrm{stat}} 120 \pm_{\mathrm{syst}} 140)$	1	$(37.3 \pm_{\mathrm{stat}} 0.5 \pm_{\mathrm{syst}} 3.8)$
Grating 5	$(18460 \pm_{\mathrm{stat}} 70 \pm_{\mathrm{syst}} 20)$	4	$(295.4 \pm_{\mathrm{stat}} 1.2 \pm_{\mathrm{syst}} 30.1)$

From calculating the aperture function for the first grating, the quotient between gap width and grating constant has also been calculated:

$$q = (0.222 \pm_{\mathrm{stat}} 0.011 \pm_{\mathrm{syst}} 0.003)\,.$$

For the analysis of the phase grating the typical intensity distributions were observed. Fits with the squared Bessel functions $J_m^2(\alpha U)$ lead to different fit parameters α and different χ^2-values collected in Table 8.

Tab. 8: Summery of the fit parameter values α in V^{-1} and χ^2-values for the fits with the squared Bessel-function $I_m = J_m^2(\alpha U)$. In addition for comparison the value of $n - f$ where n is the data point number and f is the number of fit parameters is given.

m	α in V^{-1}	χ^2-values	$n - f$
-2	0.207 ± 0.002	103	11
-1	0.229 ± 0.004	387	20
0	0.167 ± 0.002	46	20
1	0.206 ± 0.004	403	20
2	0.191 ± 0.002	69	8

Finally the ultrasound wavelength was derived from Raman-Nath-Theory:

$$\Lambda = (620 \pm 180)\,\mu\mathrm{m}.$$

5.2 Comparison with expectation

For the grating constant of the sine grating, the calculated value can be compared to the real value $1/K_S = 1016\,\mathrm{mm}^{-1}$ by using a t-value. The t-value is calculated by dividing the difference between measured and real value with the uncertainty of the measured value. A t-value smaller than two corresponds with a good measurement. For $1/K$ we get the following t-value:

$$t = 0.55.$$

This t-value approves the accuracy of this measurement.

For the other grating constants, no real values are provided, which means no comparison is possible. With $1/K$-values between $(7314 \pm_{\text{stat}} 67 \pm_{\text{syst}} 11)\,\mathrm{m}^{-1}$ and $(27701 \pm_{\text{stat}} 16 \pm_{\text{syst}} 77)\,\mathrm{m}^{-1}$, all of the magnitudes could be realistic grating constants. For the resolution a comparison is not possible either. In addition, the resolution could vary a lot because the number of visible maxima is very hard to find properly, due to a high measurement noise.

In our comparison with the Raman-Nath-Theory, it could not really be confirmed by the Bessel-function-fits. With χ^2-values much bigger than expected, only the rough trend of the data and the theory are the same. In addition the fit-parameters differ from plot to plot which should not be the case. A discussion of this problem can be found in the following subsection 5.3.

The ultrasound wavelength can be compared to the theoretical value, since we know the frequency $f = (2070 \pm 3)\,\mathrm{kHz}$ of the ultrasound wave. The sonic speed in isooctane is $c = (1070 \pm 20)\,\mathrm{m\,s^{-1}}$ [6]. By multiplying the two values and using Gaussian propagation of uncertainties we get:

$$\Lambda_{\text{theo}} = (565 \pm 10)\,\mathrm{\mu m}.$$

To compare the values, again a t-value can be calculated:

$$t = 0.27.$$

Again, the t-value shows that the measurements of the ultrasound wave matches the theory.

5.3 Discussion of results and uncertainties

A first important discussion point is the noise that couldn't get reduced by taking the average of three measurements. If the noise would have been of statistical nature this should have reduced the noise strongly. Instead, no noise reduction is visible. The reason has to be a systematical problem with the oscilloscope or the digital software, that produces noise of the same nature for all measurements. Zooming in in the data shows a zig zag noise that is visible on every measurement.
A possible way to reduce this noise would have been to average every value with its neighbouring values. Because the peaks are not very broad, this would have lead to falsification of position and height of the peaks, so we rejected this method. Another option would have been a fast Fourier transformation of the data to cut the high frequencies of the noise.

Another discussion point are the high uncertainties some of the peak positions have. Especially for the first grating, one of the peak position has an uncertainty being three times bigger than the value itself. By using the method of fitting a Gauss curve to the peaks to find their position, some of the smaller peaks are very hard to detect by the python fitting function. Still this method seams to be much more accurate than finding the peaks by hand.

For the aperture function some of the values are greater than one which should not be possible. The reason for this is, that the intensities are all normalised with the respect to the main maximum. For a proper calibration another measurement would have been useful without a grating on the optical bench. Than all the intensities could have been normalised to the laser intensity and no values greater than one would occur in the aperture function.

What is also important to discuss is the fact, that it is not guaranteed that the beams hits the grating in a perfect 90° angle. Since for the theoretical expectation the beam is assumed to hit the grating perpendicularly, this can be a main source of error. During the measurement it has been tried to get a beam hitting the grating as perpendicular as possible, but still small variations in the angle can not be excluded. Because this aspect has not been examined while measuring, it is not possible to make a statement on how much this aspect influenced the measurement in this case.

The last and most important discussion point is the fact, that almost all the χ^2-values are way too large. That means that the Bessel-function-fits to the data were not very good and the Raman-Nath-Theory could not be directly confirmed. There are three main factors that lead to the high χ^2-values that are possible.
The first one is, that the uncertainties on the intensities are still very underestimated with 10%. Having higher uncertainties would lead to a better χ^2-value and so a better accordance to the theory. Especially because the rough trend still fits the theory, this is a very possible reason.
Another plausible problem could be the method used for finding the data points. Only finding the peaks for the highest voltage and not for every single voltage and that extending the model on the other voltage-measurements could lead to the problem that not every peak is hitted directly. On the other hand also the uncertainty would have decreased leading to only a tiny approvement of the χ^2-value.
A last important reason is the already discussed noise that could be reduced. Due to this noise, also the amplitude of the peaks is changed slightly which leads to more inaccurate measurements.

Improvement of the measurements would be possible by having digital filters for the noise. In addition to that, one could have performed more measurements to also reduce the statistical noise better. For the second part it would have been interesting to also perform the phase-grating-measurement with higher voltages to get more values for the Bessel-function. It would have been insightful to see the main peak disappear and reappear. Of course measurements could also been improved by hitting the diodes more direct or adjusting the mirrors to get an even better centred beam.

References

[1] M. Köhli: *Ultraschall - Versuchsanleitung Fortgeschrittenen Praktikum Teil 1*, (Freiburg im Breisgau: 2012)

[2] Lutz Lefèvre: *Beugung am Amplitudengitter und Phasengitter - Versuchsaufbau für das Fortgeschrittenen-Praktikum*, (Freiburg im Breisgau: 1977)

[3] Melles Griot, Inc.: *Green, Yellow, Orange and Red Self-Contained Helium Neon Lasers (Data Sheet provided in the folder for the experiment)* www.mellesgriot.com (visited at: 25.08.2022)

[4] Dr. Christof Bartels, Dr. Lukas Bruder, Dr. Thomas Pfohl: *Datenanalyse Teil A - Skript zur Vorlesung am 06.09.2021*, (Freiburg im Breisgau: 2021/22)

[5] Dr. Christof Bartels, Dr. Lukas Bruder, Dr. Thomas Pfohl: *Datenanalyse Teil B - Skript zur Vorlesung am 28.02.2022*, (Freiburg im Breisgau: 2021/22)

[6] W.M.Haynes: *CRC Handbook of Chemestry and Physics - 95th Edition*, (Boca Raton: CRC Press, 2014), p. 2449

7 Attachment

7.1 Tables and graphics

Tab. 9: Values for the linear regression to calibrate the oscilloscope with the grating R ($1/K_R = 80\,\mathrm{cm}^{-1}$). The first column shows the given order m of the maximum, the second column lists the expected values for $\sin(\theta_m)$ and in the last column the time t of the peaks from the grating are listed.

m	$\sin(\theta_m)$	time t in ms
-3	-0.015	0.262 ± 0.003
-2	-0.010	0.3384 ± 0.0008
-1	-0.005	0.4140 ± 0.0005
0	0.000	0.4906 ± 0.0004
1	0.005	0.5676 ± 0.0005
2	0.010	0.6424 ± 0.0015
3	0.015	0.723 ± 0.005
4	0.020	0.797 ± 0.002
5	0.025	0.878 ± 0.003

Tab. 10: In the table the positions of the five main peaks $\sin(\theta_m)$ of the phase grating interference patter are listed.

m	$\sin(\theta_m)$
-2	-0.0021 ± 0.0015
-1	-0.0010 ± 0.0005
0	0.0000 ± 0.0003
1	0.0010 ± 0.0005
2	0.002 ± 0.002

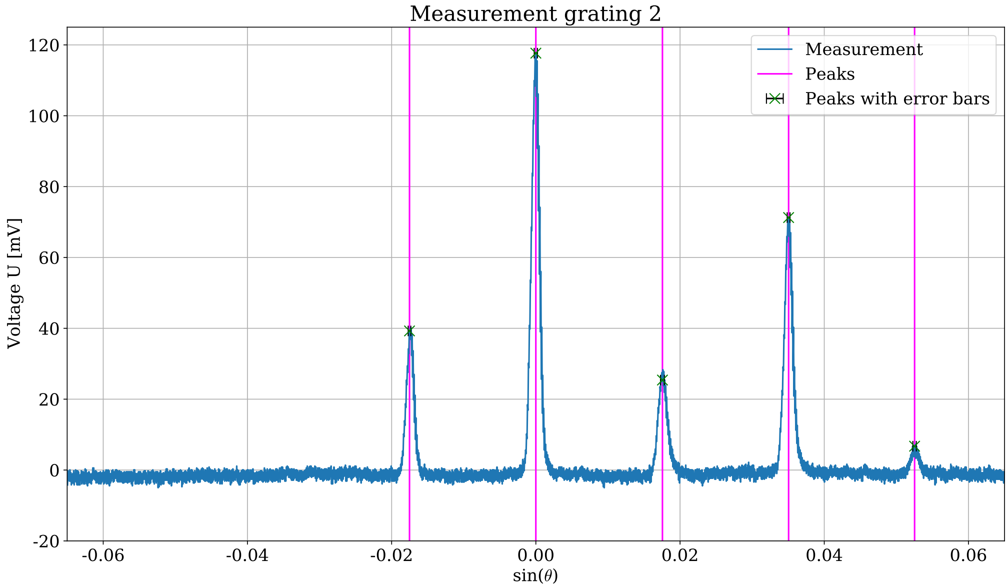


Fig. 17: In this graphic the Voltage U in mV is plotted against the time t in ms for the second grating. You can also see the 5 marked peaks in the interference pattern. Additionally the position of the peaks is marked by a cross with error bars.

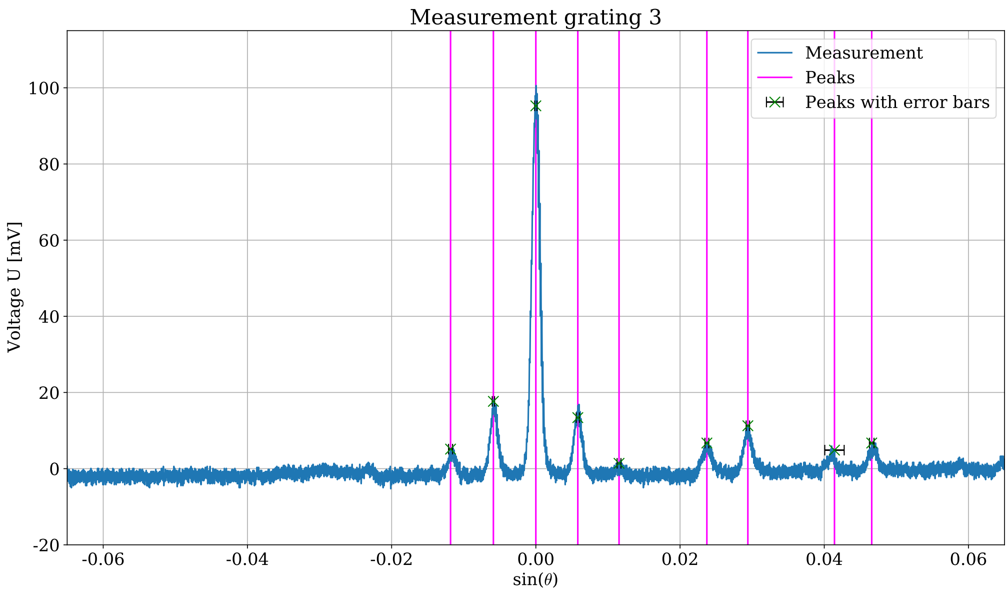


Fig. 18: In this graphic the Voltage U in mV is plotted against the time t in ms for the third grating. You can also see the 9 marked peaks in the interference pattern. Additionally the position of the peaks is marked by a cross with error bars.

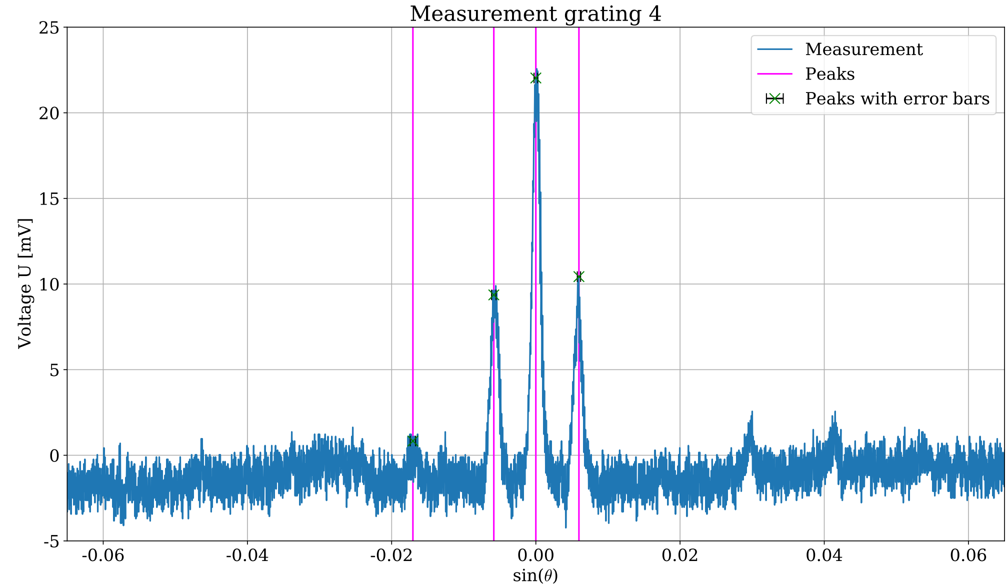


Fig. 19: In this graphic the Voltage U in mV is plotted against the time t in ms for the fourth grating. You can also see the 4 marked peaks in the interference pattern. Additionally the position of the peaks is marked by a cross with error bars.

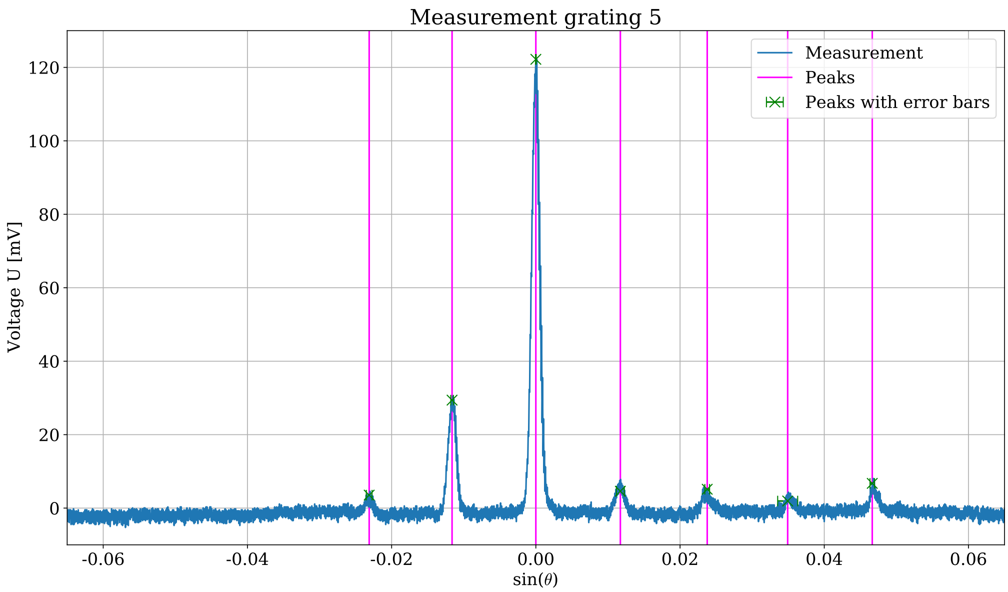


Fig. 20: In this graphic the Voltage U in mV is plotted against the time t in ms for the fifth grating. You can also see the 7 marked peaks in the interference pattern. Additionally the position of the peaks is marked by a cross with error bars.

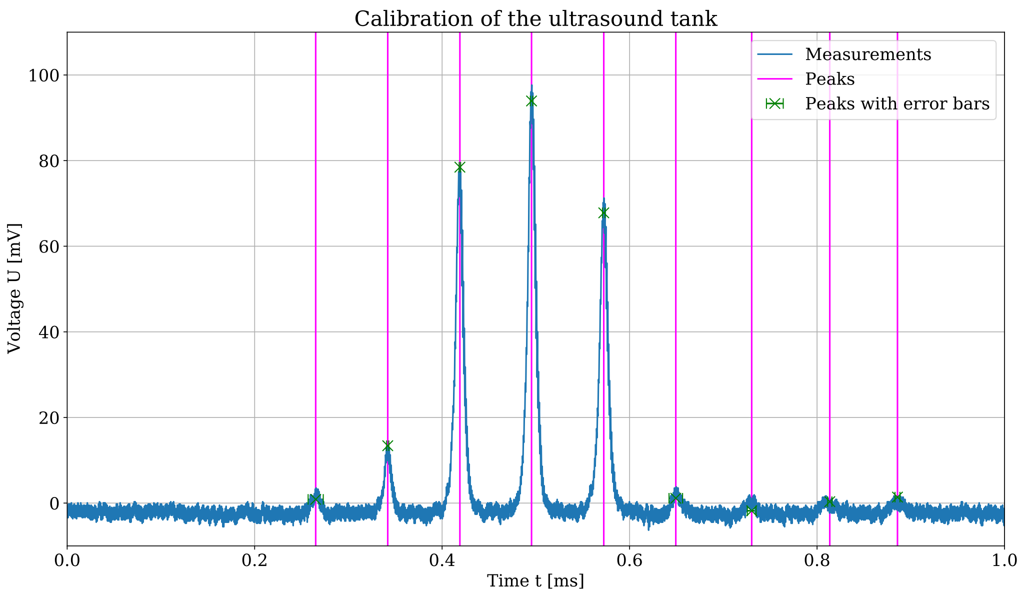


Fig. 21: In this graphic the Voltage U in mV is plotted against the time t in ms for the calibration grating for the ultrasound tank. You can also see the 9 marked peaks in the interference pattern. Additionally the position of the peaks is marked by a cross with error bars.

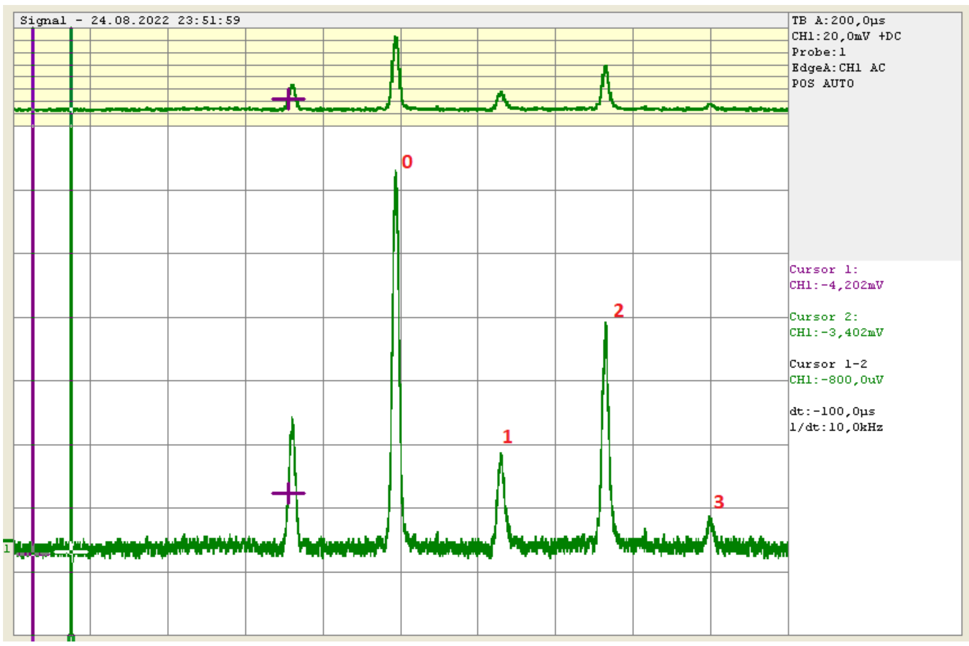


Fig. 22: Screenshot of the oscilloscope for counting the orders of the maxima for the second grating. The x-axis is the time-axis and the y-axis the intensity of the interference pattern. In red the three visible peaks are enumerated.

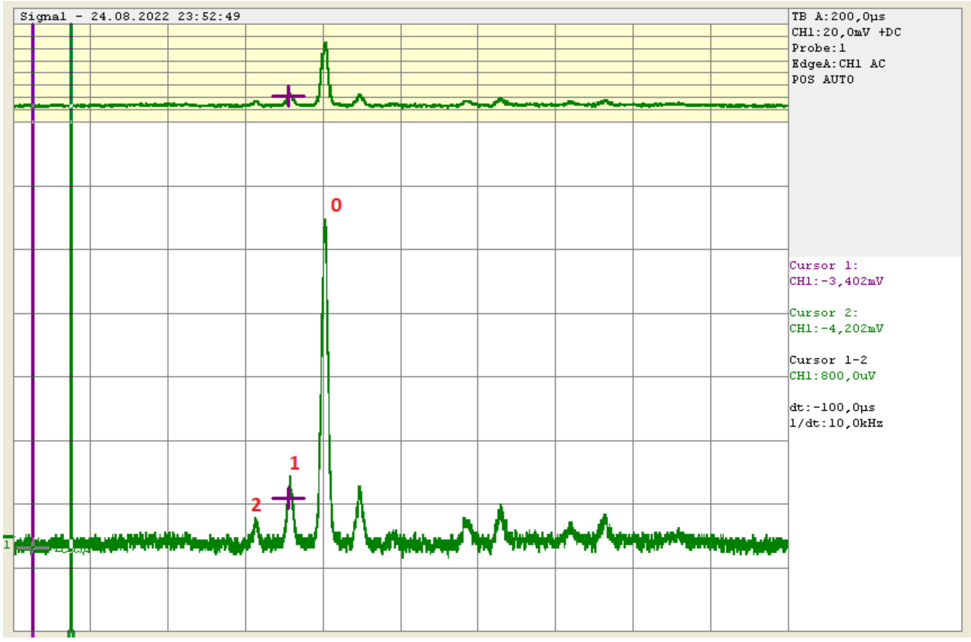


Fig. 23: Screenshot of the oscilloscope for counting the orders of the maxima for the third grating. The x-axis is the time-axis and the y-axis the intensity of the interference pattern. In red the two visible peaks are enumerated.

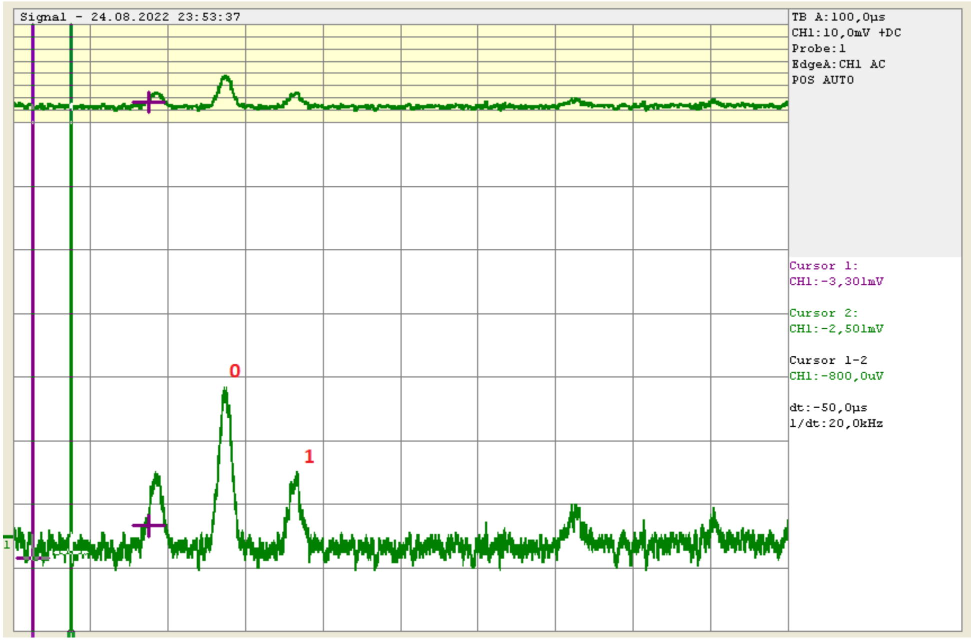


Fig. 24: Screenshot of the oscilloscope for counting the orders of the maxima for the fourth grating. The x-axis is the time-axis and the y-axis the intensity of the interference pattern. In red the one visible peak is enumerated.

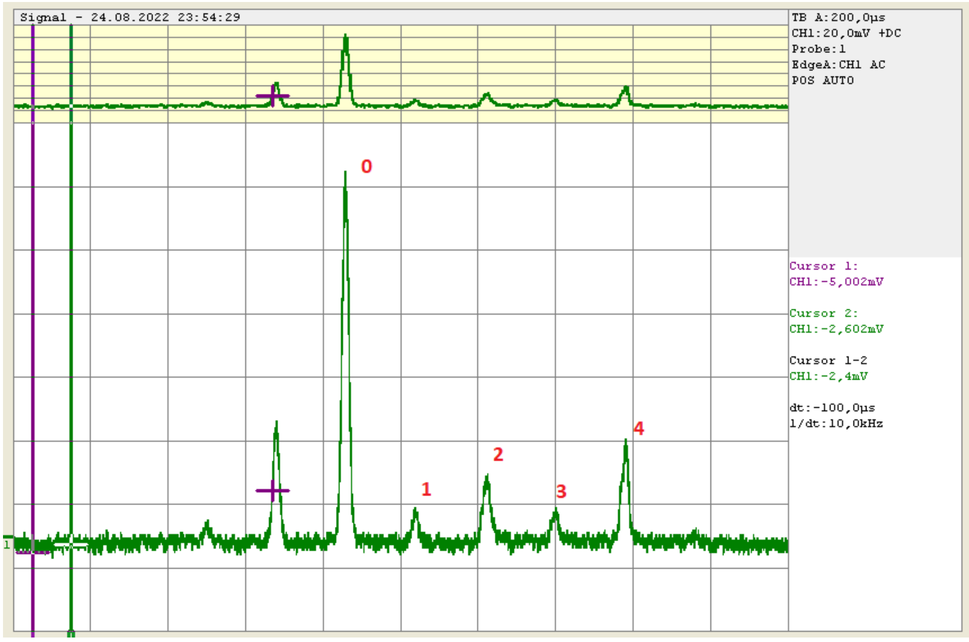


Fig. 25: Screenshot of the oscilloscope for counting the orders of the maxima for the fifth grating. The x-axis is the time-axis and the y-axis the intensity of the interference pattern. In red the four visible peaks are enumerated.

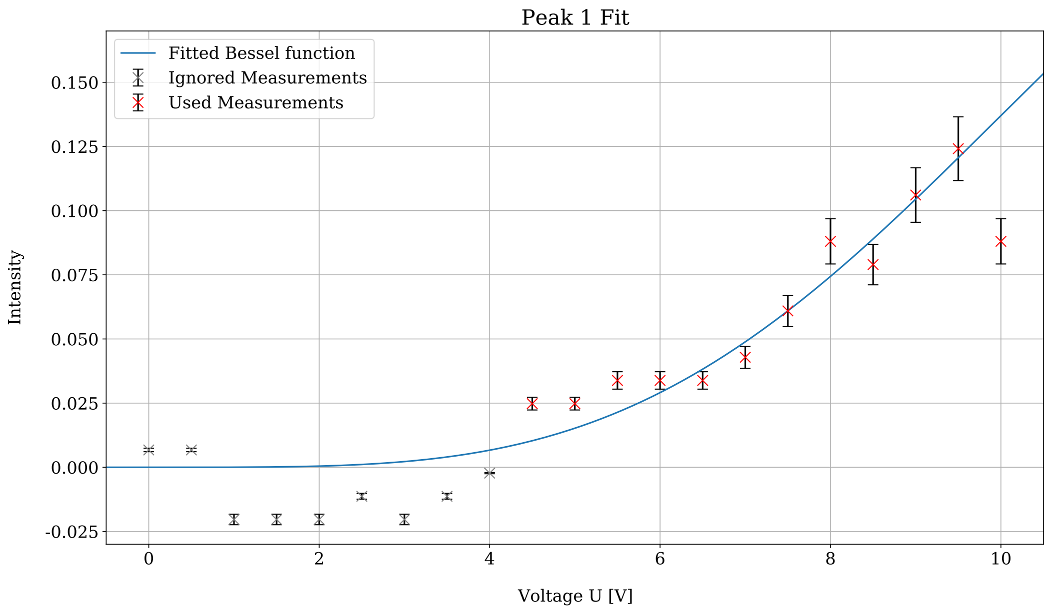


Fig. 26: In this graphic the intensities I of the secondary maximum 2 on the left are plotted against the voltage U in V. One can also see the fitted Bessel function for which only the red data points are used.

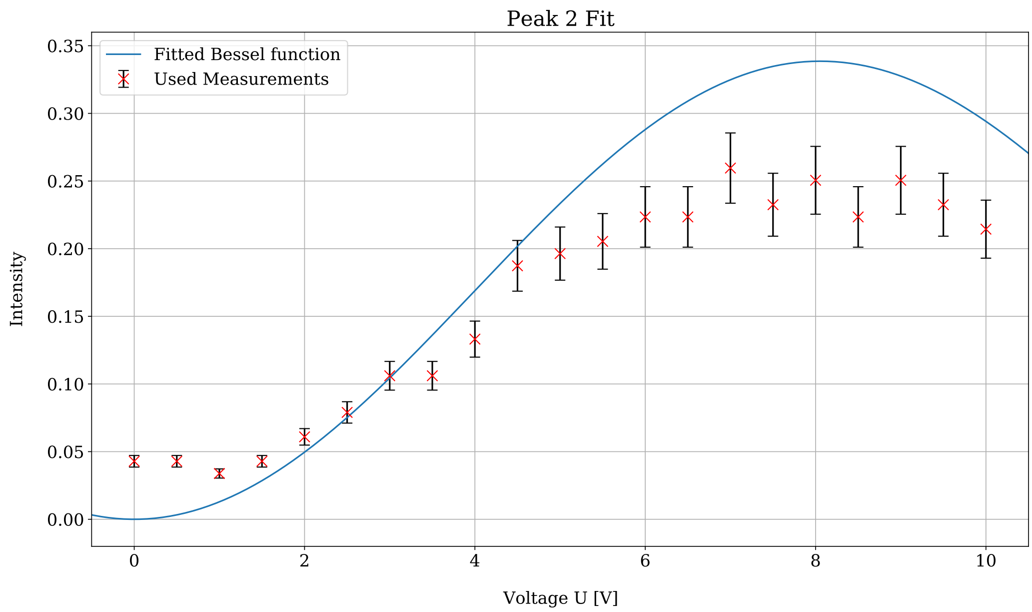


Fig. 27: In this graphic the intensities I of the secondary maximum 1 on the left are plotted against the voltage U in V. One can also see the fitted Bessel.

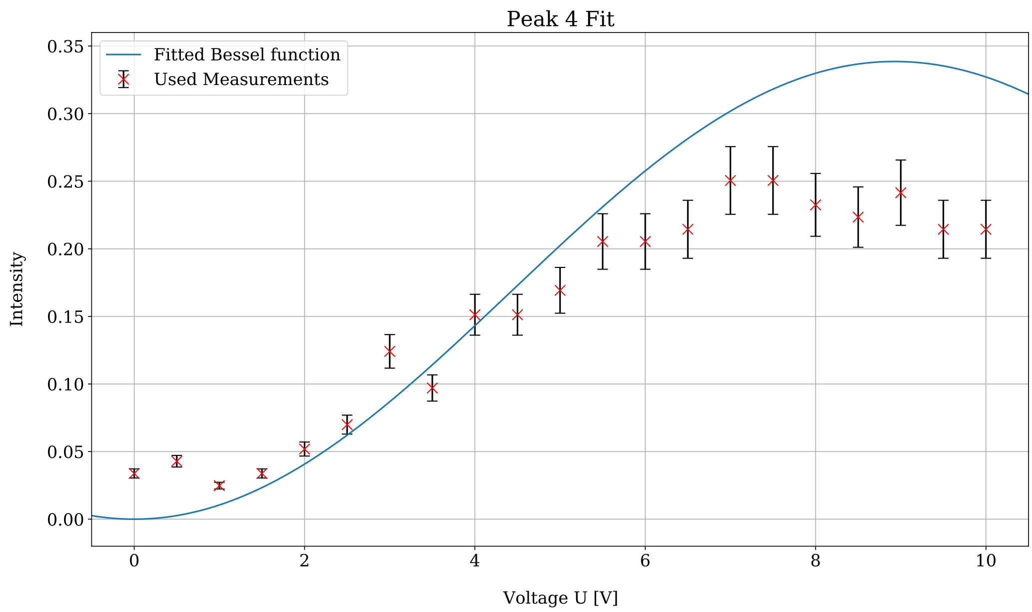


Fig. 28: In this graphic the intensities I of the secondary maximum 1 on the right are plotted against the voltage U in V. One can also see the fitted Bessel function

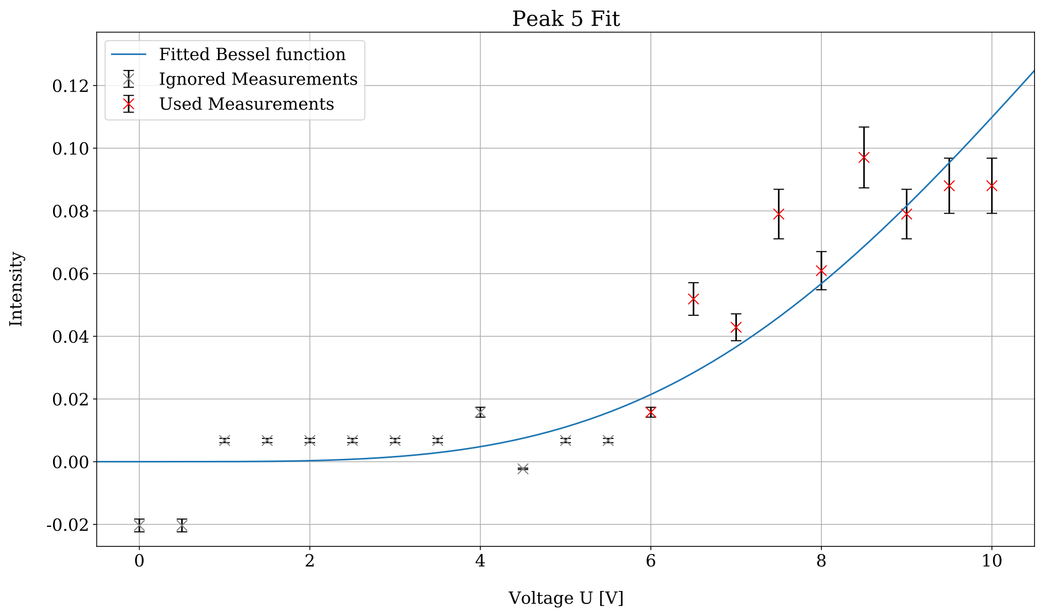


Fig. 29: In this graphic the intensities I of the secondary 2 maximum on the right are plotted against the voltage U in V. One can also see the fitted Bessel function for which only the red data points are used.

7.2 Lab notes

Experiment: Ultrasound – Phase grating

(1) Measuring the sine-grating

Setup:

$\lambda_{Laser} = 632{,}8\,nm$

$\rightarrow K_S = 1016\,\frac{L}{cm}$

$a = (6{,}4 \pm \frac{0{,}3}{\sqrt{6}})\,cm$ (distance grating / screen)

$x = (10{,}6 \pm \frac{0{,}4}{\sqrt{6}})\,cm$ (distance first max)

(2) Measuring the amplitude gratings

Setup:

L_1 : 50 mm

L_2 : 100 mm

L_3 : 300 mm

After setup: nothing changed / touched

1. Calibration with grating 5 : $K = 80\,\frac{L}{cm}$
2. measurement of the gratings
3. measurement of resolution

} Data on USB

↳ screenshots to count m_{max}

↳ N by measuring the width of the beam on the millimeter-screen : $c = (4{,}0 \pm \frac{1{,}0}{\sqrt{6}})\,mm$

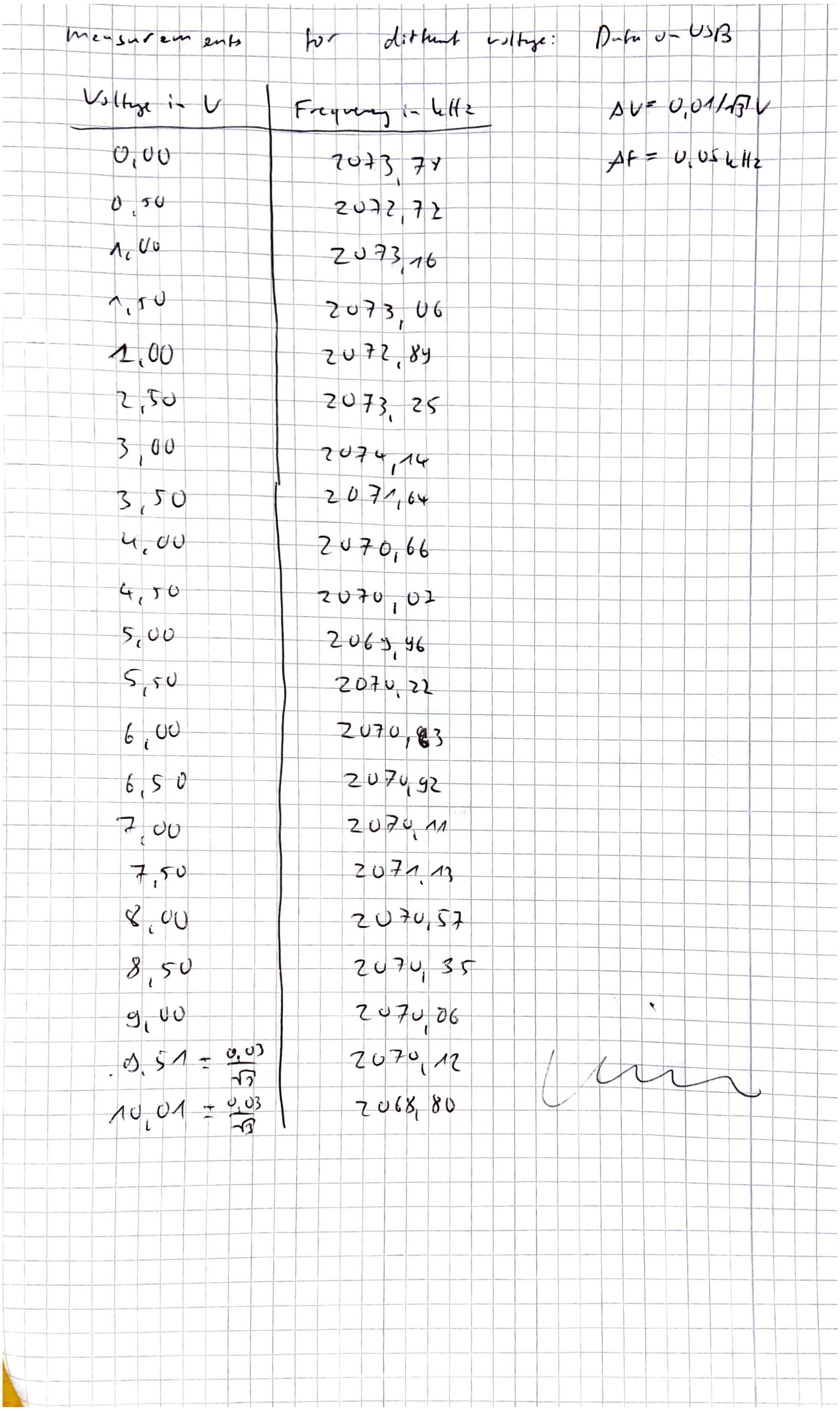
(3) + (4): only data analysis and discussion

(5) Setup

Motor frequency 12,5 Hz

measurement with grating 5 to find the wave length of the ultrasound wave ⇒ Data on USB

Fig. 30: Lab notes - page 1



Measurements for diffrent voltage: Data on USB

Voltage in V	Frequency in kHz
0,00	2073,78
0,50	2072,72
1,00	2073,16
1,50	2073,06
2,00	2072,89
2,50	2073,25
3,00	2074,14
3,50	2071,64
4,00	2070,66
4,50	2070,02
5,00	2069,96
5,50	2070,22
6,00	2070,83
6,50	2070,92
7,00	2070,11
7,50	2071,13
8,00	2070,57
8,50	2070,35
9,00	2070,06
9,51 $\pm \frac{0,03}{\sqrt{3}}$	2070,12
10,01 $\pm \frac{0,03}{\sqrt{3}}$	2068,80

$\Delta V = 0,01/\sqrt{3}\,V$

$\Delta f = 0,05\,kHz$

Fig. 31: Lab notes - page 2